Physics 130 General Relativity Seminar

Assignment 1 January 21, 2013

General topic: **Getting Started with Geometry and Review of Past**

General relativity is a theory of gravitation and to understand the background to the theory we have to look at how theories of gravitation developed. Aristotle's notion of the motion of bodies impeded understanding of gravitation for a long time. He believed that force could only be applied by contact; force at a distance being impossible, and a constant force was required to maintain a body in uniform motion.

Copernicus's view of the solar system was important as it allowed sensible consideration of gravitation. Kepler's laws of planetary motion and Galileo's understanding of the motion and falling bodies set the scene for Newton's theory of gravity which was presented in the *Principia* in 1687. Newton's law of gravitation is expressed by

$$F = G\frac{M_1 M_2}{d^2}$$

where F is the force between the bodies of masses M_1, M_2 and d is the distance between them. G is the universal gravitational constant.

After receiving their definitive analytic form from Euler, Newton's axioms of motion were reworked by Lagrange, Hamilton, and Jacobi into very powerful and general methods, which employed new analytic quantities, such as potential, related to force but remote from everyday experience. Newton's universal gravitation was considered proved correct, thanks to the work of Clairaut and Laplace. Laplace looked at the stability of the solar system in 1799. In fact the so-called three-body problem was extensively studied in the 19th Century and was not properly understood until much later. The study of the gravitational potential allowed variations in gravitation caused by irregularities in the shape of the earth to be studied both practically and theoretically. Poisson used the gravitational potential approach to give an equation which, unlike Newton's, could be solved under rather general conditions.

Newton's theory of gravitation was highly successful. There was little reason to question it except for one weakness which was to explain how each of the two bodies knew the other was there. Some profound remarks about gravitation were made by Maxwell in 1864. His major work *A dynamical theory of the electromagnetic field* (1864) was written

> *... to explain the electromagnetic action between distant bodies without assuming the existence of forces capable of acting directly at sensible distances.*

At the end of the work Maxwell comments on gravitation.

> *After tracing to the action of the surrounding medium both the magnetic and the electric attractions and repulsions, and finding them to depend on the inverse square of the distance, we are naturally led to inquire whether the attraction of gravitation, which follows the same law of the distance, is not also traceable to the action of a surrounding medium.*

However Maxwell notes that there is a paradox caused by the attraction of like bodies. The energy of the medium must be decreased by the presence of the bodies and Maxwell said

> *As I am unable to understand in what way a medium can possess such properties, I cannot go further in this direction in searching for the cause of gravitation.*

In 1900 Lorentz conjectured that gravitation could be attributed to actions which propagate with the velocity of light. Poincar, in a paper in July 1905 (submitted days before Einstein's special relativity paper), suggested that all forces should transform according the Lorentz transformations. In this case he notes that Newton's law of gravitation is not valid and proposed gravitational waves which propagated with the velocity of light.

In 1907, two years after proposing the special theory of relativity, Einstein was preparing a review of special relativity when he suddenly wondered how Newtonian gravitation would have to be modified to fit in with special relativity. At this point there occurred to Einstein, described by him as the *happiest thought of my life*, namely that an observer who is falling from the roof of a house experiences no gravitational field. He proposed the *Equivalence Principle* as a consequence:-

… we shall therefore assume the complete physical equivalence of a gravitational field and the corresponding acceleration of the reference frame. This assumption extends the principle of relativity to the case of uniformly accelerated motion of the reference frame.

After the major step of the equivalence principle in 1907, Einstein published nothing further on gravitation until 1911. Then he realized that the bending of light in a gravitational field, which he knew in 1907 was a consequence of the equivalence principle, could be checked with astronomical observations. He had only thought in 1907 in terms of terrestrial observations where there seemed little chance of experimental verification. Also discussed at this time is the gravitational redshift, light leaving a massive body will be shifted towards the red by the energy loss of escaping the gravitational field.

Einstein published further papers on gravitation in 1912. In these he realized that the Lorentz transformations will not apply in this more general setting. Einstein also realized that the gravitational field equations were bound to be non-linear and the equivalence principle appeared to only hold locally.

This work by Einstein prompted others to produce gravitational theories. Work by Nordström, Abraham and Mie was all a consequence of Einstein's, so far failed, attempts to find a satisfactory theory. However Einstein realized his problems.

