E/M Experiment: Electrons in a Magnetic Field.

PRE-LAB

You will be doing this experiment before we cover the relevant material in class. But there are only two fundamental concepts that you need to understand. First, moving charges, which could be due to a current in a wire, create magnetic fields. For a current I going around a circular loop of wire of radius R, the strength of the magnetic field along the axis of the circular loop (z = 0 at the center of the circular loop and is positive above the loop and negative below the loop) is given by

$$B_z = \frac{\mu_0 I}{2} \frac{R^2}{\left(R^2 + z^2\right)^{3/2}}.$$

This equation assumes SI units (for cgs units, see Eq. (41) of Chapter 6 in Purcell), so the current is in amperes, distances are in meters, and the magnetic field is in tesla (T). The constant $\mu_0 = 4\pi \times 10^{-7}$ N/A^2. Notice that along the axis of the circular loop, the magnetic field is parallel to the axis. Its relationship to the current in the circular loop is given by a right hand rule. Curl the figures of your right hand around the circular loop so they point in the direction of the current; your thumb then gives the direction of the magnetic field along the axis of the circular loop. If instead of a single circular loop there are N turns of a coil in the form of a circular loop, then the magnetic field is simply N times the magnetic field due to a single circular loop.

Pre-lab Question: Imagine two identical circular loop coils (N turns of radius R) carrying a current I in the same direction. These two coils are parallel to each other and separated by a distance L (see Fig. 2). Using superposition and the above relationship, find the magnetic field halfway between the two coils along the axis of the coils. Then set the distance L equal to R and verify Eq. (5).

The other fundamental concept is that a charge q moving with velocity $\vec{v}$ in an magnetic field $\vec{B}$ experiences a force given by (using SI units)

$$\vec{F} = q\vec{v} \times \vec{B}.$$

The corresponding equation using cgs units is Eq. (1) of Chapter 5 in Purcell. Notice the cross-product in the equation. This means that the force on the moving charge is perpendicular to both its velocity and the magnetic field. Also, if the charge is moving parallel to the magnetic field, there is no force.

I. INTRODUCTION

The discovery of the electron as a discrete particle carrying charge is credited to the British physicist J. J. Thomson (1856-1940). This work was the very beginning of the modern search for fundamental particles. His studies of cathode rays (streams of electrons) culminated in 1897 with his quantitative observations of the deflection of these rays in magnetic and electric fields. As we will find in this lab, this deflection provides a key to finding the value of *e/m*. Later, Robert Millikan (1868-1953) was able to measure the charge of the electron. Thus, these two experiments determine the mass of the electron. Thomson's work also formed the basis of the mass spectrometer, which was further developed by Al Neir.

Our experiment is a descendant of J. J. Thomson's original experiment; however, we will study the deflection of electrons in a magnetic field only, rather than both electric and magnetic fields. The basic experiment consists of a beam of electrons that are accelerated from rest through a potential difference *V* so that their final (non-relativistic) velocity can be determined from energy conservation:

$$\tfrac{1}{2}\mathrm{m}\mathrm{v}^2 = \mathrm{eV} \,. \qquad (1)$$

This is accomplished by heating a metal filament to a sufficiently high temperature that electrons "boil off" from the surface, and accelerating these electrons to the inner surface of the conducting "can" that surrounds it (Fig. 1). Some of the electrons pass through a slit in the can.

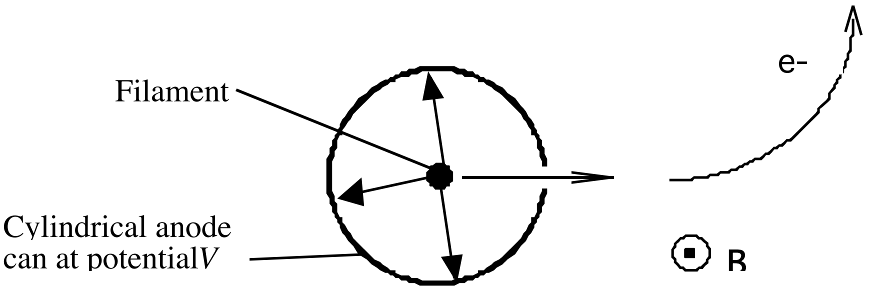

Figure 1. Electrons are accelerated through a potential difference *V* (the filament is grounded) and guided into circular motion by the magnetic field **B** (pointing out of the page).

