Electricity and Magnetism Assignments

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Assignment 1

Readings - Lectures #1 and #2, Purcell - Chapter 1

Purcell Problems:

1.05 Field due to a semicircle

1.09 Potential energy of a sphere of charge

1.11 2 charges on the x-axis

1.16 Sphere of charge with a spherical hollow

1.17 Flux through a cube

1.18 2 infinite sheets of charge

1.19 Infinite plane + infinite layer

1.24 Electric field due to a rod

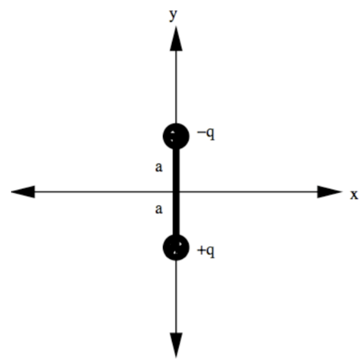
1.29 Hole in a spherical shell

1.30 Energy stored in two concentric shells

Extra Problems

1. Electric field from an electric dipole

A pair of charges lie in the $x - y$ plane. The $+q$ charge is a $(0, -a)$ and the $-q$ charge is at $(0, a)$ as in figure below:



(a) Evaluate the electric field (magnitude and direction) at a point $(d, 0)$. Show that for $d \gg a$, $|\vec{E}| \propto d^{-3}$.What is the directions in this limit? Assume $d > 0$.

(b) Evaluate the electric field (magnitude and direction) at a point $(0, d)$. Find the magnitude and direction when $d \gg a$.

(c) How much work is required to move a particle with charge q' from $(d, 0)$ to $(0, d)$? Do not assume that $d \gg a$.

2. Partial Derivatives - For all parts of this problem use the function:

$$f(x,y,z) = \tanh\left(x^2 + y^2 + z^2\right) \text{ with } \tanh(u) = \frac{\sinh(u)}{\cosh(u)} = \frac{e^u - e^{-u}}{e^u + e^{-u}}$$

(a) Compute the partial derivatives

$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial x}$$

(b) Define the radial displacement vector as $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$. Calculate the following quantity for f given above and then express in terms of $\vec{r}$.

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial x}\hat{z}$$

Assignment 2

Readings - Lecture #2, Purcell - Chapter 2

Purcell Problems:

2.01 Line integrals and gradients

2.04 $\vec{E}$and ρ from ϕ

2.08 Cylindrical charge distribution

2.12 Potential from a triangle

2.17 Prove a vector identity

2.19 How does the charge distribute?

2.20 Potential of a sphere

2.27 Energy stored and work done

2.29 Two charged nonconducting spherical shells

2.30 Potential of a cube

Extra Problems

1. Energy of a radial charge distribution - A spherically symmetric charge distribution has charge density

$$\rho = \begin{cases} \rho_0 \frac{r}{a} & r < a \\ 0 & r \geq a \end{cases}$$

(a) Find the electric field $\vec{E}$ everywhere

(b) Find the electrostatic potential ϕ everywhere

(c) Determine the energy needed to assemble the charge distribution using 2 different approaches.

2. Electrostatic potentials

(a) Find the electric field $\vec{E}$ from the electrostatic potential

$$\phi = \frac{\alpha z}{r}$$

where α is a constant and r is the distance from the origin.

(b) An electrostatic potential has the form:

$$\phi = \begin{cases} -2\pi a l(x + l/4) & x < -l/2 \\ 2\pi a x^2 & /2 < x < l/2 \\ 2\pi a l(x - l/4) & l/2 < x \end{cases}$$

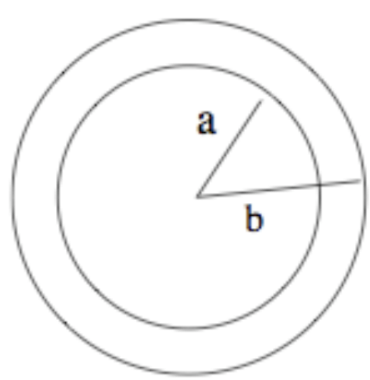
where a and l are constants. Find the charge distribution which gives this potential.

