Notes on Weinberg
The First Three Minutes

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Chapter 1

Introduction:The Giant and the Cow

There exist endless numbers of ancient myths and stories about the origin of the universe. Most present the reader with new problems to answer equal in number to the answers they purport to give. The complications that are raised can be summarized in the words - complicated initial conditions or why or how did the conditions necessary for the story to work occur in nature - do we always need a god to intervene to make things work, and so on.

Science and scientists have worried about the question from the beginnings of science. No progress could be made in any scientific explanations because the experimental data was non-existent and there were no theoretical foundations that could be applied. This all changed in the latter half of the 20^{th} century. A theory of the early universe now exists - the so-called big-bang theory (modified by inflation) along with specific recipes for the contents of the universe. This is the theory we will be discussing.

We start with a summary of the history of the early universe as it is presently understood by modern theory. We will explain all the details during our discussions.

It all started with an *explosion*. This is not any ordinary explosion as might occur today, which would have a point of origin(center) and would spread out from that point. The *explosion* we are proposing occurred simultaneously everywhere, filling all space(infinite in extent or cyclic) from the beginning, with every particle of matter rushing apart from every other particle.

At about 1/100 of a second, the earliest time that could be talked about in 1977(we will push that number closer to zero later with modern developments), the temperature of the universe was about 10^{11} degrees Centigrade. This is much hotter than the center of even the hottest star - no ordinary components of matter as we know them - molecules, atoms, nuclei - could hold together at this temperature. At this time, the matter that was rushing apart consisted of electrons(mass and negative charge), positrons(anti-electrons)(mass and positive charge), neutrinos(3 kinds and almost massless; no charge) and photons(massless; no charge). The number and average energy of the photons in the early universe was about the same as for electrons, positrons and neutrinos. These particle - electrons, positrons, neutrinos and photons - were continually being created out of pure energy and then after short lives being annihilated again. The numbers that existed were not set initially but fixed by a balancer between processes of creation and annihilation. A calculation then shows that the density of this cosmic soup at a temperature of 10^{11} degrees Centigrade was about 4×10^9 times the density of water!. There also existed a small contamination of heavier particles (protons and neutrons) - about one proton and one neutron for every 10^9 photons or electrons or positrons or neutrinos. As we will see later this ratio is measurable in the so-called cosmic radiation background we will discuss later.

As the *explosion* continued, the temperature dropped reaching 3×10^{10} degrees Centigrade after about 1/10 of a second; 10^{10} degrees Centigrade after about one second; and 3×10^9 degrees Centigrade after about 14 seconds. At this point the temperature was low enough that electrons and positrons began to annihilate faster than they could be recreated from photons and neutrinos. The extra energy released in the annihilation processes temporarily slowed the cooling process but the temperature continued to drop reaching 10^9 degrees Centigrade after about 3 minutes. At this point the temperature was low enough for protons and neutrons to begin to form complex nuclei such as deuterium and then to form the most stable of light nuclei, helium.

At the end of three minutes the contents of the universe were mostly in the form of photons, neutrinos and antineutrinos. There was a small amount of nuclear material - 73% hydrogen(protons) and 27% helium(alpha particles) along with an equally small number of electrons left over from the electron-positron annihilation processes. The matter continued to rush apart becoming steadily cooler and less dense, Much later, after a few hundred thousand years, it became cool enough for electrons to join with nuclei to form atoms of hydrogen and helium. The resulting gas would begin to form clumps under the influence of gravitation(where did that come from?) which would ultimately condense to form the galaxies and stars of the present universe. All the ingredients for these processes was created in the first three minutes!

There is much vagueness in this theory about the details of the beginning (less than 1/100 of a second). Also some initial conditions seem to need precise values without any explanation of how and why. We will see have modern developments straighten out these difficulties later. The reason the big-bang theory is preferred over all others is because of experimental evidence supporting it - in the end that is the most important thing! We embark on a process where we will fill in the details - experimental evidence and theoretical considerations. After understanding the period beyond 1/100 of a second will will endeavor to update Weinberg's work to include *inflation theory* so that we can talk about the period before 1/100 of a second. In the end experimental observations will have be our main guide for how to proceed - that is the way of theoretical physics. Finally, we will look at the future of the universe - what will happen!

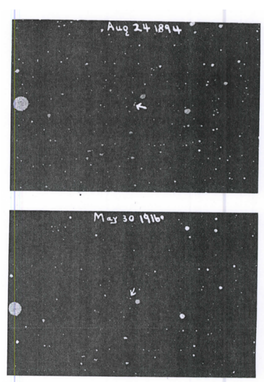
Chapter 2

The Expansion of the Universe

Look at the night sky. On the short time scale of our lives it looks like a unchanging universe! Other than local movements within our local solar system, the stars seem motionless. The stars, however, are moving, with speeds as high as a few hundred kilometers per second. Thus, in a year a fast star might travel on the order of $10^{10}\,km$. The distance to the nearest star is $4.2\,ly$ or

$$4.2\,ly \frac{c \cdot 1\ year}{ly} \frac{365 \times 24 \times 3600\,seconds}{1\,year} = 1.19 \times 10^{17}\,km$$

So this so-called *proper motion* of a star is a negligible fraction of the distance between stars(0.00001%). That is why it looks like things are not changing! A relatively fast star is Barnard's star ($56 \times 10^{12}\,km$ away moving at $89\,km/sec$ or $2.8 \times 10^{9}\,km$ per year so that its apparent position shifts in one year by an angle of 0.0029 degrees as shown below.



Proper motion of Barnard's Star: The position of Barnard's star (indicated by white arrow) is shown in two photographs taken 22 years apart. The change in the position of Barnard's star relative to the brighter background stars is readily apparent. In these 22 years, the direction to Barnard's star changed by 3.7 minutes of arc; thus the "proper motion" is 0.17 minutes of arc per year. (Yerkes Observatory Photograph)

Even this extreme case is just detectable! Other images of the seemingly unchanging universe are shown below.

Figure 2.1: The Milky Way in Sagittarius

Figure 2.2: The Spiral Galaxy: M104

Figure 2.3: The Andromeda Galaxy: M31

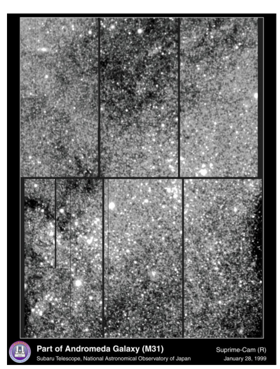


Figure 2.4: Details of the Andromeda Galaxy: M31

This appearance of an unchanging universe is an illusion! Observational evidence(as we will see) indicates that the universe is in a state of violent explosion where galaxies are rushing apart at speeds approach the speed of light. If we extrapolate their motion back in time, then in the past everything was much closer - in fact so close, they could not have had separate existence - this is the so-called *early universe* that we will be discussing to start with.

Observational evidence for the expansion of the universe comes from the fact that it is possible to determine the motion of a luminous body in a direction along a line of sight more accurately than motion at right angles to the line of sight. This ability relies on the Doppler effect that we have discussed earlier. We found these results earlier. If a source of light waves has a period of T seconds and the source is moving away from the observer at velocity V, then the period T' seen by the observer is given by

$$\frac{T'}{T} = \sqrt{\frac{c+V}{c-V}}$$

Since wavelength and period are related by $\lambda = cT$ and $\lambda' = cT'$ we then have

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{c+V}{c-V}}$$

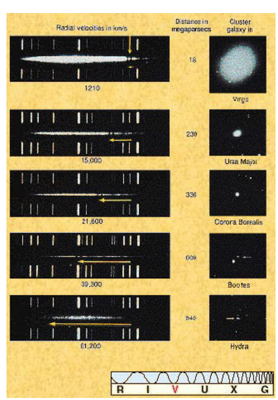
For $V \ll c$ we then have

$$\frac{\lambda'}{\lambda} \approx 1 + \frac{V}{c}$$

which corresponds to a shift of wavelengths toward the red end of the spectrum. If the source were moving toward the observer instead we have

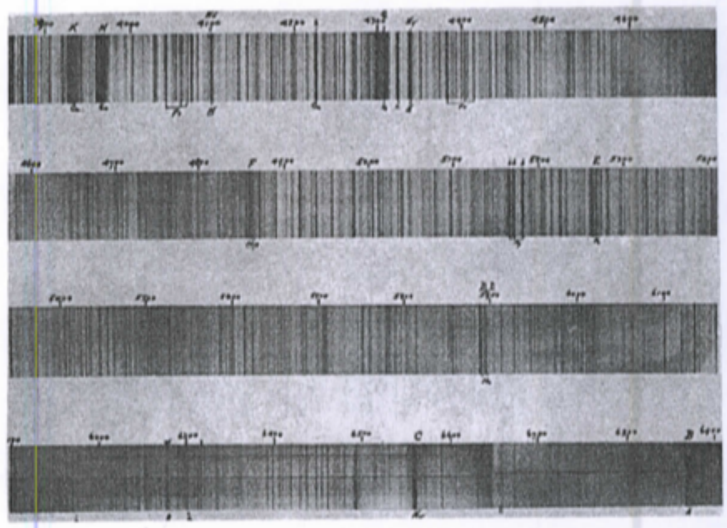
$$\frac{\lambda'}{\lambda} \approx 1 - \frac{V}{c}$$

which corresponds to a shift of wavelengths toward the blue end of the spectrum. Some examples are shown below.



If one looks at the spectrum of a star(the sun) we have as shown below

Solar Spectrum Made with the 13-foot Spectroheliograph



The Spectrum of the Sun: This photograph shows light from the sun, broken up into its various wavelengths by a 13-foot focus spectrograph. On the average, the intensity at different wavelengths is about the same as would be emitted by any totally opaque (or "black") body at a temperature of 5800° K. However, the vertical dark "Fraunhofer" lines in the spectrum indicate that light from the sun's surface is being absorbed by a relatively cool and partly transparent outer region, known as the reversing layer. Each dark line arises from the selective absorption of light at one definite wavelength; the darker the line, the more intense the absorption. Wavelengths are indicated above the spectrum in Angstrom units (10^{-8}cm). Many of these lines are identified as due to absorption of light by specific elements, such as calcium (Ca), iron (Fe), hydrogen (H), magnesium (Mg), sodium (Na). It is partly through the study of such absorption lines that we can estimate cosmic abundances of the various chemical elements. Corresponding absorption lines in the spectra of distant galaxies are observed to be shifted from their normal positions toward longer wavelengths; it is from this red shift that we infer the expansion of the universe. (Hale Observatories Photograph)

The Doppler effect is dramatically important for astrophysical observations when it is applied to the study of individual spectral lines. Frauenhofer had already discovered that when light is passed through a slit and then a glass prism, the resulting spectrum of colors is crossed by hundreds of dark line each one an image of the slit (see above). The dark lines each correspond to a definite wavelength. They are identical for many different sources of light and are produced by selective absorption of light of definite wavelengths as the light passes through the stellar atmosphere. Each line corresponds to the existence of a specific chemical element in the atmosphere. The actual shift in these dark lines due the movement of the source relative to the observer together with a known spectrum for the same source at rest allows one to determine the speed of the source.