After passing through the slit, the electrons enter a region of uniform magnetic field **B**. In Fig. 1 the **B** field is pointing out of the page. The force on the electron is given by the Lorentz equation

$$\mathbf{F} = -e\,\mathbf{v} \times \mathbf{B}. \qquad (2)$$

If both **F** and **v** are in the plane perpendicular to **B**, motion will be confined to that plane. The resulting motion is circular with radius of curvature

$$r = m_e v/eB. \qquad (3)$$

From Eqs. 1 and 3 we can solve for e/m_e:

$$\frac{e}{m_e} = \frac{2V}{B^2 r^2}\,. \qquad (4)$$

This result assumes that the magnetic field is uniform; to produce such a field, you will use a pair of current-carrying coils, each containing N loops and carrying current *I.* The coils are arranged as shown in Fig. 2 on a common axis so that the coil radius equals the separation between the coils ($R = L$). Two such coils are sometimes called Helmholtz coils.

Using the Biot-Savart Law one can show that the total magnetic field at the midpoint of the axis between the coils is directed along the axis of the coils and has magnitude

$$B = \frac{8\mu_0 N I}{R\sqrt{125}}\,. \qquad (5)$$

Equation 5 allows us to determine the value of B for the apparatus. We can therefore use Eq. (4) to calculate the charge-to-mass ratio for the electron. If we happen to know e from some other source (like Millikan's oil drop experiment), we can calculate the mass of a single electron!

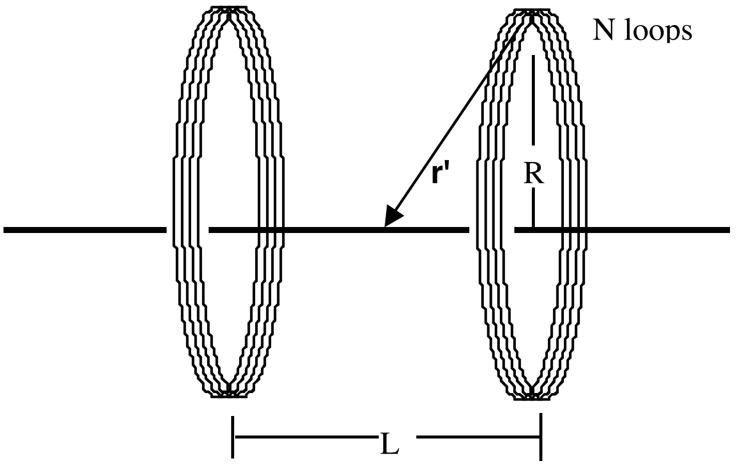

Fig. 2. Helmholtz coils. Each coil has N turns and radius R.

II. EXPERIMENTAL APPARATUS

The apparatus consists of a specialized vacuum tube inside a set of Helmholtz coils. One quick note applies: the beam that you will see does show the path that electrons travel in the vacuum tubes. However, keep in mind that we are not seeing the electrons themselves. There is a very small amount of helium gas in each tube. When electrons collide with the atoms of the gas, the gas glows. (If you want to know how **that** works, just ask your instructor.) So, you are actually seeing the glow of gas atoms that have been excited by collisions with electrons.

