Physics 14 Quantum Physics Assignment #13 Spring 2005

Readings for week #9 of Quantum Mechanics

Chapter 6 - Boccio Textbook
Chapters 6,7 in French

Extra Problems

EP-7. Polaroids - Imagine a situation in which a photon in the $|x\rangle$ state strikes a vertical polaroid. Clearly the probability of the photon getting through the vertical polaroid is 0. Now consider the case of two polaroids with the photon in the $|x\rangle$ state striking a polaroid at $45°$ and then striking a vertical polaroid.

(a) Show that the probability of the photon getting through both polaroids is 1/4.

Consider now the case of three polaroids with the photon in the $|x\rangle$ state striking a polaroid at $30°$ first, then a polaroid at $60°$ and finally a vertical polaroid.

(b) Show that the probability of the photon getting through all three polaroids is 27/64.

EP-8. Calcite - A photon is polarized at an angle θ to the optic axis is sent in the z direction through a slab of calcite $10^{-2}\,cm$ thick in the z direction. Assume that the optic axis lies in the x-y plane. Calculate, as a function of θ, the transition probability for the photon to emerge left circularly polarized. Sketch the result. Let the frequency of the light be given by $c/\omega = 5000\,\mathring{A}$, and let $n_e = 1.50$ and $n_o = 1.65$ for the calcite.

EP-9. Turpentine - Turpentine is an "optically active" substance. If we send plane polarized light into turpentine then it emerges with its plane of polarization rotated. Specifically, turpentine induces a left-hand rotation of about $5°$ per cm of turpentine that the light traverses. Write down the transition matrix that relates the incident polarization state to the emergent polarization state. Show that this matrix is unitary. Why is that important? Find its eigenvectors and eigenvalues, as a function of the length of turpentine traversed.

EP-10. What QM is all about - Photons polarized at $30°$ to the x-axis are sent through a y-polaroid. An attempt is made to determine how frequently the photons that pass through the polaroid, pass through as "right circularly polarized photons" and how frequently they pass through as "left circularly polarized photons". This attempt is made as follows:

First, a prism that passes only right circularly polarized light is placed between the source of the $30°$ polarized photons and the y-polaroid, and it is determined how frequently the $30°$ photons pass through the y-polaroid. Then this experiment is repeated with a prism that passes only left circularly polarized photons instead of the one that passes only right.

Show by explicit calculation that the sum of the probabilities for passing through the y-polaroid measured in these two experiments is different from the probability that one would measure if there were no prism in the path of the photon and only the y-polaroid.

Relate this experiment to the two-slit diffraction experiment.

EP-11. Photons and polarizers

A photon polarization state for a photon propagating in the z-direction is given by

$$|\psi\rangle = \sqrt{\frac{2}{3}}|x\rangle + \frac{i}{\sqrt{3}}|y\rangle$$

(a) What is the probability that a photon in this state will pass through a polaroid with its transmission axis oriented in the y-direction?

(b) What is the probability that a photon in this state will pass through a polaroid with its transmission axis y' making an angle ϕ with the y-axis ?

(c) A beam carrying N photons per second, each in the state $|\psi\rangle$, is totally absorbed by a black disk with its normal to the surface in the z-direction. How large is the torque exerted on the disk? In which direction does the disk rotate? REMINDER: The photon states $|R\rangle$ and $|L\rangle$ each carry a unit $\hbar$ of angular momentum parallel and antiparallel, respectively, to the direction of propagation of the photon.

EP-12. Time evolution

The matrix representation of the Hamiltonian for a photon propagating along the optic axis (taken to be the z-axis) of a quartz crystal using the linear polarization states $|x\rangle$ and $|y\rangle$ as a basis is given by

$$\hat{H} = \begin{pmatrix} 0 & -iE_0 \\ iE_0 & 0 \end{pmatrix}$$

(a) What are the eigenstates and eigenvalues of the Hamiltonian?
(b) A photon enters the crystal linearly polarized in the x direction, that is, $|\psi(0)\rangle = |x\rangle$. What is $|\psi(t)\rangle$, the state of the photon at time t ? Express your answer in the $\{|x\rangle, |y\rangle\}$ basis.
(c) What is happening to the polarization of the photon as it travels through the crystal?

EP-13 - Time development redux

Consider an energy operator $\hat{H}$ with three eigenvectors given by the equations

$$\hat{H}|E=+10\rangle = +10|E=+10\rangle$$

$$\hat{H}|E=-10\rangle = -10|E=-10\rangle$$

where $\{|E=+10\rangle, |E=-10\rangle\}$ form an orthonormal basis.

Now suppose that we are investigating some physical system and want to predict future value of a "spin" operator $\hat{S}$ with eigenvectors

$$\hat{S}|S=+1\rangle = \hat{S}\left(\frac{1}{\sqrt{2}}\left[|E=+10\rangle + |E=-10\rangle\right]\right) = +\left(\frac{1}{\sqrt{2}}\left[|E=+10\rangle + |E=-10\rangle\right]\right)$$
$$\hat{S}|S=-1\rangle = \hat{S}\left(\frac{1}{\sqrt{2}}\left[|E=+10\rangle - |E=-10\rangle\right]\right) = -\left(\frac{1}{\sqrt{2}}\left[|E=+10\rangle - |E=-10\rangle\right]\right)$$

where the eigenvectors of the "spin" operator have been written in the "energy" basis.

Suppose, in addition, that a physical system is initially prepared to be in the state

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}\left[|E=+10\rangle + |E=-10\rangle\right] = |S=1\rangle$$

that is, in an eigenvector of the "spin" operator.

(a) Is this state an eigenvector of the energy operator?

What is the probability that we will measure energy values $+10 \; or \; -10$ in this initial state? What is the probability that we will measure spin values $+1 \; or \; -1$ in this initial state ?

The time development operator is given by

$$\hat{T} = e^{-i\hat{H}t/\hbar}$$

(b) Determine

$$\hat{T}|E=+10\rangle = ? \qquad \hat{T}|S=+1\rangle = ?$$
$$\hat{T}|E=-10\rangle = ? \qquad \hat{T}|S=-1\rangle = ?$$

(c) Using (b) determine $|\psi(t)\rangle = \hat{T}|\psi(0)\rangle$ = the state of the system at a later time.

What is the probability that we will measure spin values $+1 \; or \; -1$ at this later time? You must calculate the quantities

$$P(S=1;t) = \left|\langle S=1|\psi(t)\rangle\right|^2 = \left|\langle S=1|\hat{T}|\psi(0)\rangle\right|^2$$
$$P(S=-1;t) = \left|\langle S=-1|\psi(t)\rangle\right|^2 = \left|\langle S=-1|\hat{T}|\psi(0)\rangle\right|^2$$

Will the spin value ever "flip" from $S=+1$ to $S=-1$?