Physics 008 **Problem Set #11** **Fall 2009**

My Lectures from : Purcell Chapter 9
Web Notes : Lecture Notes #8 and #9
Other Notes: EM_Waves

Purcell Problems:

9.01 What is magnetic field?
9.02 rms magnetic field strength
9.03 Where is the proton - electric field only?
9.04 Where is the proton - magnetic field also?
9.05 Satisfy Maxwell's equations 1
9.06 Satisfy Maxwell's equations 2
9.07 Plane EM wave

1. A spherical wave - Let the electric field be given by

$$\vec{E}(r,\theta,\phi,t) = A\frac{\sin\theta}{r}\left[\cos(kr-\omega t) - \frac{1}{kr}\sin(kr-\omega t)\right]\hat{\phi}$$

where $\omega / k = c$.

(a) Show that $\vec{E}$ obeys all four of Maxwell's equations in vacuum, and find the associated magnetic field.

(b) Calculate the Poynting vector. Avergage $\vec{S}$ over a full cycle to get the intensity vector $\vec{I}$.

(c) Integrate $\vec{I}\cdot d\vec{a}$ over a spherical surface to determine the total power radiated.

2. Discovery of magnetic charge - This problem will explore some of the similarities between electric and magnetic fields. Let us say that magnetic charge has been discovered, called magnetic monopoles. We will work in a system where the units of magnetic charge density μ are chosen so that $\nabla\cdot\vec{B} = 4\pi\mu$.

(a) When the monopole is in motion, there is a magnetic current density $\vec{L} = \mu\vec{v}$ analogous to the electric current density.

Write down the continuity equation for magnetic charge in differential form.

(b) Write down the new Maxwell's equations in differential form, including the effects of these monopoles.

3. Pair of electric and magnetic fields - A pair of electric and magnetic fields are given by:

$$\vec{E} = E_0 \cos(\alpha y - \gamma z + \delta t)\hat{x}$$

and

$$\vec{B} = B_0 \cos(\alpha y - \gamma z + \delta t)(\hat{y} + \hat{z})$$

By substituting into Maxwell's equations in vacuum, derive the conditions that the constants α, γ, δ, E_0, and B_0 must obey to satisfy them. Is this a legitimate electromagnetic wave? Why?