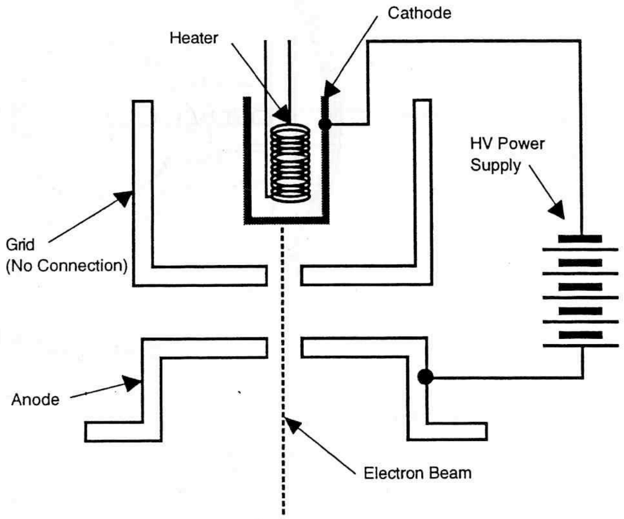

Figure 3. Circuit inside the e/m vacuum tube.

The circuit inside the electron beam vacuum tube is shown in Figure 3. You can see the heater filament for the cathode. As the electrons on the cathode are heated (energized), they become easier to remove from the cathode using a high accelerating voltage. So these electrons are drawn from the cathode through the grid slit towards the anode, where most are stopped by the anode plate. However, some pass through via a hole in the plate. This forms the electron beam which passes into the bulb.

A diagram of the entire apparatus is shown in Figure 4. The Helmholtz coils have **N = 130** turns and a radius *approximately* **R = 15.2 cm**. They provide an applied magnetic field perpendicular to the electron beam velocity. As a result, the electron beam follows a circular path. Within the tube, a glass scale indicates the ***diameter*** of the circular beam.

(1) The *heater current* is generated inside the apparatus to heat the cathode. It turns on when you turn on the unit. ***Leaving the heater current turned on for prolonged periods of time will shorten the life of the bulb. To save bulb lifetime, make sure you turn off the unit as soon as you finish your measurements.***

(2) The *accelerating voltage*, V, is generated by the high-voltage supply in the apparatus, and is controlled by the knob on the left-hand side of the front panel. The typical range is **V = 200 to 500 Volts**.

(3) The *Helmholtz coil current* is also generated by a power supply in the unit, and is controlled by the knob on the left-hand side of the front panel. The coil current should lie in the range **1 to 2.5 A**.

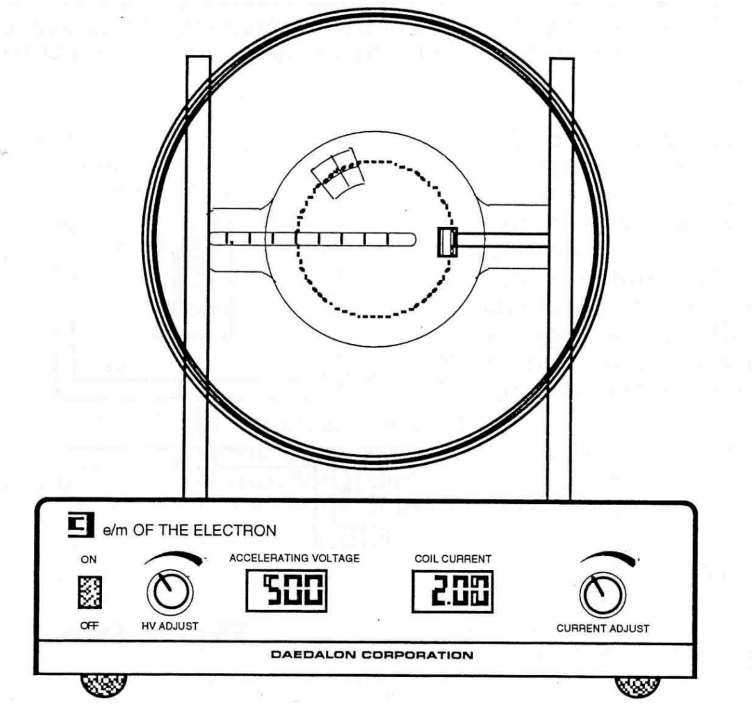

Figure 4. The e/m apparatus. The electrons emerge from the filament and accelerating voltage and curve in the presence of a magnetic field. Note the radius scale marker in the bulb. The axis of the Helmholtz coils is perpendicular to the page.