When you look at the night sky two objects of great cosmological importance can been seen. As seen in Fig 2.1 above, we have the band of light called the Milky

Way galaxy stretching across the sky. It represents a flat disk of stars (we are in the disk). Seen from outside we have a similar galaxy in Fig 2.2 (M104) seen edge-on. Finally in Figs 2.3 and 2.4 we have our nearest neighbor galaxy - the Andromeda galaxy (M31) and a close-up showing stellar details. Such galaxies are of the order of $100,000$ light-years in diameter, of order $10,000$ light-years in thickness, have typical masses of order 10^{11} solar masses (even more when we include dark matter, if it exists). Massive telescope were eventually able to distinguish spiral and elliptical galaxies. Within these galaxies are variable stars (Cepheid variables) which have a definite relationship between their period and their absolute luminosity, which is the total radiant power emitted by an astronomical object in all directions. Absolute luminosity is observable only at the star. Away from the star, we only measure apparent luminosity, which is the radiant power received at the distant detector (reduced by distance - $1/r^2$ fall off - the so-called inverse square law. With Cepheid variables, we measure the period T, which tells us the absolute luminosity $L_0 = f(T)$. We then measure the apparent luminosity $L = L_0/d^2$,where $d =$ the distance between us and the star and thus we can determine d.

$$L = \frac{L_0}{d^2} = \frac{f(T)}{d^2} \to d = \sqrt{\frac{f(T)}{L}}$$

This type of calculation showed that most observed galaxies were outside our own galaxy (it was thought early on that they were just objects within our galaxy). Thus, it was found that an enormous number of galaxies like our own fill the universe to great distances in all directions.

At the same time the light from these galaxies was also measured to have shifted spectra indicating the galaxies were moving away or toward the earth. Some examples are shown in a figure above. At first it was thought that these might be merely relative velocities, reflecting a motion of our own solar system toward some galaxies and away from others. However, this explanation quickly became untenable as more and more of the larger spectral shifts were discovered - all toward the red end of the spectrum(moving away). It seemed that, except for a few galaxies in our local neighborhood, all other galaxies are generally rushing away from our own. This does not place our galaxy at some special place of central importance - it seems to say that the universe has undergone some sort of explosion in which every galaxy is rushing away from every other galaxy.

In 1929, Hubble discovered that the red shift of galaxies increases roughly in proportion to their distance from us. It does not matter who the *us* is - the universe looks the same from the viewpoint of any observer - all the galaxies are rushing away with speeds proportional to distance. - this is the so-called Cosmological Principle(valid in the universe on the large scale - about equal to the distance between clusters of galaxies - $10^8\ ly$). In fact, one can use the principle to show that the relative speed of any two galaxies must be proportional to the distance between just as Hubble discovered from observations.

To see this consider the figure below

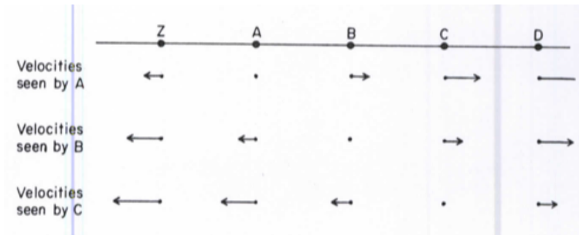


Figure 1. *Homogeneity and the Hubble Law.* A string of equally spaced galaxies Z, A, B, C, . . . are shown, with velocities as measured from A or B or C indicated by the lengths and directions of the attached arrows. The principle of homogeneity requires that the velocity of C as seen by B is equal to the velocity of B as seen by A; adding these two velocities gives the velocity of C as seen by A, indicated by an arrow twice as long. Proceeding in this way, we can fill out the whole pattern of velocities shown in the figure. As can be seen, the velocities obey the Hubble law: the velocity of any galaxy as seen by any other is proportional to the distance between them. This is the only pattern of velocities consistent with the principle of homogeneity.

Look at the three typical galaxies A, B, and C strung out along a straight line as in the figure. Suppose that the distance between A and B is the same as the distance between B and C. Whatever the speed of B as seen from A, the Cosmological Principle requires that C should have the same relative speed relative to B. C, however, is twice as far away from A as is B, is also moving twice as fast relative to A as is B and so on for any other galaxies in the chain. Alternatively we have

$$\frac{\lambda'}{\lambda} = 1 - \frac{V}{c}$$

$$z = \text{ redshift } = \frac{\lambda' - \lambda}{\lambda} = \frac{\lambda'}{\lambda} - 1 = \frac{V}{c}$$

so that $z = \frac{V}{c} \propto$ distance. Philosophers love this result - they would ask - why should any part of the universe or any direction be any different from any other? The answer, observationally, seems to be - they are not!

So there is nothing special about mankind's location in the universe. The universe is *isotropic* around us(our galaxy) and it must be isotropic around any other galaxy. Now, any point in the universe can be carried to any other point by a series of rotations around fixed centers as shown below.

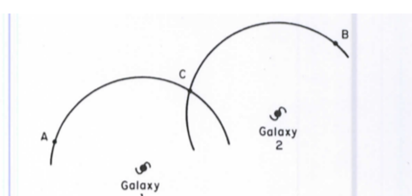
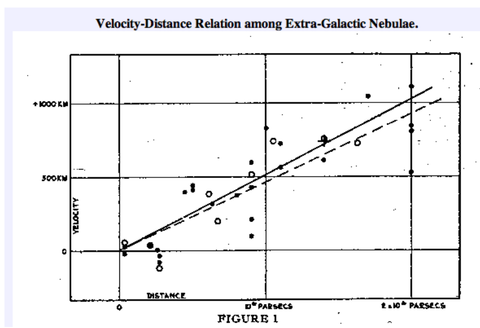


Figure 2. *Isotropy and Homogeneity.* If the universe is isotropic about both galaxy 1 and galaxy 2, then it is homogeneous. In order to show that conditions at two arbitrary points A and B are the same, draw a circle through A around galaxy 1, and another circle through B around galaxy 2. Isotropy around galaxy 1 requires that conditions are the same at A and at the point C where the circles intersect. Likewise, isotropy around galaxy 2 requires that conditions are the same at B and C. Hence they are the same at A and B.

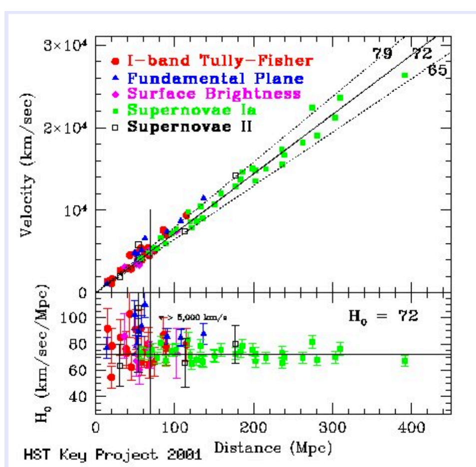
so if the universe is isotropic around every point, it is necessarily also homogeneous!

We also note that we used the Newtonian velocity addition rule in our discussion so the result is only valid for $V \ll c$. This was OK for Hubble, because none of the galaxies he observed were moving very fast. At some point in our discussions we will have to incorporate the relativistic velocity addition rules we have derived so that velocities do not exceed the speed of light, i.e., we will have use general relativity. Long before that the entire concept of distance will begin to fail and we will have deal with that also.

Hubble found distances using apparent luminosity of Cepheid variable stars and velocities using Doppler shifts and concluded (1929) that there is a *roughly linear relation between velocities and distances.* His actual data as shown below



would not, however, allow you to draw such conclusions - he knew what he wanted to be true and drew the corresponding conclusions. Modern data does not have any problem as shown below.



So Hubble drew the correct conclusions, but using a very dangerous and almost dishonest process, which in many other similar cases has led science down blind allies and disastrous paths. The proportionality constant is called the *Hubble constant H.*

What does all this say about the origin of the universe? If everything is rushing apart, then at some earlier time they must have been closer together. If the velocities had been constant throughout this time, then the time taken for any pair of galaxies to reach their present separation is just the present distance divided by their relative velocity. With velocity proportional to present separation this time would then be the same for all pairs of galaxies - they must have all been close together at some time in the past! Using the Hubble constant of $H = 15 \times 10^6\, km/sec/10^6\, ly$ the time since the galaxies began to move apart would be $1/H = 20 \times 10^9\, years$. This number(age) calculated from the Hubble constant is called the *characteristic expansion time*. Since, as we will see, the galaxy velocities have not been constant for all time(they have been slowing down), the true age of the universe must be less than the *characteristic expansion time* for this value of the Hubble constant.

This result is usually taken to mean that the universe size is increasing. All this means is that the separation distance between any pair of distant galaxies is increasing. We want to avoid thinking of the universe as some volume in 3-dimensional space and thus being mislead about what is really happening. We note at this point, that we must be careful about drawing conclusions from one set of experiments. It is difficult to measure velocity versus distance(uncertainties about extragalactic distance scales or even what they mean are the main concern), redshifts may come other sources (gravity) than just motion, etc. The calculated age need to be confirmed in other unrelated experiments and so on. One example is the age of stuff in our galaxy is estimated to be about $10-15\times 10^9\, years$ from measurements of the relative abundances of radioactive isotopes and from calculations concerning stellar evolution. There is no connection between these new results and redshifts and since the calculated ages are very similar, the presumption is strong that the age of the universe deduced from the Hubble constant really does represent a true beginning. These ideas then are the beginning of the so-called big-bang cosmology as a theory of the universe.

Our picture of the universe so far is one of an expanding swarm of galaxies. Light has only played the role of a messenger telling us about the distances and velocities of the galaxies. It turns out, however, that conditions in the early universe were very different - light was the dominant constituent of the universe - ordinary matter was a negligible contamination. Let us therefore look more closely at the redshift in terms of the behavior of light waves in an expanding universe. This will be very important later on.

Consider a light wave traveling between two galaxies. The separation between the galaxies equals the light travel time times the speed of light and the increase in the separation during the light transit time equals the light transit time times the relative velocity of the galaxies. If we calculate the fractional increase in separation we have

$$f = \frac{\text{increase in separation}}{\text{mean separation during increase}} = \frac{v_{rel} \times t_{transit}}{c \times t_{transit}} = \frac{v_{rel}}{c}$$

that is, the transit time cancels out!. This same ratio, as we saw earlier, gives the fractional increase in wavelength of light during the journey. Therefore, *the wavelength of any light ray increases in proportion to the separation between the galaxies as the universe expands*. If the wavelengths of light appear to *stretch* (confirmed by observation), then we conclude that the universe itself is also large by the same amount, i.e., if the wavelength of light increases by 10% during its transit from some galaxy to earth, then during the same time the universe has gotten 10% larger.