(c) Give the electric field of the charge distribution you found in part(b)

3. Electric field, potential and flux - A hollow spherical shell carries charge density

$$\rho = \frac{k}{r^2}$$

in the region $a \le r \le b$ (see figure).



(a) Find the electric field $\vec{E}$ everywhere in space.

(b) Find the potential ϕ everywhere in space.

(c) Calculate the flux:

(i) through the concentric sphere with radius $r_1 > a$

(ii) through the concentric sphere with radius $a < r_2 < b$

(iii) through the concentric sphere with radius $r_3 < a$

(iv) through the nonconcentric sphere with radius $r_4 = 2b$ centered at any arbitrary point on the outer surface of the shell.

Assignment 3

Readings - Lecture #2 and #3, Purcell - Chapter 3

Purcell Problems:

3.03 Where does the field line go?

3.04 Images

3.05 More images

3.06 Solved all problems?

3.08 Three conducting plates

3.09 Charge in a corner

3.11 Two capacitors

3.16 Parallel-plate capacitor

3.18 What is the potential?

3.23 Coaxial cylinders

3.24 Two parallel plates

3.26 Capacitance coefficients

Assignment 4

Readings - Lecture #3 and #4, Purcell - Chapter 4

Purcell Problems:

4.16 Input resistance

4.17 Internal resistance

4.18 Maximum power

4.19 Ohmmeter

4.20 Black box

4.21 Thevenin equivalence

4.22 Greatest power

4.25 Relaxation time

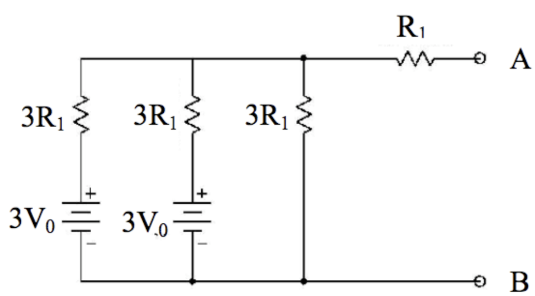
4.31 Resistor cube

4.32 infinite network

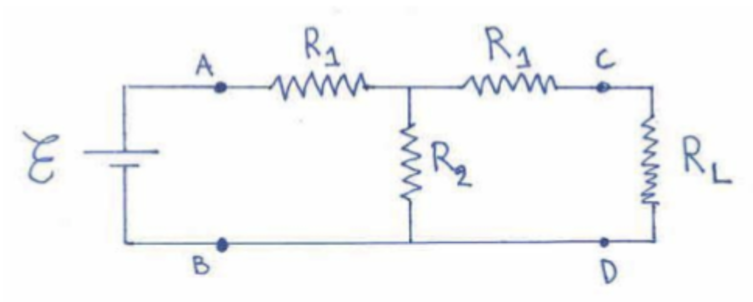
4.33 Minimum power

Extra Problems

1. Consider the circuit shown below:



Calculate the Thevenin equivalent circuit.



2. Consider the network shown above

(a) Introduce on the figure arrows indicating the emf and the currents flowing in the branches.

(b) Write down a system of equations using Kirchoff's laws that enables you to solve for all unknown currents.

(c) Solve for the unknown currents.

(d) What if the effective resistance that the emf "sees", i.e., what is the total resistance to the "right" of points A and B

(e) In what follows, assume that R_1 and R_2 are magically adjusted so that the effective resistance just calculated is equal to R_L.

(1) Find R_2 in terms of R_1 and R_L.

(2) Find the voltage drop on R_L(i.e., $V_C - V_D$ as identified on the figure) as a function of $\mathcal{E}$,R_1 and R_2.

