Physics 14 Quantum Physics Assignment #8 Spring 2005

Readings for week #4 of Quantum Mechanics

Pages 85-112 in Boccio Wave Mechanics Notes
Chapter 3 - pp 105-123 in French Textbook

Exercises #11, #12, #13, #14, #15, and #16(end of Lecture Notes)

[11] Let $A(\vec{x},\vec{p}) = \sum_{\alpha=1}^{N} f_\alpha(\vec{x}) g_\alpha(\vec{p})$ be some physical quantity where $f_\alpha(\vec{x})$ and $g_\alpha(\vec{p})$ are **real** functions of $\vec{x}$ and $\vec{p}$. Show that

$$A_{op} = \frac{1}{2}\sum_{\alpha=1}^{N}\left[f_\alpha(\vec{x}) g_\alpha(\vec{p}_{op}) + g_\alpha(\vec{p}_{op}) f_\alpha(\vec{x})\right]$$

is an acceptable hermitian operator corresponding to $A(\vec{x},\vec{p})$.

[12] Using $\left[x_i, x_j\right] = 0 = \left[p_{iop}, p_{jop}\right]$ and $\left[x_i, p_{jop}\right] = i\hbar\delta_{ij}$

(a) Show that $\left[L_{xop}, L_{yop}\right] = i\hbar L_{zop}$, $\left[L_{yop}, L_{zop}\right] = i\hbar L_{xop}$, $\left[L_{zop}, L_{xop}\right] = i\hbar L_{yop}$

(b) Find $\left[\vec{L}_{op}^2, L_{iop}\right]$ for $i = 1,2,3$ where $\vec{L}_{op}^2 = L_{xop}^2 + L_{yop}^2 + L_{zop}^2$

(c) Find $\left[L_{zop}, x_i\right], \left[L_{zop}, p_{iop}\right]$ for $i = 1,2,3$

[13] Let $\psi(\vec{x},t_0) = \psi_0(\vec{x})$ be an eigenfunction of L_z with eigenvalue $\ell\hbar$.

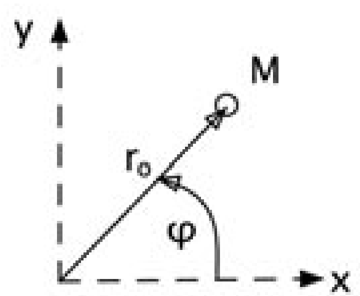
(a) Find $\langle x\rangle, \langle y\rangle, \langle p_x\rangle, \langle p_y\rangle$ at time t_0

(b) Show that $\left\langle L_x^2\right\rangle = \left\langle L_y^2\right\rangle$ at time t_0. HINT: consider $L_x\left[L_y, L_z\right]$ and $\left[L_z, L_y\right]L_x$

(c) Show that neither (ΔL_x) nor (ΔL_y) can be zero at t_0 for $\ell \neq 0$

(d) Show that $\left\langle \vec{L}\cdot\vec{L}\right\rangle = \left\langle L_x^2 + L_y^2 + L_z^2\right\rangle \geq \ell(\ell+1)\hbar^2$

[14] **Rigid Rotator**: Classical model - a particle of mass M lies in the $x-y$ plane and rotates about the origin at a fixed distance r_0 as shown in the figure:



We are therefore neglecting the zero-point vibrations in the r direction and in the z direction.

We have $E = \frac{1}{2}mv^2$ with $L_z = mvr_0$

so that $E = \frac{L_z^2}{2I}$ where $I = mr_0^2$ = constant moment of inertia

Therefore, $H_{op} = \frac{L_{zop}^2}{2I} = -\frac{\hbar^2}{2I}\frac{\partial^2}{\partial\varphi^2}$

(a) Find the energy eigenvalues and eigenfunctions
(b) Suppose the particle has a charge q. If energy is conserved, how can the particle make a transition from one energy level to another energy level?

[15] Consider 1-dimensional motion (x-axis). Let $A = x$. The eigenvalue equation is $x\varphi(x) = X\varphi(x)$, X = constant eigenvalue of position.

(a) Show that $u_X(x)$ and $u_{X'}(x)$ are orthogonal for $X \neq X'$. Show that the eigenfunctions are $u_X(x) = N_X\delta(x - X)$ with X any real in $[-\infty, +\infty]$. Determine N_X so that $u_X(x)$ is normalized to $\delta(X' - X)$
(b) Expand the wave function $\psi(x, t_0)$ in terms of $u_X(x)$ and comment
(c) Following the recipe in the notes, calculate $\wp(x, t_0)$ and compare with postulate 1.

[16] Consider a free particle (no forces present) moving in one-dimension (x-axis) with $H_{op} = \frac{p_{xop}^2}{2m}$.

(a) Find the eigenfunctions and the spectrum of H_{op}
(b) Normalize the eigenfunctions to $\delta(\sqrt{E} - \sqrt{E'})$

Textbook Problems:

3.06 - Energy of particles in the nucleus
3.10 - Normalization of a wave function

Extra Problems

EP-4 - Square Wave Packet

Consider a free particle, initially with a well-defined momentum p_0, whose wave function is well approximated by a plane wave. At $t = 0$, the particle is **localized** in a region $-a/2 \le x \le a/2$, so that its wave function is

$$\psi(x) = \begin{cases} Ae^{ip_0x/\hbar} & -a/2 \le x \le a/2 \\ 0 & otherwise \end{cases}$$

(a) Find the normalization constant A and sketch $\mathrm{Re}(\psi(x)), \mathrm{Im}(\psi(x)), |\psi(x)|^2$.
(b) Find the momentum space wave function $\tilde{\psi}(p)$ and show that it too is normalized.
(c) Find the uncertainties Δx and Δp at this time. How close is this to the minimum uncertainty wave function.