So far we have only be concerned with *kinematic* effects - the description of the motion without any consideration of the forces that produce the motion. Physical theory also wants to understand the dynamics of the universe - in this case that is a study of the cosmological role of the gravitational force between astronomical bodies. Newton was the first to tackle this problem. He thought that matter had to be distributed throughout an infinite space, which is so difficult to work with that it stifled all progress until Einstein developed general relativity as we have discussed. Einstein's equations admitted solutions that predicted the existence of a redshift proportional to distance - the so-called *de Sitter model*. It was probably this theory that pushed Hubble to say his data had the same property. The problem was that solutions of this type were *static* - unchanging and that is not the universe we now know from observations (they did not know back then). We need solutions which are not static, but are homogeneous and isotropic. Finally in 1922 Friedmann found such a solution to Einstein's equations - it provided the mathematical background for most modern cosmological theories.

There are two types of Friedmann models. If the average density of the matter of the universe is less than or equal to a certain critical value, then the universe must be spatially infinite. In that case, the present expansion of the universe will go on forever.On the other hand, if the density of the matter of the universe is greater than this critical value, then the gravitational field produced by matter curves the universe back on itself - it is finite though unbounded, like the surface of a sphere. In this case, the gravitational fields are strong enough eventually to stop the expansion of the universe - the universe will ultimately implode back to indefinitely large density. This critical density turns out to be proportional to the square of the Hubble constant which is about $5 \times 10^{-30}\, gm/cm^3$ or about 3 hydrogen atoms per cubic meter of space.

The motion of a typical galaxy in the Friedmann models is just like that of a stone thrown upward from the surface of the earth. If the stone is thrown fast enough or equivalently if the mass of the earth is small enough, then the stone will gradually slow down but will nevertheless escape to infinity. This corresponds to the case of of a universe density less than the critical density. On the other hand, if the stone is thrown with too little speed, then it will rise to a maximum height and then plunge back downward, which correspond to the universe density above the critical value. In these models the galaxies are not rushing apart because some mysterious force is pushing them apart(stone is not being repelled by the earth) - the galaxies are moving apart because they were thrown apart by some sort of *explosion* in the past!

Many of the detailed properties of the Friedmann models can be calculated without using the full blown general relativity. In order to calculate the motion of a galaxy relative to our own, draw a sphere with our galaxy at the center and the other galaxy on the surface. The motion of the galaxy on the surface of the sphere depends only on the mass of the matter inside the sphere - the outside matter has no effect! This result was known to Newton and still hold in Einstein's theory(Birkhoff's theorem - see below). The only requirement for the theorem to hold is homogeneous and isotropic matter distribution.

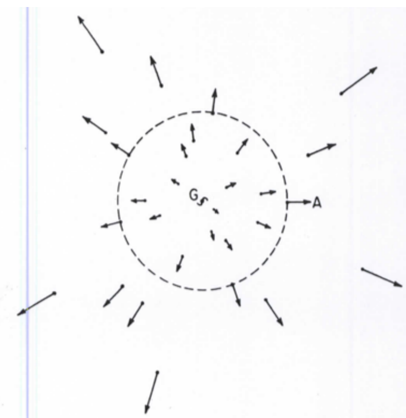


Figure 3 *Birkhoff's Theorem and the Expansion of the Universe.* A number of galaxies are shown, together with their velocities relative to a given galaxy G, indicated here by the lengths and directions of the attached arrows. (In accordance with the Hubble law, these velocities are taken to be proportional to the distance from G.) Birkhoff's theorem states that in order to calculate the motion of a galaxy A relative to G, it is only necessary to take into account the mass contained within the sphere around G that passes through A, shown here by the dashed line. If A is not too far from G, the gravitational field of the matter within the sphere will be moderate, and the motion of A can be calculated by the rules of Newtonian mechanics.

We can use the theorem to calculate the critical density of the Friedmann models. As in the figure above, we can use the mass within the sphere to calculate the velocity that the galaxy on the surface would have to have to escape to infinity. This escape velocity is proportional to the radius of the sphere (larger radius means larger mass inside the sphere so that escape velocity is larger). On the other hand, Hubble's law says that the actual velocity of the galaxy is proportional the radius of the sphere(distance from us) also. Therefore the ratio of the actual velocity to the escape velocity does not depend on the radius of the sphere. Therefore, depending on the values of the Hubble constant and the cosmic density every galaxy which moves according to Hubble's law will either exceed escape velocity (go to infinity) or be less than escape velocity(fall back toward us). The critical density is the value of the cosmic density at which the escape velocity equals the Hubble law velocity. Let us derive the result now.

For a sphere of galaxies of radius R we have a total mass (using cosmic density ρ)

$$M = \frac{4\pi R^3}{3}\rho$$

The gravitational potential energy of any galaxy (mass m) at the surface of the sphere is

$$U = -\frac{mMG}{R} = -\frac{4\pi m R^3 \rho G}{3} \quad , \quad G = 6.67 \times 18^{-8}\, cm^3/gm\, sec^2$$

The velocity of this galaxy is given by Hubble's law as $V = HR$. Therefore the kinetic energy is

$$K = \frac{1}{2}mV^2 = \frac{1}{2}mH^2R^2$$

The total energy is then

$$E = K + U = mR^2\left[\frac{1}{2}H^2 - \frac{4}{3}\pi\rho G\right]$$

which is a constant during the motion of the galaxy. The condition $E = 0$ correspond to just enough energy to escape to infinity. Thus the critical density condition is

$$\frac{1}{2}H^2 - \frac{4}{3}\pi\rho_{crit}G = 0 \rightarrow \rho_{crit} = \frac{3H^2}{8\pi G}$$

This result. although calculated without general relativity, is still valid in a relativistic universe if the density ρ is interpreted as the total energy density divided by c^2 instead of just the mass density. Putting in numbers we find $\rho_{crit} = 4.5 \times 10^{-30}\, gm/cm^3$.

The detailed time dependence of the size of the universe can also be worked out - very mathematical. For once let me show you the equations from GR:

$$R\dot{\rho}c^2 = -3(\rho c^2 + p))\dot{R} \quad , \quad p = \text{ pressure}$$

The pressure is given by $w\rho c^2$ where $w = 0$ for ordinary matter and $w = 1/3$ for radiation. The results of solving these equations is shown below.

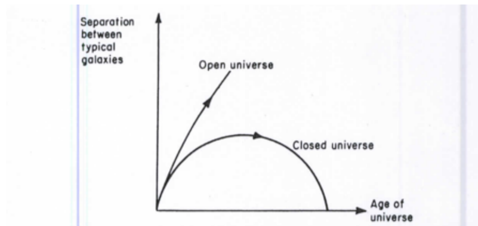
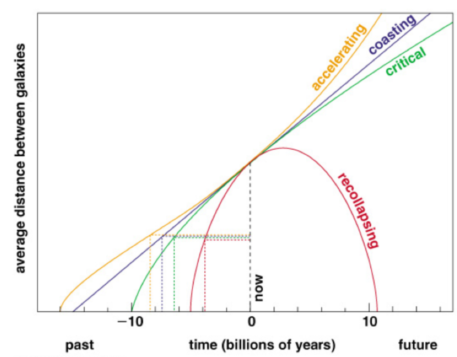


Figure 4. *Expansion and Contraction of the Universe.* The separation between typical galaxies is shown (in arbitrary units) as a function of time, for two possible cosmological models. In the case of an "open universe," the universe is infinite; the density is less than the critical velocity; and the expansion, though slowing down, will continue forever. In the case of a "closed universe," the universe is finite; the density is greater than the critical density; and the expansion will eventually cease and be followed by a contraction. These curves are calculated using Einstein's field equations without a cosmological constant, for a matter-dominated universe.

and even fancier



Note the three solutions - open, closed and flat universes. There is one result that will be very important to us later which we will derive now.

If we go back to the total energy result we derived earlier and now put in the time dependence we have

$$E = mR^2(t)\left[\frac{1}{2}H^2(t) - \frac{4}{3}\pi\rho(t)G\right]$$

where now all the variables take on their values at the particular time t. The energy, however, must be constant. As we will show shortly, as $R(T) \to 0$, $\rho(t)$ increase at least as fast as $1/R^3(t)$, so that $\rho(t)R^2(t)$ grows at least as fast as $1/R(t)$ as $R(T) \to 0$. In order to keep the energy constant, the two terms in the bracket must nearly cancel. Therefore for $R(T) \to 0$ we have

$$\frac{1}{2}H^2(t) \to \frac{4}{3}\pi\rho(t)G$$

The characteristic expansion time is the reciprocal of the Hubble constant or

$$t_{exp}(t) = \frac{1}{H(t)} = \sqrt{\frac{3}{8\pi\rho(t)G}}$$

Now let us return to see how the density ρ varies with distance R.

If the mass density is dominated by the mass of nuclear particles (the matter-dominated era), then the total mass within a comoving sphere of radius $R(t)$ is proportional to the number of such particles within the sphere - this must remain constant.

$$\frac{4}{3}\rho(t)R^3(t) = \text{ constant}$$

which says that

$$\rho(t) \propto \frac{1}{R^3(t)}$$

If, instead, the mass density is dominated by the mass equivalent to the energy of radiation (radiation-dominated era), then $rho(t)$ is proportional to the fourth power of the temperature (Stefan-Boltzmann law). The temperature, however, varies like $1/R(t)$, so then

$$\rho(t) \propto \frac{1}{R^4(t)}$$

In order to be able to deal with both cases in our discussions we will write

$$\rho(t) \propto \frac{1}{R^n(t)}$$

with

$$n = \begin{cases} 3 & \text{matter-dominated era} \\ 4 & \text{radiation-dominated era} \end{cases}$$

Our earlier conclusion that $\rho(t)$ increases at least as fast as $1/R^3(t)$ is now clear.

Now the Hubble constant is proportional to $\sqrt{\rho}$ and therefore

$$H(t) \propto \frac{1}{R^{n/2}(t)}$$

The velocity of a typical galaxy is then

$$V(t) = H(t)R(t) \propto \frac{1}{R^{1-n/2}(t)}$$

Since the velocity is the derivative of the distance, the mathematics of calculus says that if the the velocity is proportional to some power of the distance as it is above, then the time it takes to go from one point to another is given by

$$t_1 - t_2 = \frac{2}{n}\left[\frac{R(t_1)}{V(t_1)} - \frac{R(t_2)}{V(t_2)}\right] = \frac{2}{n}\left[\frac{1}{H(t_1)} - \frac{1}{H(t_2)}\right]$$

Finally we have

$$t_1 - t_2 = \frac{2}{n}\sqrt{\frac{3}{8\pi G}}\left[\frac{1}{\sqrt{\rho(t_1)}} - \frac{1}{\sqrt{\rho(t_2)}}\right]$$

Therefore, whatever the value of n, the time elapsed is proportional to the change in the inverse square root of the density. For example, during the who of the radiation-dominated era after the annihilation of electrons and positrons, the energy density was given by

$$\rho = 1.22 \times 10^{-35} \left[T(^\circ K)\right]^4 \; gm/cm^3$$

with n=4. Therefore the time required to cool from 10^8 degrees to 10^7 degrees was $t = t_1 - t_2 = 1.90 \times 10^6 \, sec = 0.06 \, years$.