Assignment 5

Readings - Lecture #4a, #4b and #4c, Purcell - Chapter 10.1-10.4

Purcell Problems:

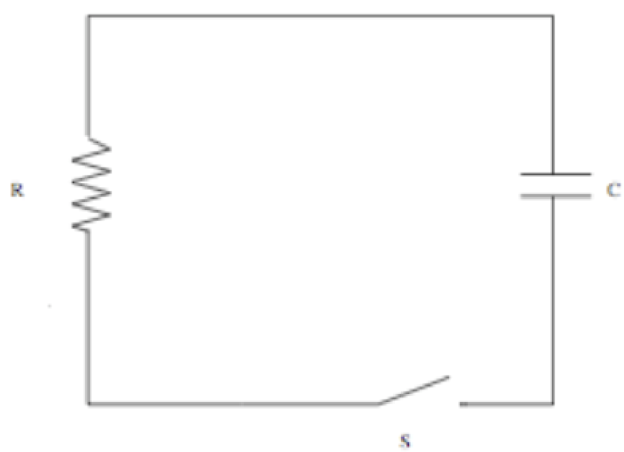
10.3 Dipole moments

10.6 Parallel plate capacitor

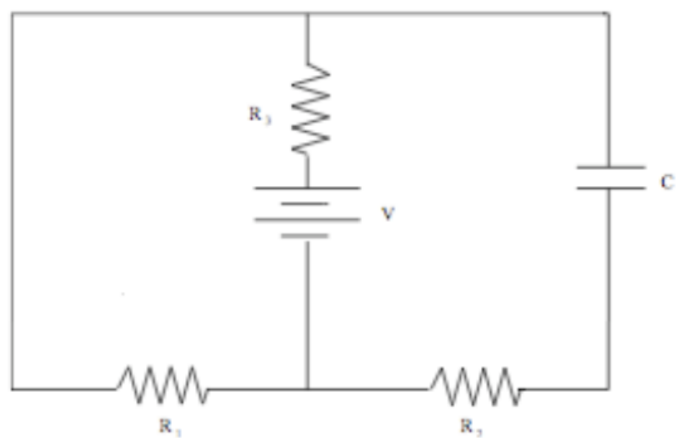
10.14 Three capacitors

Extra Problems

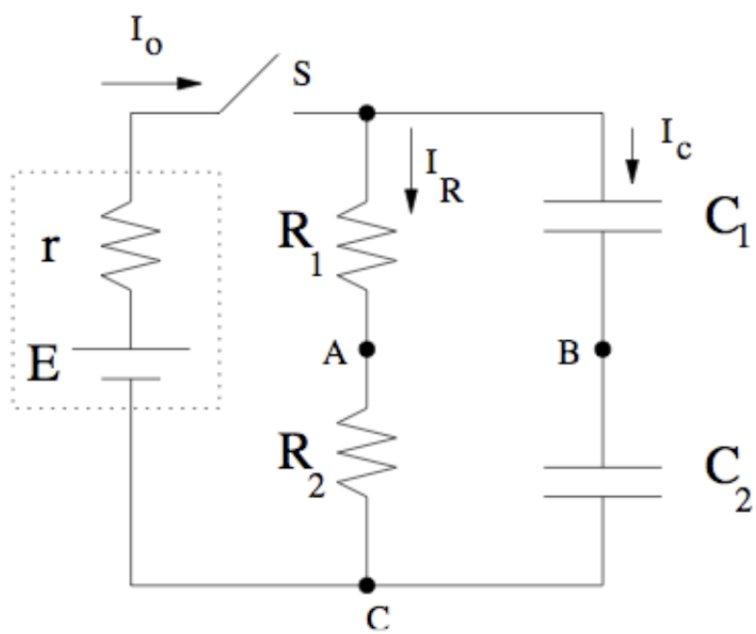
1. Consider the simplest RC circuit as shown below. Suppose that a charge Q_0 is stored on the capacitor initially. Show that the total energy dissipated in the resistor after the switch is closed equals the energy that was stored in the capacitor before the switch was closed.