If all accelerated systems are equivalent, then Euclidean geometry cannot hold in all of them.

Einstein then remembered that he had studied Gauss's theory of surfaces as a student and suddenly realised that the foundations of geometry have physical significance. He consulted his friend Grossmann who was able to tell Einstein of the important developments of Riemann, Ricci (Ricci-Curbastro) and Levi-Civita. Einstein wrote

… in all my life I have not laboured nearly so hard, and I have become imbued with great respect for mathematics, the subtler part of which I had in my simple-mindedness regarded as pure luxury until now.

In 1913 Einstein and Grossmann published a joint paper where the tensor calculus of Ricci and Levi-Civita is employed to make further advances. Grossmann gave Einstein the Riemann-Christoffel tensor which, together with the Ricci tensor which can be derived from it, were to become the major tools in the future theory. Progress was being made in that gravitation was described for the first time by the metric tensor but still the theory was not right. When Planck visited Einstein in 1913 and Einstein told him the present state of his theories Planck said

> *As an older friend I must advise you against it for in the first place you will not succeed, and even if you succeed no one will believe you.*

Planck was wrong, but only just, for when Einstein was to succeed with his theory it was not readily accepted. It was the second half of 1915 that saw Einstein finally put the theory in place. Before that however he had written a paper in October 1914 nearly half of which is a treatise on tensor analysis and differential geometry. This paper led to a correspondence between Einstein and Levi-Civita in which Levi-Civita pointed out technical errors in Einstein's work on tensors. Einstein was delighted to be able to exchange ideas with Levi-Civita whom he found much more sympathetic to his ideas on relativity than his other colleagues.

At the end of June 1915 Einstein spent a week at Göttingen where he lectured for six 2 hour sessions on his (incorrect) October 1914 version of general relativity. Hilbert and Klein attended his lectures and Einstein commented after leaving Göttingen

> *To my great joy, I succeeded in convincing Hilbert and Klein completely.*

The final steps to the theory of general relativity were taken by Einstein and Hilbert at almost the same time. Both had recognized flaws in Einstein's October 1914 work and a correspondence between the two men took place in November 1915. How much they learnt from each other is hard to measure but the fact that they both discovered the same final form of the gravitational field equations within days of each other must indicate that their exchange of ideas was helpful.

On the 18th November he made a discovery about which he wrote For a few days I was beside myself with joyous excitement . The problem involved the advance of the perihelion of the planet Mercury. Le Verrier, in 1859, had noted that the perihelion (the point where the planet is closest to the sun) advanced by 38" per century more than could be accounted for from other causes. Many possible solutions were proposed, Venus was 10% heavier than was thought, there was another planet inside Mercury's orbit, the sun was more oblate than observed, Mercury had a moon and, really the only one not ruled out by experiment, that Newton's inverse square law was incorrect. This last possibility would replace the $1/d^2$ by $1/d^p$, where $p = 2 + \epsilon$ for some very small number ϵ. By 1882 the advance was more accurately known, 43" per century. From 1911 Einstein had realized the importance of astronomical observations to his theories and he had worked with Freundlich to make measurements of Mercury's orbit required to confirm the general theory of relativity. Freundlich confirmed 43" per century in a paper of 1913. Einstein applied his theory of gravitation and discovered that the advance of 43" per century was exactly accounted for without any need to postulate invisible moons or any other special hypothesis. Of course Einstein's 18 November paper still does not have the correct field equations but this did not affect the particular calculation regarding Mercury. Freundlich attempted other tests of general relativity based on gravitational redshift, but they were inconclusive.

Also in the 18 November paper Einstein discovered that the bending of light was out by a factor of 2 in his 1911 work, giving 1.74". In fact after many failed attempts (due to cloud, war, incompetence etc.) to measure the deflection, two British expeditions in 1919 were to confirm Einstein's prediction by obtaining 1.98" ± 0.30" and 1.61" ± 0.30".

On 25 November Einstein submitted his paper The *field equations of gravitation* which give the correct field equations for general relativity. The calculation of bending of light and the advance of Mercury's perihelion remained as he had calculated it one week earlier.