We can derive a more general result. The time required for the density to drop to a value ρ from a value very much greater than ρ is

$$t = \frac{2}{n}\sqrt{\frac{3}{8\pi\rho G}} \begin{cases} t_{exp}/2 & \text{radiation-dominated era} \\ 2t_{exp}/3 & \text{matter-dominated era} \end{cases}$$

Thus, for example, at $3000^\circ \, K$ the mass density of photons and neutrinos was

$$\rho = 1.22 \times 10^{-35} \left[3000\right]^4 \; gm/cm^3 = 9.9 \times 10^{-22} \, gm/cm^3$$

This is much less than the density at $10^{8\circ} \, K$ that the time require for the universe to cool from very high early temperature to $3000^\circ \, K$ is given by $(n = 4)$, $t = 2.1 \times 10^{13} \, sec = 680000 \, years$.

Finally, since the density is proportional to $1/R^n$, the time is proportional to $R^{n/2}$ or

$$R \propto t^{2/n} = \begin{cases} t^{1/2} & \text{radiation-dominated era} \\ t^{2/3} & \text{matter-dominated era} \end{cases}$$

Now one way to tell whether or not galactic velocities exceed the escape velocity is to measure the rate at which they are slowing down. If this deceleration is less(or greater) than a certain amount, then the escape velocity is (or is not) exceeded. In practice, this means that one must measure the curvature of the graph of redshift versus distance for very distant galaxies as shown below.

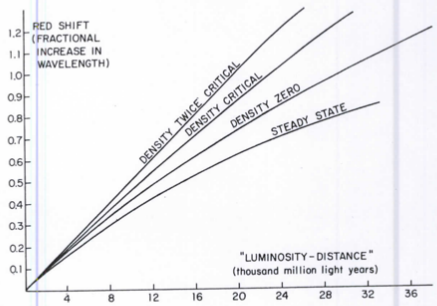


Figure 5. *Red Shift vs. Distance.* The red shift is shown here as a function of distance, for four possible cosmological theories. (To be precise, the "distance" here is "luminosity distance"—the distance inferred for an object of known intrinsic or absolute luminosity from observations of its apparent luminosity.) The curves labeled "density twice critical," "density critical," and "density zero" are calculated in the Friedmann model, using Einstein's field equations for a matter-dominated universe, without a cosmological constant; they correspond respectively to a universe that is closed, just barely open, or open. (See figure 4.) The curve marked "steady state" will apply to any theory in which the appearance of the universe does not change with time. Current observations are not in good agreement with the "steady-state" curve, but they do not definitely decide among the other possibilities, because in non-steady-state theories galactic evolution makes determination of distance very problematical. All curves are drawn with the Hubble constant taken as 15 kilometers per second per million light years (corresponding to a characteristic expansion time of 20,000 million years), but the curves can be used for any other value of the Hubble constant by simply rescaling all distances.

As one proceeds from a more dense finite universe to a less dense infinite universe, the curve of redshift versus distance flattens out at very large distances. By the 1970's the program to determine this graph had not produced conclusive results(The situation is very different now as we will see later).

The problem is in estimating the distances. What is distance when we cannot directly use a *ruler*? Close to us we can use geometry - measure angles and determine distance directly - this only works, however, for small distances - difficult to measure angles precisely. We then compare the direct distances to the distance calculated with Cepheid variables in some overlap region (this calibrates the Cepheid variable distance) and then use Cepheid variables to go further out in distance. Eventually, however, it becomes impossible to pick out any Cepheid variable star from within a galaxy. We could use the same technique substituting entire galaxies butwe don't know the absolute luminosity of a typical galaxy. In fact, a distant (and older) galaxy may not have the same properties has closer(younger) galaxies due to galactic evolution. So distance measurement is very difficult to do and is a major limitation.

IN the 1970's the best inference from the data was that the deceleration of distant galaxies seems very small, which would mean that they are moving at more than the escape velocity(I note, as we will see later, that they actually seem to be accelerating according to modern data). Generally, the uncertainties in astrophysical type measurement are large and make drawing conclusions difficult. Luckily for us, we do not have to pin down the large-scale geometry of the universe to draw conclusions about its beginning. The reason is that the universe has a *horizon* and this horizon shrinks rapidly as we look back toward the beginning. No signal can travel faster than light so at any time we can only be affected by events occurring close enough so that a ray of light would have had time to reach us since the beginning of the universe - the events that could affect us at any time are those in our past - in our backward light-cone - the set of past events that are timelike related to our present event. Any event that occurred beyond this distance could have no effect on us - it is beyon the *horizon*, i.e., there is at any time t after the beginning a horizon at a distance of order ct, from beyond which no information could yet have reached us. Since $R(t)$ vanishes less rapidly as $t \to 0$ than the distance to the horizon, at a sufficiently early time any given *typical* particle is beyond the horizon. If the universe is now 10^{10} years old, the horizon distance is now about 3×10^{10} light-years. But when the universe was a few minutes old, the horizon was only at a distance of a few light-minutes - less than the present distance from the earth to the sun. The entire universe, of course, was smaller than also (the separation between bodies decreases). However, as we look back toward the beginning, the distance to the horizon shrinks faster than the size of the universe, i.e as we found earlier, the size of the universe is proportional to $t^{1/2}$ or $t^{2/3}$ while the distance to the horizon is proportional to the time - thus, for earlier and earlier times, the horizon encloses a smaller and smaller portion of the universe as shown below.

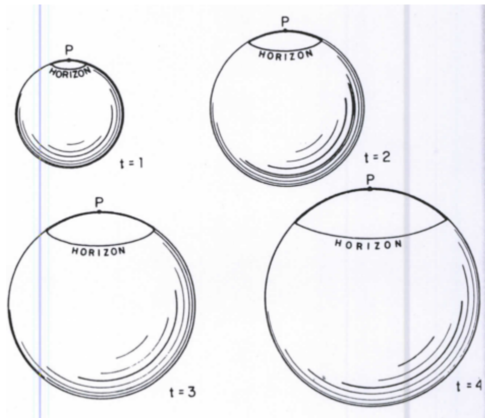


Figure 6. *Horizons in an Expanding Universe*. The universe is symbolized here as a sphere, at four moments separated by equal time intervals. The "horizon" of a given point P is the distance from beyond which light signals would not have had time to reach P. The part of the universe within the horizon is indicated here by the unshaded cap on the sphere. The distance from P to the horizon grows in direct proportion to the time. On the other hand, the "radius" of the universe grows like the square root of the time, corresponding to the case of a radiation-dominated universe. In consequence, at earlier and earlier times, the horizon encloses a smaller and smaller proportion of the universe.

As a consequence of this closing in of horizons in the early universe, the curvature of the universe as whole makes less and less difference as we look back to earlier and earlier times. Thus, we can still deal with the beginning without knowing the large-scale geometry!

We now have a view of the universe that is as simple as it is grand. The universe is expanding uniformly and isotropically - all galaxies see the same things in all directions. As the universe expands, the wavelengths of light rays are stretched out in proportion to the distance between the galaxies. The expansion was not thought to be dues to any cosmic repulsion in the 1970s but completely due to an initial explosion (modern data is not so sure). The expansion velocities in the 1970s seemed to be slowing down(slowly) under the influence of gravity suggesting a low matter density and a gravitational field too week to ever stop the expansion. Extrapolation backwards suggest an age of the universe on the order of $(10 - 20) \times 10^9$ years.

Chapter 3

The Cosmic Microwave Background

We now look at a very different set of observations. We stop looking at light emitted in the last few hundred million years from galaxies like our own, but instead look at the diffuse background of radio static noise left over from near the beginning of the universe. It is an interesting story. In 1964 Bell Labs had an antenna built for radio communication with the Echo satellite - it was a 20 foot horn reflector with ultra-low noise, which attracted radio astronomers to the instrument. Penzias and Wilson attempted to use the antenna to measure the intensity of radio waves emitted from our galaxy at high galactic latitudes (out of the plane of the Milky Way). A very difficult measurement. The radio waves from our galaxy are more like noise - much like the static on a radio during a thunderstorm. This radio noise is hard to distinguish from always present electrical noise produced by random electron motion in the antenna and other connected electrical circuits or from radio noise picked up by the antenna from the earth's atmosphere.

If one is studying a small (localized) source like a star or a distant galaxy, then one can point the antenna at the star and receive signals, then point the antenna away from the star and receive background signals (noise, etc) only and then subtract them out of the star signal (since they are the same no matter where we point the antenna). Penzias and Wilson, however, were trying to measure the signal coming from our own galaxy - in effect, from the sky itself - there is no way to subtract off the spurious signals. They therefore set out to identify and minimize all electrical noise produced in the antenna receiving system. To study the noise from the antenna structure they set the antenna to look at radio waves of a short wavelength $7.35\,cm$(these are called microwaves). They thought that radio noise from the galaxy would be negligible at this wavelength and therefore all the signal would be from the antenna structure. Noise from the earth's atmosphere would be easy to measure and subtract off since it could be easily identified by the dependence of the signal on direction (it depends on the thickness of the atmosphere along the antenna direction). Once having understood the noise this way, they could then proceed to study the radio waves from the galaxy which were expected to be at wavelength 21 cm (characteristic of hydrogen which is the dominant stuff in the universe).

To their surprise, they found a very large signal of microwave noise at 7.35 cm that was independent of direction. They also found that htis *static* did not vary with tome of day or, as the year went on, with the season. IT could not be coming from our galaxy since they would have already seen it in measurement taken on the Andromeda galaxy (almost the same as the Milky Way galaxy). They lack of any dependence on direction suggested that the radio noise was coming from a much larger volume of the universe than just our own galaxy.

Still thinking it was somehow antenna noise, they took the apparatus apart, cleaned it, and rebuilt it - no effect - the mysterious noise was still there undiminished.

Now any body at any temperature above absolute zero always emits radio noise (due to the thermal motions of the electrons within the body) The radio noise at any wavelength depends only on the temperature - the higher the temperature, the more intense the static. Thus, one can state an equivalent temperature for any radio source. Penzias and Wilson found an equivalent temperature for their observed noise between 2.5 and 4.5 degrees above absolute zero or about 3.5 degrees Kelvin or $3.5^\circ K$. This was much greater than expected, but very low in absolute terms. They hesitated publishing their results. They certainly did not think that this was the most important cosmological advance since the discovery of red shifts!

Several theories existed at the time of the measurements that, although not completely correct, had the right idea about what was happening. Most of the theories realized that if there had not existed an intense background of radiation during the first few minutes of the universe, then nuclear reactions would have proceeded so rapidly that a large fraction of the hydrogen present would have been *cooked* into heavier elements - this could not have happened because about 3/4 of the present universe is hydrogen. The rapid nuclear cooking could have been prevented only if the early universe was filled with intense radiation having an enormous temperature at very short wavelengths - this would blast apart any nuclei that formed as fast as they formed.

As we will see later, this radiation has survived the expansion of the universe but its equivalent temperature decreased as the universe expanded in inverse proportion to the size of the universe. Therefore, the present universe should be filled with radiation, but with an equivalent temperature significantly less than its value in the first few minutes. The first theoretical estimates showed the expected temperature to be in the same range as that measured by Penzias and

Wilson. Thus, the temperature recorded by the antenna is not the temperature of the present universe, but seems to be the temperature that the universe had near its beginning - greatly reduced by expansion.