2. Consider the circuit shown below. Suppose that the capacitor is initially discharged. The switch is closed at $t = 0$ (circuit as shown). Find the charge $I_s(t)$ through the battery, i.e., the middle branch, as a function of time, and the charge on the capacitor $Q(t)$.



3. A battery E with internal resistance r, two resistors $R_1 = 10r$ and $R_2 = 5r$ and two capacitors C_1 and C_2 with $C_1 = 2C_2$ are arranged as shown below. The capacitors are initially uncharged. Express all your answers in terms of E, r and C_2..



(a) At $t = 0$, the switch S is closed. What is the potential at A with respect to C, i.e., $V_{AC} = V_A - V_C$, and what is the potential at B with respect to C, i.e., $V_{BC} = V_B - V_C$?

(b) After infinite time has elapsed(and with switch S remaining closed) what is $V_{AC} = V_A - V_C$ and $V_{BC} = V_B - V_C$?

(c) Write down a set of independent equations that will yield the solutions for the currents flowing in the three branches of the circuit, i.e., $I_0(t)$, $I_R(t)$, $I_C(T)$. Do NOT solve them.

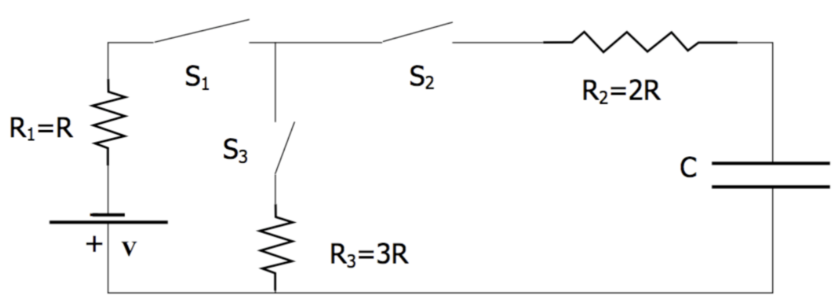
We now short-circuit points A and B by connecting them with a resistanceless conducting wire.

(d) Will there be any current flowing through it(yes/no) and in what direction?

(e) What will be the final $V_{AC} = V_A - V_C$ and $V_{BC} = V_B - V_C$?

(f) What is the total charge the passed through the short-circuiting wire? Is this consistent with the answer to (d)?

4. Consider the circuit below.



Initially all switches are open and the capacitor C is discharged.

(a) At time $t = t_0$, we close S_1 and S_2 simultaneously.

(b) At time $t = t_1 \gg t_0$, we close S_3(with S_1 and S_2 still closed)

(c) At time $t = t_2 \gg t_1$, we open S_1(S_2 and S_3 still closed)

Sketch how the following quantities vary with time:

(a) V_C (potential across the capacitor)

(b) I_{R_2} (current through resistor R_2)

(c) V_{R_3} (potential across resistor R_3)

Assignment 6

Readings - Lecture #4a, #4b and #4c, Purcell - Chapter 7.9,8.1-8.5

Purcell Problems:

7.13 RL circuit

7.17 RL circuit

8.2 RC circuit

8.3 RLC circuit

8.4 RLC circuit

8.10 Real impedance

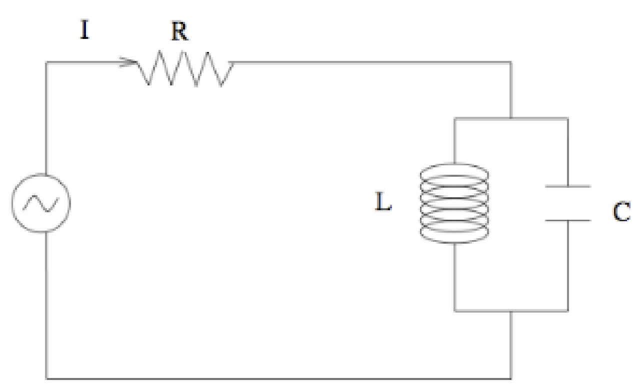
8.12 Out of phase

8.13 Voltage difference is zero

8.14 Equivalent circuits

Extra Problems

1. Consider the circuit shown.

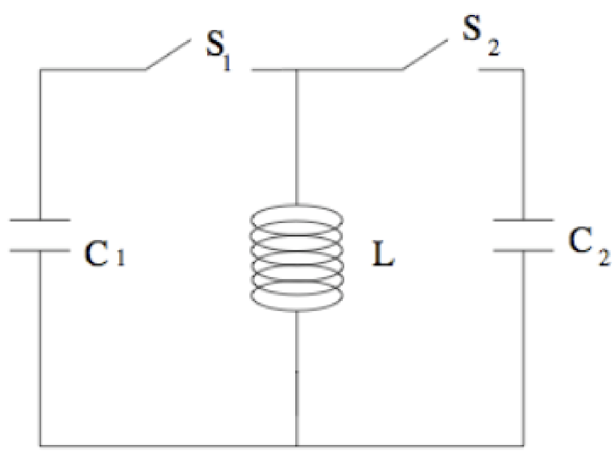


(a) What is the complex impedance of the circuit elements?

(b) The AC voltage is given as $V_0 \cos(\omega t)$. What is the current I (the actual current, not the complex current) flowing through the circuit? Find the phase angle.

(c) Explain the high and low frequency behavior of the phase shift of the current in terms of the currents through each of the circuit elements.

2. Consider the circuit shown below, where C_1 is initially charged to 75 volts. Suppose that C_1 is 10000 μF, C_2 is 3000 μF and L is 15 H. Explain how to open and close the switches so as to discharge C_1 and charge C_2. Starting at $t = 0$, you should give explicitly times for opening and closing each switch. What is the final voltage across C_2?



Assignment 7

Readings - Lecture #5b and #5c, Review Relativity from Mechanics class.

Extra Problems

1. What is that spaceship doing? - Two radar pulses sent out from the earth at 6:00 AM and 8:00 AM one day bounce off an alien spaceship and are detected on earth at 3:00 PM and 4:00 PM. You are not sure, however, which reflected pulse corresponds to which emitted pulse, Is the spaceship moving toward earth or away? If its speed is constant (but less than c), when will it (or did it) pass by the earth? Drawing a spacetime diagram will make this problem easy.

2. More events in different frames - In the solar system frame, two events are measured to occur 3.0 hr apart in time and 1.5 hr apart in space. Observers in an alien spaceship measure the two events to be separated by only 0.5 hr in space. What is the time separation between the events in the alien's frame?

3. They are simultaneous somewhere - The space and time coordinates of two events as measured in a frame S are as follows:

$$\text{EVENT 1: } x_1 = x_0\ ,\ t_1 = x_0/c \quad , \quad \text{EVENT 2: } x_2 = 2x_0\ ,\ t_2 = x_0/2c$$

(a) There exists a frame in which these two events are simultaneous. Find the velocity of that frame(S') with respect to S.

(b) What is the value of at which both events occur in the new frame S' ?

4. Can you save them? - In 2095 a message arrives at earth from the growing colony at Tau Ceti (11.3 ly from earth). The message asks for help in combating a virus that is making people seriously ill (the message includes a complete description of the virus genome). Using advanced technology available on earth, scientists are quickly able to construct a drug that prevents the virus from reproducing. You have to decide how much of the drug can be sent to Tau Ceti.

The space probes available on short notice could either boost 200 g of drug (in a standard enclosure) to a speed of 0.95c, 1 kg to a speed of 0.90c, 5 kg to a speed of 0.80, or 20 kg to a speed of 0.60c relative to the earth.

The only problem is that a sample of the drug in a standard enclosure at rest in the lab is observed to degrade due to internal chemical processes at a rate that will make it useless after 5.0 years. Is it possible to send the drug to Tau Ceti? If so, how much can you send?