III. PROCEDURE

Turn on the power switch. The unit will perform a 30-second self-test, indicated by the digital display values changing rapidly. During the self-test, the controls are locked out, allowing the cathode to heat to the proper operating temperature. When the self-test is complete, the display will stabilize and show “000.” Although the unit is now ready for operation, a 5-10 minute warm-up time is recommended before taking careful measurements.

Turn the “Voltage Adjust” control (which provides the accelerating voltage) up to 200 V and observe the bottom of the electron gun. The bluish beam will be traveling straight down to the envelope of the tube.

Turn the “Current Adjust” control (which provides the current to the Helmholtz coils) up and observe the circular deflection of the beam. When the current is high enough, the beam travels in a complete circle within the bulb. The diameter of the beam can be measured using the centimeter scale markings inside the tube; the markings fluoresce when struck by the electron beam.

The experiment on our apparatus is slightly complicated due to the presence of Earth’s magnetic field. As this field is weak compared to the field generated by the Helmholtz coils, we could ignore this effect as a first approximation. However, let us take a moment to orient the apparatus and try to reduce the effect of the ambient field as much as we can.

Get a compass from your instructor and use it to locate geomagnetic North. Align the axis of the Helmholtz coils so it is parallel to the compass needle.

In your notebook, make a sketch of the relative positions of the electron beam and the geomagnetic field. What are the relative positions of the geomagnetic field and the field generated by the Helmholtz coils? Explain why orienting the coils parallel to the compass needle reduces the effect of the ambient field on your measurements.

Return the compass, and obtain a bar magnet from your instructor. Predict the direction of change of the electron beam if you hold up the magnet to the apparatus such that the north-south axis of the magnet is perpendicular to the electron beam. Test your prediction. Was your prediction correct? If not, can you explain the difference? Using the field demonstrators, observe the shape of the magnet's field. Does this help explain what you observed?

Return the magnet to your instructor. Now you are ready to collect some data!

For your accelerating voltage of 200V, measure the current in the Helmholtz coil for 8 fluorescent markings (8 beam diameters). Don't forget to include uncertainties on all your measurements! (The criteria you use to decide when the beam hits a fluorescent marking are a bit subjective, so the uncertainty in the coil current is higher than the accuracy implied by the current readout.)

Repeat your measurements using at least two more accelerating voltages in the range of 200V to 500V.

When all of your data have been collected, switch off the apparatus.

Measure the internal and external diameter of the Helmholtz coils on several axes. They may not be quite round and the two coils may not be quite the same. Average your measurements and determine the standard error.

IV. ANALYSIS

Create a KaleidaGraph table with the r values you used in the first column. The second column should be the corresponding I values for one accelerating voltage. The third column should be the corresponding I values for another accelerating voltage. Continue to create columns for each of the accelerating voltages you used. Then for each accelerating voltage, create a column of the corresponding B values.

Plot B vs. 1/r for all of your data. According to Eq. (4), your data should lie along multiple straight lines (one for each accelerating voltage used). Fit the data to a straight line for each accelerating voltage, and from the slope of each line determine the value of e/m_e. Include an error estimate for each value of e/m_e. It is instructive to make a column of error estimates to go along with each column of B values. You can then plot these as error bars to see if they are consistent with how far the data points are from the straight line.

Do your values agree with the accepted value of $(1.75881 \pm 0.00093) \times 10^{11}$ C/kg? If you had to come up with a single value for e/m_e based on your measurements, what would it be? Is it consistent with the accepted value? What were the largest sources of error? How might the experiment be improved in order to reduce the uncertainty in the value you obtain for e/m_e? Does the presence of the earth’s magnetic field reveal itself in any way on your plot?

LAB NOTEBOOK CHECKLIST

You should have:

- table of your data
- final values for e/m_e with uncertainty
- plot of B vs. 1/r
- sketch of apparatus alignment with geomagnetic field
- comments on agreement with accepted value, sources of error