Is this microwave radiation actually left over from the beginning of the universe? To answer this question we need to discuss what we expect theoretically: What are the general properties of the radiation that *should* be filling the universe if current cosmological ideas are correct? We will need to consider what happens to radiation as the universe expands - not only at the time of nucleosynthesis(at the end of three minutes) but also in all the time that has elapsed since then.

We need to give up the classical picture of radiation in terms of electromagnetic waves and switch to the quantum view that radiation consists of particles known as photons. An everyday light wave contains an enormous number of photons traveling along together. Classically, the energy is assumed to be a continuous quantity. If, however, we were to measure the enrgy of a light wave with extreme precision, we would find that it always comes in multiples of a definitie quantity, which is the energy of a single photons in the wave, i.e., $E_{wave} = NE_{photon}$ where $E_{photon} = \hbar\nu = \hbar c/\lambda$ where N is a very large number (10^{15} per cm^3), $\nu =$ the frequency of the wave, $\lambda =$ the wavelength of the wave and $c = \lambda\nu$. We note that a typical frequency is $\nu = 10^{14}\, sec^{-1}$ and therefore $E_{photon} = \hbar\nu \approx 10^{-20}\, Joules$, which is extraordinarily small. We know there are photons from the interaction of light with atoms which usually take place one photon at a time and cannot be explained except via the photon idea. Photons have zero mass and zero charge, carry a definite energy $E_{photon} = \hbar\nu$ and momentum $p = E_{photon}/c$.

What happens to an individual photon as it travels through the universe? Not much as it turns out. The light from objects as much as 10^{10} light-years away seems to reach us easily. This indicates that whatever matter may be present in the universe it is sufficiently transparent so that photons can travel for very large distances(for a large fraction of the of the universe) without being scattered or absorbed.

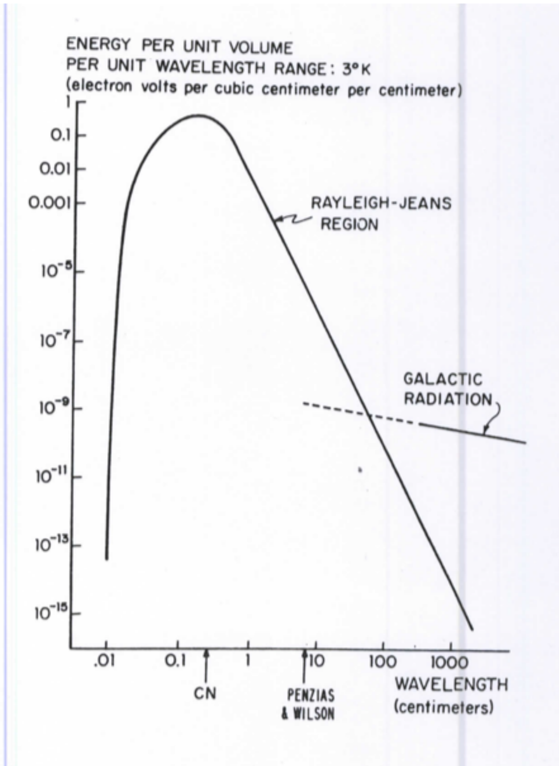
The redshift data tells us that the universe is expanding - the universe must have been more compressed in the past than now. In general, if you compress matter its temperature will rise As we will find later, there was a period which probably lasted for the first 700000 years of the universe when the contents were so hot and dense that the formation of stars and galaxies was impossible - matter remained as nuclei and electrons. Under these extreme conditions a photon could not travel very far with out being scattered or absorbed(as it can now). The mean free time between photon interactions with matter was much shorter than the characteristic time for the universe expansion. Therefore, even though the universe was expanding very rapidly at first, to an individual photon or electron or nucleus the expansion was taking a large amount of time - time enough for each particle to be scattered or absorbed and reemitted many times as the universe expanded. Any system like this where individual constituents have time for many interactions usually comes to a state of equilibrium. The properties of such a state are not determined by the initial conditions but instead by the requirements needed to maintain the equilibrium. Each of the particles is constantly changing in this state, but all the statistical variables such as distributions(probability) of energy or position are remaining essentially constant. This type of equlibrium is called *thermal equilibrium.* because it is characterized by a definite temperature which is essentially uniform throughout the system.

Most physical systems in the world are far from thermal equilibrium - the surface of the earth certainly is not a state of equilibrium. However, at the center of a star, there is nearly perfect thermal equilibrium and thus we can estimate the conditions are like with great confidence. The universe is certainly not in a state of perfect thermal equilibrium - it is expanding! However, as we mentioned the times scale of interaction in the early universe compared to the time scale of expansion - interactions taking place on a much faster time scale than the expansion - was such that the universe could be regarded as evolving *slowly* from one state of nearly perfect thermal equilibrium to another. It turns out that the properties of any system in thermal equilibrium are entirely determined once we specify the temperature of the system and the densities of a few conserved quantities. Thus, the universe preserves only a very limited memory of its initial state. This is not good if we want to know what the actual big-bang was like, but is great if we want to infer the course of events after the beginning without having to make too many arbitrary assumptions.

We now ask - What are the general properties of radiation in thermal equilibrium with matter?

By the 1890s it was know that the properties of radiation in a state of thermal equilibrium with matter depend only on the temperature - the amount of radiation energy per unit volume in a small range of wavelengths is given by a universal formula which involves only the wavelength and the temperature. This same formula, as it turns out, gives the intensity of the radio noise in terms of the equivalent temperature. For various obscure reasons this radiation became known as *black-body radiation.*

A full theoretical understanding of black-body radiation require quantum mechanics, which we will not go into here.Weinberg gives some ideas about it in the text. The characteristic black-body radiation distribution called the Planck distribution takes the form shown below.



This is a plot of energy density per unit wavelength range as a function of wavelength (log-log scales) for black-body radiation with a temperature of $3^\circ K$. For a temperature which is greater than $3^\circ K$ by a factor f, it is only necessary to reduce the wavelength scale by a factor $1/f$ and increase the energy density scale by a factor f^5 and the curve is exactly the same! The arrows indicate the Penzias and Wilson measurement.

Returning to our discussion of the observed microwave radiation, the universe must have been so hot and dense there were only free nuclei and electrons and the scattering of photons by free electrons maintained a thermal equilibrium between matter and radiation. As time passed, the universe expanded and cooled eventually reaching a temperature ($3000^\circ K$) cool enough to allow the combination of nuclei and electrons into atoms. The sudden disappearance of the free electrons broke the thermal contact between radiation and matter and the radiation continued to expand freely. At that moment the energy of the radiation at all wavelengths was determined by the conditions of thermal equilibrium - the Planck distribution for a temperature of $3000^\circ K$. the typical photon wavelength would have been $0.0001\,cm$ or $10000\,\mathring{A}$ and the average distance between photons was roughly the typical wavelength.

What has happened to the photons since then? Individual photons would not be created or destroyed - the average distance between them would increase in proportion to the size of the universe - in proportion to the average distance between typical galaxies. The associated cosmological redshift cause the wavelength of any individual photon to increase in proportion to the size of the universe. The photons would thus remain one typical wavelength apart just as for black-body radiation - the radiation filling the universe would continue to be black-body radiation as the universe expanded event though it is no longer in thermal equilibrium with matter. We give the mathematical proof below.

The Planck distribution gives the energy du of black-body radiation per unit volume, in a narrow range of wavelengths from λ to $\lambda + d\lambda$, as

$$du = \frac{\frac{8\pi hc}{\lambda^5} d\lambda}{e^{hc/k\lambda T} - 1}$$

where T is the temperature; k is Boltzmann's constant $(1.38 \times 10^{-16}\, erg/^\circ K)$; c is the speed of light; e is a numerical constant (3.718); and h is Planck's constant$(6.625 \times 10^{-27}\, erg{-}sec)$. The Planck formula for du reaches a maximum at a wavelength $\lambda = 0.2014052 hc/kT$ and then fall steeply off for decreasing wavelengths. The total energy density for all wavelengths is

$$u = \int_0^\infty \frac{\frac{8\pi hc}{\lambda^5} d\lambda}{e^{hc/k\lambda T} - 1} = \frac{8\pi^5 (kT)^4}{15(hc)^3} = 7.56464 \times 10^{-15} [T(^\circ K)]^4\, erg/cm^3$$

which is the Stefan-Boltzmann law we mentioned earlier.

The Planck distribution can easily be interpreted in terms of photons. Each photon has an energy $E = hc/\lambda$. Hence the number dN of photons per unit volume in black-body radiation in a narrow range of wavelengths from λ to $\lambda + d\lambda$ is

$$dN = \frac{du}{hc/\lambda} = \frac{\frac{8\pi}{\lambda^4} d\lambda}{e^{hc/k\lambda T} - 1}$$

and the total number of photons is

$$N = \int_0^\infty dN = 60.42198 \left(\frac{kT}{hc}\right)^3 = 20.28 [T(^\circ K)]^3\, photons/cm^3$$

and the average photon energy is

$$E_{photon} = \frac{u}{N} = 3.73 \times 10^{-16} [T(^\circ K)]\, ergs$$

Now let us consider what happens to black-body radiation in an expanding universe. Suppose the size of the universe changes by a factor f. As we saw earlier, wavelengths will change in proportion to the size of the universe, $\lambda' = f\lambda$. After the expansion, the energy density du' in the new wavelength range λ' to $\lambda' + d\lambda'$ is less than the original energy density du in the old wavelength range λ to $\lambda + d\lambda$ for two different reasons:

1. Since the volume of the universe has increased by a factor f^3, as long as no photons have been created or destroyed, the number of photons per unit volume has decreased by a factor $1/f^3$.

2. The energy of each photon is inversely proportional to its wavelength and is therefore decreased by a factor $1/f$.

It then follows that the energy density is decreased by an overall factor $1/f^3$ times $1/f$ or $1/f^4$. Thus,

$$du' = \frac{1}{f^4} du = \frac{\frac{8\pi hc}{\lambda^5 f^4} d\lambda}{e^{hc/k\lambda T} - 1}$$

If we rewrite this formula in terms of the new wavelength λ', it becomes

$$du' = \frac{\frac{8\pi hc}{\lambda'^5} d\lambda'}{e^{hcf/k\lambda' T} - 1}$$

but this is exactly the same as the old formula for du in terms of λ and $d\lambda$, except that T has been replace by a new temperature $T' = T/f$. Thus, we conclude that freely expanding black-body radiation remains described by the Planck formula, but with a temperature that decreases in inverse proportion to the scale of the expansion.

Penzias and Wilson found a temperature of about $3^\circ K$, which would be expected if the universe had expanded by a factor of 1000 since the time when the temperature was high enough ($3000^\circ K$) to keep matter and radiation in thermal equilibrium. If this interpretation is correct, then the $3^\circ K$ radio static is by far the most ancient signal received by astronomers - it was emitted long before the light from the most distant galaxies that we can see!