5. The Strange World of Relativity - Solve this problem with the Lorentz transformation equations and with a spacetime diagram. At noon a rocketship passes the earth with a velocity of 0.8c. Observers on the ship and on earth agree that it is noon.

(a) At 12:30 PM as read by a rocketship clock, the ship passes an interplanetary navigational station that is fixed relative to the earth and whose clocks read earth time. What time is it at the station?

(b) How far from earth (in earth coordinates) is the station?

(c) At 12:30 PM rocketship time the ship reports by radio back to earth. When (earth time) does the earth receive the signal?

(d) The station on earth replies immediately. When (by rocket time) is the reply received?

6. Red or Green? - There is a spaceship shuttle service from the earth to Mars. Each spaceship is equipped with two identical lights, one at the front and one at the rear. The spaceships normally travel at a speed v_0, relative to the earth, such that the headlight of a spaceship approaching the earth appears to be green ($\lambda = 5\times10^{-7}$ m) and the taillight of a departing spaceship appears to be red ($\lambda = 6\times10^{-7}$ m).

(a) What is the value of v_0?

(b) One spaceship accelerates to overtake the spaceship ahead of it. At what speed must the overtaking spaceship travel (relative to the earth) so that the taillight($\lambda = 6\times10^{-7}$ m) of the Mars-bound spaceship ahead of it looks like a headlight ($\lambda = 5\times10^{-7}$ m)?

7. Center-of-Mass Frame - In the laboratory frame a particle of rest mass m_0 and a speed v is moving towards a particle of rest mass m_0 that is at rest. What is the speed of the inertial frame in which the total momentum of the system is zero? This frame is called the **center of mass** or **zero momentum** frame.

8. Proton Dynamics - A proton with a kinetic energy of 10^{10} eV collides with a proton at rest. Find

(a) the velocity of the center of mass

(b) the total momentum and total energy in the laboratory frame

(c) the kinetic energy of the particles in center of mass frame

9. Total energy - Two particles of rest mass m_0 approach each other with equal and opposite velocity v in the laboratory frame. What is **the total energy** of one particle as measured in the rest frame of the other?

10. Relativistic collision - A particle of rest mass m_1 and velocity v_1 collides with a stationary particle of mass m_2 and is absorbed by it. What is the velocity and rest mass of the final compound system?

11. Relativistic decays - A strange, neutral particle called a kaon (K_S^0) can decay into two charged pions($\pi^{\pm}$). The kaon has a mass of 498 MeV/c^2, and the pions each have a mass of 140 MeV/c^2.

(a) If the K_S^0 is at rest in the lab frame when it decays, what is the speed of the resulting pions?

(b) Now assume that the kaon is travelling at 0.99c with respect to the lab frame. What is the greatest speed that one of the pions can have in the lab frame? The least speed?

12. Threshold energy - A reaction that can produce antiprotons in an accelerator is

$$p + p \rightarrow p + p + p + \bar{p}$$

where the first proton is a part of a beam, the second is at rest in the target, and $\bar{p}$ is an antiproton. Both the proton and the antiproton have the same rest mass ($= 0.94 \times 10^9$ eV). At threshold, all four final particles have essentially zero velocity with respect to each other. What is the beam energy in that case?

Assignment 8

Readings - Lecture #4, #5c and #6, Purcell - Chapter 5

Purcell Problems:

5.02 Electrons on a filament

5.03 Properties of an electron beam

5.05 Sheet of charge

5.08 Charged particle interaction

5.09 Deflection plates

5.10 Two massive particle passing

5.13 What is the electron doing?

5.14 What does the field look like?

5.17 Must have a magnetic field also

5.18 Composite line of charge

5.19 Field of rebounding proton

Assignment 9

Readings - Lecture #6, Purcell - Chapter 6

Purcell Problems:

6.04 Bent wire

6.05 Three wires

6.08 Wire bent in right angle

6.12 Strange bent wire

6.16 Power of superposition

6.17 Solenoids

6.22 Magnetic torque

6.25 Vector potential given a field

6.26 Vector potential for a wire

6.28 Proton path of motion

6.32 Cathode ray tube

Extra Problems

1. Transformation of fields - A very large sheet of charge lies in the $x-y$ plane of frame F. The charge per unit length is σ. In frame F', this sheet moves to the right with speed v.