Subsequent measurements have shown that this temperature is characteristic of all wavelengths as expected. However, all the measurements were on the long wavelength end of the Planck distribution for $3^\circ K$, i.e., above the maximum which occurs for at $0.1\,cm$. In order to confirm that we really have black-body radiation, we need to check at all wavelengths- not just radio/microwave radiation but also infrared radiation. Unfortunately, the earth's atmosphere which is nearly transparent at wavelengths above $0.3\,cm$ becomes increasingly opaque at shorter wavelengths so ground-based astronomy will not work. Continued experimentation required getting above the atmosphere. This was done using rockets and balloons and black-body radiation at about $3^\circ K$ was confirmed to wavelengths as low as $0.06\,cm$. To use a satellite, was a very difficult undertaking since the detectors had to be cooled to liquid helium temperature (comparable to the black-body temperature itself). This was easier to do on earth and in balloons and rockets. Another reason to go to satellites was to get above the atmosphere. We have using the idea that all the radiation was isotropic - same in all directions up to to now. This was need to be consistent with the Cosmological Principle. One, however, wants to investigate the possible directionality of the radiation. With the atmosphere in the way this is almost impossible since the atmosphere has it own directional effects that would mask it.

Why do we want to look for direction dependence?

There might be fluctuations in the intensity with small changes in direction caused by the actual lumpiness of the universe either at the time the radiation was emitted or since then. Galaxies in early stages might show up as hot spots. Also there is a small directionality due to the earth's motion through the universe. If the earth were moving at about $300\,km/sec$ with respect to the average matter in the universe (around the sun + sun around center of galaxy + galaxy motion), then we would have a Doppler effect where the wavelength from ahead or astern would be decreased or increased respectively by about 0.1%. Non-satellite instruments do not have this kind of accuracy. Since this book was written, a Swarthmore graduate John Mather has led a project (COBE) to measure both the background radiation at all wavelengths and the fluctuations in the background. We present below the COBE results which we will talk about later. Also included are later WMAP and PLANCK data.

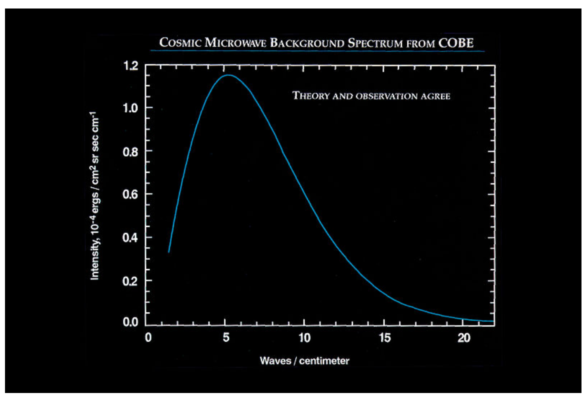


Figure 3.1: COBE Black-Body Data

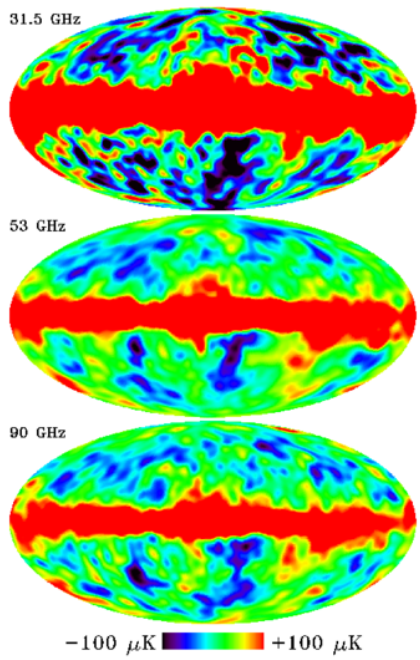


Figure 3.2: COBE Full Sky - Different Wavelengths - Data

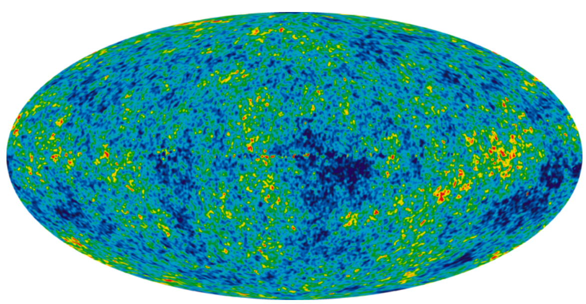


Figure 3.3: WMAP Full Sky Data

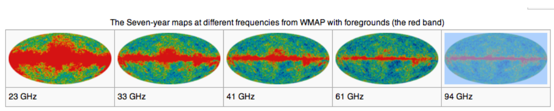


Figure 3.4: WMAP Full Sky - Different Wavelengths - Data

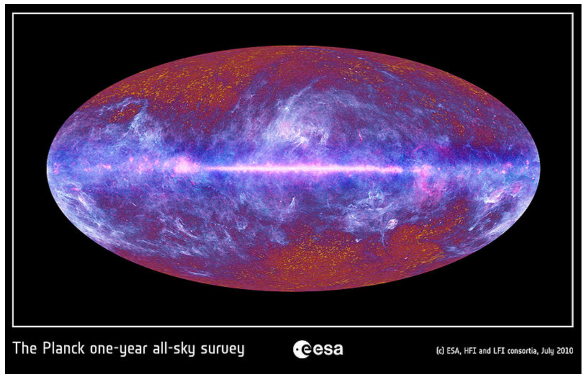


Figure 3.5: PLANCK Full Sky Data

Clearly it is exactly Black-Body radiation and there are fluctuations to be understood.

What cosmological insight can we draw from the particular temperature value $3^\circ K$? It will allow us to determine one crucial number that we will need to follow the history of the first three minutes.

At $3^\circ K$ there are 550000 photons per liter. The density of nuclear particles (neutrons and protons) in the present universe is between 6 and 0.03 particles per 1000 liters. Thus there are between 10^{10} and 2×10^{10} photons for every nuclear particle in the universe today. This number has been roughly constant for a long time. No creation of either has taken place during the expansion after $3000^\circ K$. This is the most important quantitative conclusion we can draw from the early data. For practical purposes we will assume the number is 10^9 photons per nuclear particle.

The differentiation of matter into galaxies and stars could not have begun until the time when the cosmic temperature became low enough for electrons to be captured into atoms. In order for gravity to produce the clumping of matter into isolated fragment it is necessary for gravity to overcome the the pressure of matter and the associated radiation. The gravitational force within a clump increases with the size of the clump while the pressure does not depend on the size; hence at any given density and pressure there is a minimum mass which is susceptible to gravitational clumping. This is called the *Jeans mass*. It turns out that the Jeans mass is proportional the $(pressure)^{3/2}$ (see derivation below).

In order tofor a clump of matter to form a gravitationally bound system, it is necessary for its gravitational potential energy to exceed its internal thermal energy. The gravitational potential energy of a clump of radius r and mass M

is of order

$$U_{grav} \approx -\frac{GM^2}{r}$$

The internal energy per unit volume is proportional to the pressure p, so the total internal energy is of order

$$E_{int} \approx pr^3$$

The gravitational clumping should be favored if

$$\frac{GM^2}{r} \gg pr^3$$

But for a given density ρ we can express r in terms of M through the relation

$$M = \frac{4\pi}{3}\rho r^3$$

The condition for gravitational clumping can therefore be written

$$GM^2 \gg p\left(\frac{M}{\rho}\right)^{4/3}$$

or $M \geq M_J$ where M_J is the quantity know as the *Jeans mass*:

$$M_J = \frac{p^{3/2}}{G^{3/2}\rho^2}$$

For example, just before the the free electrons and the free nuclei combined into hydrogen(recombination), the mass density was $9.9 \times 10^{-22}\, gm/cm^3$ (calculated earlier) and the pressure was $p \approx \rho c^2/3 = 0.3\, gm/cm\, sec^2$ so that the Jeans mass is $M_J = 9.7 \times 10^{51}\, gm = 5 \times 10^{18} M_\odot$, where $M_\odot$ is one solar mass. Note that our galaxy mass is about $10^{11} M_\odot$. Galaxies are not massive enough to have formed at this time or before. After recombination, the pressure dropped by a factor of 10^9, so the Jeans mass dropped to $M_J = 1.6 \times 10^5 M_\odot$ and galaxies were able to form. We know they can form but we still do not know how they form.

So before recombination, there were no stars or galaxies in the universe, only an ionized and undifferentiated soup of matter and radiation.

Another remarkable consequence of the large ratio of photons to nuclear particles is that there must have been a time, not too far in the past, when the energy of radiation was greater than the energy contained in the matter of the universe. The energy in the mass of a nuclear particle $mc^2 \approx 9.4 \times 10^8\, eV$. The average energy of a $3°\, K$ black-body radiation photon is about $0.0007\, eV$ so that even with 10^9 photons per neutron or proton most of the energy of the present universe is in the form of matter, not radiation. However, at earlier times, the temperature was higher, so the energy of each photon was higher, while the energy of the nuclear particles is unchanged. With 10^9 photons per neutron or proton, in order for the radiation energy to exceed the energy of matter we must have a temperature of about $4000^\circ K$ This is the temperature that marks the transition between a *radiation-dominated* era in which most of the energy in the universe was in the form of radiation, and the present *matter-dominated* era in which most of the energy is in the masses of the nuclear particles.

It is striking that this transition $4000^\circ K$ occurred about the same time as the content of the universe were becoming transparent to radiation $3000^\circ K$ We do not know why. It is also not clear which change occured first (the numbers used to calculate the transition temperature are fairly uncertain).

None of these uncertainties will affect our study. We only need to know the early era was radiation-dominated with only a small contamination of matter.

Chapter 4

Recipe for a Hot Universe

We now know the universe is expanding and that it is filled with a universal background of radiation, now at a temperature of about $3^\circ K$. This radiation was left over from a time when the universe was effectively opaque, when it was 1000 times smaller(the average distance between a typical pair of particles was a 1000 times smaller) and hotter than at present. In order to account for the first three minutes, we must now use theory to look back even earlier when the universe was even smaller and hotter, to discover the physical conditions that prevailed.

During the radiation-dominated era, the numbers of photons are so large and the energy so large, that we can consider the universe as if it were filled only with radiation (assume essentially no matter). We note that the radiation-dominated era began at the end of three minutes when the temperature dropped below 10^9 degrees Kelvin. Prior to this period, matter was important, but not the kind of ordinary matter we usually talk about.

We need to find out how the temperature was related to the size of the universe so that we can figure out how hot things were at any given moment. We know that if the radiation were expanding freely (no interactions), then the wavelength of each photon is stretched out(redshift) in proportion to the size of the universe as the universe expands. We also know that the average wavelength of black-body radiation is inversely proportional to its temperature. Thus the temperature would have decreased in inverse proportion to the size of the universe, just as it is doing now. However, the expansion was not free during this era - the photons were experiencing rapid collisions with the small number of electrons and nuclear particles - remember the universe was essentially opaque during this era. Luckily for us, however, most of the time the photons were freely traveling between collision (such a small amount of matter), and therefore they essentially behaved as if they were free as far as the temperature change due to the expansion of the universe was concerned and the earlier argument still works!