(a) What is the electric field in the rest frame F(above and below the sheet)?

(b) What is the electric field in the frame F'(above and below the sheet)?

(c) What is the magnetic field in the frame F'(above and below the sheet)?

(d) Show that the results of (b) and (c) are consistent with the Lorentz transformations for electric and magnetic fields (Eq 60 of Purcell Chapter 6).

2. Electric and magnetic forces - Two infinite lines of charges with charge per unit length λ_0 in their rest frame are separated by a distance d. These charges are moving in a direction parallel to their length with speed v.

(a) In the rest frame, what is the electric force per unit length that the top line feels due to the bottom line? Give both direction and magnitude.

(b) Repeat (a) in the lab frame.

(c) In the lab frame, what is the magnetic force per unit length that the top line feels due to the bottom line? Give both direction and magnitude.

(d) What is the total force per unit length in the lab frame?

Assignment 10

Readings - Lecture #7, Purcell - Chapter 7

Purcell Problems:

7.02 emf in moving loop 1

7.04 emf in moving loop 2

7.09 Mutual inductance of two rings

7.11 Coils and inductance

7.13 RL circuit

7.14 Crossbar sliding on rails

7.15 Induced alternating voltage

7.18 Get the total charge

7.21 Mutual inductance of coil within coil 7.22 How much angular momentum?

7.23 Energy density

7.29 Measuring speed of light

Assignment 11

Readings - Lecture #8 and #9, Purcell - Chapter 9

Purcell Problems:

9.01 What is magnetic field?

9.02 rms magnetic field strength

9.03 Where is the proton - electric field only?

9.04 Where is the proton - magnetic field also?

9.05 Satisfy Maxwell's equations 1

9.06 Satisfy Maxwell's equations 2

9.07 Plane EM wave

Extra Problems

1. A spherical wave - Let the electric field be given by

$$\vec{E} = A\frac{\sin\theta}{r}\left[\cos(kr - \omega t) - \frac{1}{kr}\sin(kr - \omega t)\right]\hat{\phi}$$

where $\omega/k = c$.

(a) Show that $\vec{E}$ obeys all four of Maxwell's equations in vacuum, and find the associated magnetic field.

(b) Calculate the Poynting vector. Average $\vec{S}$ over a full cycle to get the intensity vector $\vec{I}$.

(c) Integrate $\vec{I} \cdot d\vec{a}$ over a spherical surface to determine the total power radiated.

2. Discovery of magnetic charge - This problem will explore some of the similarities between electric and magnetic fields. Let us say that magnetic charge has been discovered, called magnetic monopoles. We will work in a system where the units of magnetic charge density μ are chosen so that $\nabla \cdot \vec{B} = 4\pi\mu$.

(a) When the monopole is in motion, there is a magnetic current density $\vec{L} = \mu\vec{v}$ analogous to the electric current density. Write down the continuity equation for magnetic charge in differential form.

(b) Write down the new Maxwell's equations in differential form, including the effects of these monopoles.

2. Pair of electric and magnetic fields - A pair of electric and magnetic fields are given by:

$$\vec{E} = E_0 \cos(\alpha y - \gamma z + \delta t)\hat{x}$$

and

$$\vec{B} = B_0 \cos(\alpha y - \gamma z + \delta t)(\hat{y} + \hat{z})$$

By substituting into Maxwell's equations in vacuum, derive the conditions that the constants α , γ, δ, E_0, and B_0 must obey to satisfy them. Is this a legitimate electromagnetic wave? Why?