As we look earlier in the evolution of the universe, we will come to a time when the temperature was so high that photon-photon collisions began to produce material particles out of pure energy. The particle produce this way will turn out to be just as important as the radiation during the first three minutes - both for determining the rates of nuclear reactions and in determining the rate of expansion of the universe itself. What is the temperature threshold for this particle production to occur? The process is understood in terms of the quantum theory of light. Two photons can collide, disappear and all their energy and momentum reappear in the production of material particles($E = mc^2$). Ordinary nuclear reactions also use this form of energy conversion but generally only a small fraction of the mass is converted. In order for two photons to produce two material particles of mass m the energy of each photon must be greater than or equal to mc^2.

Can photons have this energy ? or What is the characteristic energy of individual photons in the radiation field? We can estimate by a rule from Statistical Mechanics, namely $E_{char} \approx kT$ where k is Boltzmann's constant and $k = 0.00008617\, eV/{}^\circ K$ and T is the temperature in degrees Kelvin. Therefore, at $3000^\circ K$ when the contents of the universe were just becoming transparent, each photon had a characteristic energy of $0.26\, eV$. This is the characteristic energy of reactions involving atoms - that is why the radiation was able to prevent the recombination of nuclei and electrons into atoms.

To produce particles, however, the characteristic energy needs to larger than mc^2 and the corresponding threshold temperature would be mc^2/k. The electron e^- and the positron e^+(antiparticle) are the particles with the smallest mass. The rest energy mc^2 of the electron(positron) is $0.511002\, MeV$. Therefore the threshold temperature at which photon can have this much energy is $mc^2/k = 6 \times 10^9\, {}^\circ K$(NOTE: $T_{center\ of\ sun} = 15 \times 10^6\, {}^\circ K$).

At this temperature or higher, electrons and positrons would have been freely created in photon collisions and would be present in very large numbers. That is why we do not see electrons and positrons popping out of empty space whenever the sunlight is bright! Similar rules apply for all other particles and their associated antiparticles.

The table below summarizes some facts for several particles.

Class	Name	Symbol	Energy(MeV)	$T_{thresh}(10^9\,^\circ K)$	Species	Mean Life(sec)
	Photon	γ	0	0	1	stable
Lepton	Neutrinos	$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$	0	0	2	stable
Lepton	Electron	e^-, e^+	0.5110	5.930	7/4	stable
Lepton	Muon	μ^-, μ^+	105.6	1226.2	7/2	2.197×10^{-6}
Hadron	π mesons	π^0	135.0	1566.2	1	0.8×10^{-16}
Hadron	π mesons	π^+, π^-	139.6	1566.2	2	2.6×10^{-8}
Hadron	Proton	$p, \bar{p}$	938.3	10888	7/2	stable
Hadron	Neutron	$n, \bar{n}$	939.6	10903	7/2	920

Table 4.1: Properties of Some Elementary Particles

The particles that are present in large numbers at different times in the history of the universe can be read off the table by looking at the threshold temperature column.

The number of particles present at temperatures above the threshold temperature is governed by the thermal equilibrium - the number must have been just high enough so that precisely as many were being destroyed each second as were being created. Since the rate of 2 photons into 2 particles is the same as the rate of 2 particles into 2 photons, the condition of thermal equilibrium requires that the number of particles of each type, whose threshold temperature is below the actual temperature should be about equal to the number of photons. If number less than photons, will be created faster and number rise to balance and if number smaller, will be destroyed faster and numbers decrease! Thus, above electron threshold temperature, say at $6000^\circ K$ the number of positrons and and electron is the same as the number of photons and the universe is a soup of all three!

On the other hand, high above the threshold temperature, material particles basically have energy kT, which is much larger than mc^2 and the mass can be neglected so that material particle behave like photons. In this case, the pressure end energy density contributed by each type of material particle is proportional to T^4 in the same way as photons. The universe is then composed of a variety of types of *radiation* - one for each species - the effective species number is lusted in the table above. The energy density of the universe is then proportional to T^4 and the effective number of species with threshold temperature below the actual temperature.

When was the universe at these very high temperatures? The balance between the gravitational field (attractive) and the outward momentum pressure (repulsive) of the contents of the universe the determines the rate of expansion of the universe. It is the total energy density that is the source of the gravitational field at early times in the universe. The energy density depends essentially only on the temperature - thus the cosmic temperature can be used as a kind of clock - cooling instead of ticking as the universe expands. We showed earlier that the time required for the energy density of the universe to fall from one value to another is proportional to the difference of the reciprocals of the square roots of the energy density(proportional to number of species and T^4). Thus, as long as the temperature does not cross a threshold, the *time required for the universe to cool from one temperature to another is proportional to the difference of the inverse squares of these temperatures.* Therefore, if we start at a temperature of $10^8\,^\circ K$ then after about 700000 years the temperature would reach $3000^\circ K$ as shown below.

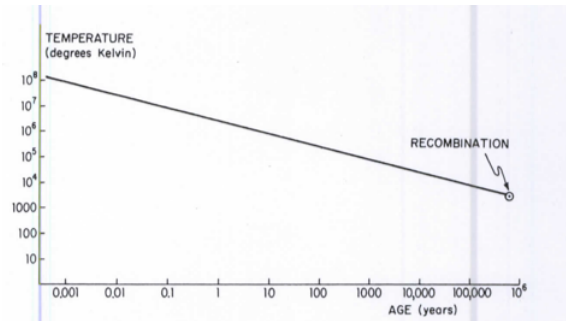


Figure 8. *The Radiation-Dominated Era.* The temperature of the universe is shown as a function of time, for the period from just after the end of nucleosynthesis to the recombination of nuclei and electrons into atoms.

During the first few minutes the number of particles and antiparticles cannot have been precisely equal. Since they would then have all annihilated when the temperature dropped below $10^9\,^\circ K$ and nothing would be left except radiation - we have evidence against this - us! There must have been an excess of particles over antiparticles that led to the matter in the present universe. What exactly were the constituents making up this excess? The list of possibilities is endless! In order to figure out what was there, we need to look in more detail at the thermal equilibrium. In a state of thermal equilibrium, there are quantities that do not change - *conserved quantities.* For example total energy - even though collisions are constantly redistributing the energy, the total remains constant - this is called a *conservation law.* For each such conservation law there exists a value that must be specified in advance in order to work out the properties of a system in thermal equilibrium - the values cannot be determined from the conditions for equilibrium. once these constant values have been specified for a system in thermal equilibrium then all of its properties are uniquely determined. Since the universe has passed through a state of thermal equilibrium, all we need to do to give a complete recipe for the contents of the universe at early times is to know what physical quantities are conserved as the universe expands and what were the values of these quantities.

It turns out that there are just three conserved quantities whose densities must be specified in our recipe for the early universe:

1. Electric Charge - Particles and antiparticles can annihilate or be created, but total charge never changes.

2. Baryon Number - The number of protons, neutrons, mesons minus the number of their antiparticles. Baryons and antibaryons can annihilate or be created, but total baryon number never changes.

3. Lepton Number - The number of electrons, muons, neutrinos minus the number of their antiparticles. Leptons and antileptons can annihilate or be created, but total lepton number never changes.

To complete the recipe for the contents of the universe at any given time, we must specify the charge, baryon number, and lepton number densities as well as the temperature at that time. The conservation laws then say that as the universe expands the three total quantities remain fixed. Thus, charge, baryon number, and lepton number densities vary with the inverse cube of the size of the universe. On the other hand, the photon density also varies with the inverse cube of the size of the universe - the photon density is proportional to T^3 and the temperature varies with inverse size of the universe. Thus, the totals for all these numbers charge, baryon, lepton, and photon which equal their respective density times the volume remain fixed and the recipe can be specified by giving the charge, baryon number, and lepton number ratios to the photon number.

The net charge of the universe must essentially be zero or the cosmic electric charge per photon is negligible. We can see this from the numerical considerations below. If the earth and the sun had an excess of positive over negative charges of about one part in 10^{36} electric force between them would be greater than the gravitational force between the earth and the sun. Gravity dominates on all scales in the universe - hence our conclusion.

The only stable baryons are the nuclear particles - the proton and the neutron and their antiparticles. As far observation is concerned there is no appreciable amount of antimatter in the universe. Thus, the baryon number is essentially the number of nuclear particles. At the present time there is one nuclear particle for every 10^9 photons in the microwave radiation background. Thus, the baryon number per photon is about 10^{-9}. This is a remarkable result. Let us see why. Consider a time in the past when the temperature was above $10^{13}\,{}^\circ K$, which is the threshold temperature for protons and neutrons. At that time, the number of nuclear particles and antiparticles was about equal to the number of photons. But baryon number is the difference between particle and antiparticle numbers. If this difference was 10^9 smaller than the photon number(as above) and hence also about 10^9 smaller then the total number of nuclear particles, then the number of nuclear particles would have exceeded the number of antiparticles by only one part in 10^9. In this view, when the universe cooled below the threshold temperature for nuclear particles, the antiparticles all annihilated with corresponding particles, leaving behind the tiny excess of particles over antiparticles as a residue which eventually turns into the world we know now.

Since the net charge in the universe is zero, their exist exactly one electron for each proton. proton make up about 87% of all nuclear particles in the present universe. If electrons were the only leptons in the present universe, then the lepton number per photon would be about the same as the baryon number per photon. However, there is another stable lepton, namely the neutrino. It is massless, has zero charge. Thus, we need to figure out the numbers of neutrinos and antineutrinos. Not easy to do! Neutrinos only interact very weakly with matter and radiation (no charge, no mass). A good number to remember - it would take 46 light-years of lead to stop a neutrino!

So the universe might be filled with neutrinos and we could not easily tell. It would easily be possible for the number of neutrino and their corresponding energies to be comparable to photons. However, once again lepton number is a difference and we still assume it is very small compared to the photon number while the number of electrons or neutrinos is about the same as the number of photons above threshold temperatures.

We now have the recipe for the contents of the early universe. Take a charge per photon equal to zero, a baryon number per photon equal to one part in 10^9. Take the lepton number per photon to be uncertain but small. Take the temperature at any given time to be greater than the temperature $3\,^\circ K$ of the present radiation background by the ratio of the present size of the universe to the size at that time. Stir well, so that the detailed distribution of particles of various types are determined by the requirements of thermal equilibrium. Place in an expanding universe, with a rate of expansion governed by the gravitational field produced by this medium. After a long enough wait, this concoction should evolve into our present universe.

Chapter 5

The First Three Minutes

We can now follow the course of cosmic evolution through its first three minutes. The time scale used will be temperature-based, i.e., we will take a picture each time the temperature drops by a factor of about 3. We start the story at 1/100 second (we will look at the earliest period later since this is the period where most of the physics was discovered after Weinberg wrote his book).

First Frame. The temperature of the universe is $10^{11}\,^\circ K$. The universe is in a state that is easy to describe. It is filled with an uniform soup of matter and radiation, each particle of which has rapid collisions with other particles. Even though it is rapidly expanding, the universe is approximately in perfect thermal equilibrium. Statistical mechanics rules - means that nothing depends on what happened before the first frame (except that the temperature value and the size value - how they got those values is a story for later). We also know the conserved quantities - charge, baryon and lepton number are all very small or zero.The most abundant particles are those with a threshold temperature below $10^{11}\,^\circ K$ - electrons, positrons, photons, neutrinos and antineutrinos. The universe is so dense that even neutrinos (remember 46 light-years of lead to stop them!) are kept in thermal equilibrium with the other particles by rapid collisions. Since the threshold temperature for electrons and positrons is well below $10^{11}\,^\circ K$ they act like pure radiation (neglect their mass) - like photons and neutrinos.

What is the energy density of the different kinds of radiation? From the table listing of effective species number we see that it turns out that the total energy density of the soup is 9/2 greater than it would be if we only had photons. The Stefan-Boltzmann law says the energy density of electromagnetic radiation at $10^{11}\,^\circ K$ is $4.72 \times 10^{44}\, eV\ per\ liter$. Thus the total energy density of the universe-soup is $21 \times 10^{44}\, eV\ per\ liter$. This is equivalent to a mass density of $3.8^9\, kg\ per\ liter$ or 3.8^9 times the density of water.

The universe is rapidly expanding and cooling at this time. The rate of expansion is given by the condition that all parts of the universe are moving at the escape velocity away from some arbitrary center. Since the universe is so dense, the escape velocity is very high and the characteristic time for expansion of the universe is about 0.02 seconds(see earlier calculations - page 16). This *characteristic expansion time* is approximately 100 times the time interval during which the size of the universe would increase by 1%. It is equal to the reciprocal of the Hubble constant at that time. During this period the small number of existing protons and neutrons will be experiencing rapid reactions with the dense radiation. The two dominant reactions are

$$\bar{\nu} + p \rightarrow e^+ + n \quad , \quad \nu + n \rightarrow e^- + p$$

which occur at equal rates. Thus, the numbers of protons and neutrons do not change. Equilibrium requires these two numbers to be about the same at this time.

Second Frame. The temperature of the universe is $3 \times 10^{10}\,{}^\circ K$. Since the first frame 0.11 seconds have elapsed. Nothing has changed qualitatively - the contents of the universe are still the same and all are still in thermal equilibrium and high above their threshold temperatures. Thus, the energy density has simply decreased like T^4 to about 3×10^7 that of the rest energy density of water. The rate of expansion has decreased like T^2 so the characteristic expansion time of the universe has now increased to 0.2 seconds. No nuclei form as yet, but the rate of neutrons to protons is now greater than for protons into neutrons and the nuclear balance has shifted from 50-50 to 38-62 neutrons-protons.

Third Frame. The temperature of the universe is $10^{10}\,{}^\circ K$. Since the first frame 1.09 seconds have elapsed. At about this time, the decreasing density and temperature have increased the mean free time (between collisions) of neutrinos so much that they are beginning to act like free particles (no interactions), no longer in thermal equilibrium with electrons, positrons or photons. They no longer play any role in the evolution of the universe except that their energy will still act as part of the source of the gravitational field. The neutrino wavelength will continue to stretch in direct proportion to the size of the universe. The total energy density has continued to decrease as T^4 to about 4×10^5 that of the rest energy density of water. The rate of expansion has decreased like T^2 so the characteristic expansion time of the universe has now increased to 2.0 seconds. The temperature is now only 2 times the electron threshold temperature - this means they are just beginning to annihilate more rapidly than they are recreated out of radiation. It is still too hot for nuclei to form. The nuclear balance has shifted to 24-76 neutrons-protons.

Fourth Frame. The temperature of the universe is $3 \times 10^9\,{}^\circ K$. Since the first frame 13.82 seconds have elapsed. We have dropped below the threshold temperature for electrons and positrons - they are rapidly disappearing as major constituents of the universe. The energy released in their annihilation slows down the cooling of the universe - neutrinos do not pick up any of this energy so they are 8% cooler than the electrons, positrons and photons. Since electrons and positrons are disappearing the energy density is slightly smaller than just the T^4 drop off.

The temperature is now low enough for stable nuclei like helium to form but it does not happen immediately because the expansion rate is still too rapid and the formation of helium requires a complex series of fast two-particle reactions.

$$\begin{aligned}
&n + p \rightarrow H^2(deuterium) + \gamma \\
&H^2 + p \rightarrow He^3 + \gamma \\
&or \\
&H^2 + n \rightarrow H^3(tritium) + \gamma \\
&He^3 + n \rightarrow He^4 + \gamma \\
&or \\
&H^3 + p \rightarrow He^4 + \gamma
\end{aligned}$$

To work the first step, the production of deuterium, must take place. Deuterium is a weakly bound structure compared to helium. It takes only 1/9 as much energy to break apart deuterium as it takes to break apart helium. At $3 \times 10^9 \,^\circ K$ nuclei of deuterium are broken apart as soon as they are formed and thus the process to produce helium cannot take place. The nuclear balance has now shifted to 17-83 neutrons-protons.

Fifth Frame. The temperature of the universe is $10^9 \,^\circ K$, which is only 70 times hotter than the center of the sun. Since the first frame 3 minutes and 2 seconds have elapsed. Electrons and positrons have mostly disappeared leaving photons and neutrinos. The energy released by electron-positron annihilations has given the photons a temperature 35% higher than that of the neutrinos. The universe is now cool enough for H^3 and He^3 and He^4 to hold together - the deuterium bottleneck, however is still effective and the heavier nuclei do not form as yet. Most particle collisions have ceased. Now however, the decay of the neutron (lifetime = 15 minutes) now starts to become important. In each 100 seconds, 10% of the remaining neutrons decay into protons. The nuclear balance has now shifted to 14-86 neutrons-protons.

A Little Later. Shortly after the fifth frame a dramatic event occurs - the temperature drops to a point at which deuterium nuclei hold together thus eliminating the bottleneck to the production of heavier nuclei. The production stop at helium however because of other bottleneck - there are no stable nuclei with 5 or 8 nuclear particles! At this point all the remaining neutrons are cooked into helium nuclei. For a density of 10^9 photons per nuclear particle, this nucleosynthesis will begin at a temperature of $9 \times 10^8 \,^\circ K$. Approximately 3 minutes and 46 seconds have passed. Neutron decay had shifted the nuclear balance to 13-87 neutrons-protons as nucleosynthesis begins. After nucleosynthesis, the fraction by weight of helium is just equal to the fraction of all nuclear particles that are bound into helium; half are neutrons and essentially all neutrons are bound into helium so that the fraction by weight of helium is twice the fraction of neutrons among nuclear particles or about 26%. If the nuclear particle density were a little higher, then nucleosynthesis would start earlier when not so many neutrons have decayed and thus more helium would have been produced (not more than 28% by weight). See the figure below.

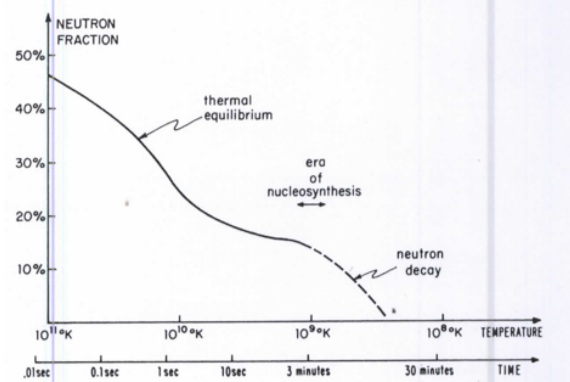


Figure 9. *The Shifting Neutron-Proton Balance.* The fraction of neutrons to all nuclear particles is shown as a function both of temperature and of time. The part of the curve marked "thermal equilibrium" describes the period in which densities and temperature are so high that thermal equilibrium is maintained among all particles; the neutron fraction here can be calculated from the neutron-proton mass difference, using the rules of statistical mechanics. The part of the curve marked "neutron decay" describes the period in which all neutron-proton conversion processes have ceased, except for the radioactive decay of the free neutron. The intervening part of the curve depends on detailed calculations of weak-interaction transition rates. The dashed part of the curve shows what would happen if nuclei were somehow prevented from forming. Actually, at a time somewhere within the period indicated by the arrow marked "era of nucleosynthesis," neutrons are rapidly assembled into helium nuclei, and the neutron-proton ratio is frozen at the value it has at that time. This curve can also be used to estimate the fraction (by weight) of cosmologically produced helium: for any given value of the temperature or the time of nucleosynthesis, it is just twice the neutron fraction at that time.

What now happens in the next frame?

Sixth Frame. The temperature of the universe is $3 \times 10^8\,^\circ K$. Since the first frame 34 minutes and 40 seconds have elapsed. Electrons and positrons are almost completely annihilated except for a small (one part in 10^9) excess of electrons needed to balance the charge of the protons. The energy released by electron-positron annihilations has given the photons a temperature 40.1% higher than that of the neutrinos. The total energy density has continued to decrease to 9.9% that of water. Of this 31% is in the form of neutrinos and 69% in form of photons. The characteristic expansion time is now $1 - 1/4$ hours. Nuclear processes have stopped - the nuclear particles are now either bound in helium nuclei or are free protons (hydrogen nuclei) with about $22 - 28\%$ by weight. The universe is still too hot for stable atoms to form.

The universe continues to expand and cool without any significant events for about $700000\,years$. At that time the temperature will have dropped enough that electrons and nuclei can form stable atoms. The disappearance of free electrons makes the universe transparent to radiation. The decoupling or radiation and matter now allows matter to begin to form stars and galaxies under the influence of gravity.

That is the theory - any theory must be testable by experimental observations. The $22 - 28\%$ by weight of helium has been confirmed not only in stars but in the universe as a whole. On the other hand, the abundance of heavier elements does vary dramatically from place to place. This is what is expected if the heavy elements were produced in stars, but helium produced in early universe before any stars existed. Small traces of deuterium have also been found. Deuterium abundance is very sensitive to the density of nuclear particles at the time of nucleosynthesis - higher densities make nuclear reactions proceed faster so all deuterium would have been cooked into helium. Calculation show that the abundance of deuterium (by weight) produced in the early universe is as shown in the table below.

Photons/nuclear particle	Deuterium abundance(parts per million)
10^8	0.00008
10^9	16
10^{10}	600

If we could determine the primordial deuterium abundance that existed before stellar cooking began we could make a precise determination of the photon-to-nuclear particle ratio. Knowing the present radiation temperature of $3\,^{\circ}K$, we could determine a precise value for the present nuclear mass density of the universe and decide whether it is open or closed.

Satellite observations indicates deuterium abundance of 20 parts per million and that leads to a present density value of 500 nuclear particles per million liters. This is very much less than the critical density for a closed universe. - therefore the universe is open and will expand forever. We will have more to say about this matter latter on - beyond what Weinberg knew about!

Some of the conclusions that Weinberg has drawn in his account of the first three minutes must seem very tenuous and based on flimsy evidence at best. But that is the way physics progresses - keep an open mind - try all kinds of theories based on one's best assumptions - test the consequences - continue until get a better and better theory. One must put forward all kinds of ideas in order to find the right ones in the end. As we will see shortly the model has its flaws and new experiments will lead to changes that make a better theory based on the old one.

Chapter 6

A Historical Diversion

Just read this historical material on your own.

Chapter 7

The First One-Hundredth Second

Here we now substitute a more modern approach due to Alan Guth.

Chapter 8

What Lies Ahead

Here we now substitute a more modern approach due to Alan Guth.