Assignment 12

Readings - Lecture #8 and #9, Purcell - Chapter 9

Purcell Problems:

9.08 An EM wave in a metal box

9.09 Energy density

9.10 Field inside capacitor

9.11 Beam energy down from orbit

9.12 Voltage standing wave ratio

9.13 Invariants

Extra Problems

1. The Poynting vector in a capacitor - A current $I = dQ/dt$ delivers charge to a parallel-plate capacitor. This capacitor has circular plates of radius a, and the plates are separated by a distance $s \ll a$ (so you can ignore fringing).

(a) Find the Poynting vector due to the electric field and the magnetic field between the capacitor plates. Give both the magnitude and the direction.

(b) Calculate the total power, $P = \int \vec{S} \cdot d\vec{a}$, flowing into the capacitor. Given the Poynting vector found in (a), what is the correct surface to use for the integral?

(c) Integrate the power over time. Assuming that the capacitor has charge 0 at $t = 0$ and has some charge level Q at a later time t, show that the total energy that flows into the capacitor is given by $U = Q^2/2C$.

2. The Poynting vector and a coaxial cable - A coaxial cable transmits DC power from a battery to a load. The cable consists of two concentric, long, hollow cylinders of zero resistance. The inner cylinder has radius a, the outer has radius b, and the length of both is l. The battery provides an emf ε between the two conductors at one end of the cable and the load is a resistance R connected between the two conductors at the other

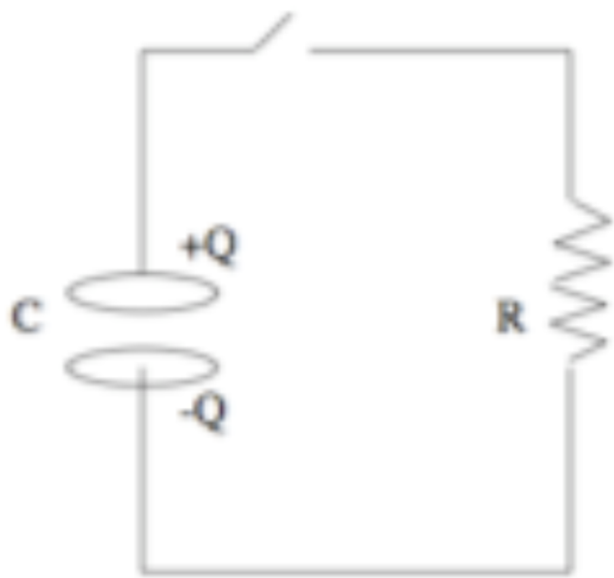
(a) How much power is dissapted in the resistor?

(b) What are $\vec{E}$ and $\vec{B}$ in the cable?

(c) What is the Poynting vector $\vec{S}$ in the cable?

(d) Show that $P = \int \vec{S} \cdot d\vec{a} = P$ (from part(a)).

3. Displacement current - Consider the circuit below:



A circular parallel-plate capacitor with radius b and spacing $s \ll b$ (so you can ignore fringing) is charged to a voltage V_0. At time $t = 0$, the switch is closed and the capacitor begins to discharge through the resistor R.

(a) Give an expression for the charge $Q(t)$ as a function of time of the positively charged plate (the upper one in the figure) of the capacitor.

(b) Find an electric field $\vec{E}(t)$ between the two capacitor plates.

(c) Find the displacement current density $\vec{J}(t)$ between the two capacitor plates.

(d) Find the magnetic field $\vec{B}(t)$ between the two plates.

4. Polarized light - For this problem, an electric field for a wave will be given. Find the associated magnetic field, the direction of propagation and the direction of polarization for the light wave.

(a) $\vec{E} = E_0\hat{y}\cos(kx - \omega t) + E_0\hat{z}\cos(kx - \omega t)$

(b) $\vec{E} = E_0\hat{x}\cos(ky - \omega t) + E_0\hat{z}\sin(ky - \omega t)$

(c) $\vec{E} = E_{0_1}\hat{x}\cos(kz - \omega t) + E_{0_2}\hat{y}\cos(kz - \omega t + \pi/6)$ where $E_{0_1} = \frac{3+\sqrt{13}}{2}E_{0_2}$