Five days before Einstein submitted his 25 November paper Hilbert had submitted a paper The foundations of physics which also contained the correct field equations for gravitation. Hilbert's paper contains some important contributions to relativity not found in Einstein's work. Hilbert applied the variational principle to gravitation and attributed one of the main theorem's concerning identities that arise to Emmy Noether who was in Göttingen in 1915. No proof of the theorem is given. Hilbert's paper contains the hope that his work will lead to the unification of gravitation and electromagnetism.

In fact Emmy Noether's theorem was published with a proof in 1918 in a paper which she wrote under her own name. This theorem has become a vital tool in theoretical physics. A special case of Emmy Noether's theorem was written down by Weyl in 1917 when he derived from it identities which, it was later realized, had been independently discovered by Ricci in 1889 and by Bianchi (a pupil of Klein) in 1902.

Immediately after Einstein's 1915 paper giving the correct field equations, Karl Schwarzschild found in 1916 a mathematical solution to the equations which corresponds to the gravitational field of a massive compact object. At the time this was purely theoretical work but, of course, work on neutron stars, pulsars and black holes relied entirely on Schwarzschild's solutions and has made this part of the most important work going on in astronomy today.

Einstein had reached the final version of general relativity after a slow road with progress but many errors along the way. In December 1915 he said of himself

> *That fellow Einstein suits his convenience. Every year he retracts what he wrote the year before.*

Most of Einstein's colleagues were at a loss to understand the quick succession of papers, each correcting, modifying and extending what had been done earlier. In December 1915 Ehrenfest wrote to Lorentz referring to *the theory of November 25, 1915.* Ehrenfest and Lorentz corresponded about the general theory of relativity for two months as they tried to understand it. Eventually Lorentz understood the theory and wrote to Ehrenfest saying *I have congratulated Einstein on his brilliant results.* Ehrenfest responded

> *Your remark "I have congratulated Einstein on his brilliant results" has a similar meaning for me as when one Freemason recognizes another by a secret sign.*

In March 1916 Einstein completed an article explaining general relativity in terms more easily understood. The article was well received and he then wrote another article on relativity which was widely read and went through over 20 printings.

Today relativity plays a role in many areas, cosmology, the big bang theory etc. and now has been checked by experiment to a high degree of accuracy.

As we will see the main features of General Relativity are:

1. Space and space-time are not rigid arenas in which events take place. They have form and structure which are influenced by the matter and energy content of the universe.

2. Matter and energy tell space (and space-time) how to curve.

3. Space tells matter how to move. In particular small objects travel along the straightest possible lines in curved space (space-time).

In curved space the rules of Euclidean geometry are changed. Parallel lines can meet, and the sum of the angles in a triangle can be more, or less than 180 degrees, depending on how space is curved. Einstein's theory gave a correct prediction for the perihelion shift of Mercury. It also explained why objects fall independent of their mass: they all follow the same straightest possible line in curved space-time. Finally, in Einstein's theory the instantaneous gravitational force is replaced by the curvature of spacetime. Moving a mass causes ripples to form in this curvature, and these ripples travel with the same speed as light. Thus, a distant mass would not feel any instantaneous change in the gravitational force, and special relativity is not violated.

Now to proceed with our study.

Part 1: **Readings** In case book has not arrived - Chapters 1-3 are on the website

http://chaos.swarthmore.edu/courses/Physics130_2013/index.html

Hartle: Ch 1 - Gravitational Physics

Hartle: Ch 2 - Geometry as Physics

Hartle: Ch 3 - Space, Time, and Gravity in Newtonian Physics

You must do the readings BEFORE attempting the problems in order to get a good grasp of the overall content of the week's material to be understood. A problem should then make you look more carefully at specific parts of the readings that are necessary for the solution of that particular problem!

Prior to discussing any problems, we will deal with any questions and/or discussion of the readings.

Admonition #1 Admonition #2

Doing homework assignments by yourself. Copying off some "smart friend" cheats the other students in the class, and it cheats you and your friend. Identical-looking assignments will be referred to me by the grader. You may discuss general physics principles behind the questions with other students – and I encourage you to participate in study groups.

Participating in class. Sitting there like a vege while other students think hard and bother to answer questions is parasitic, intellectually. Contribute.

Part 2: Problems

1. All problems will be discussed in seminar.
2. Random choice of presenter.
3. Quality/correctness of presentation = 50% Seminar grade.
4. If a problem is not solved by anyone, then it will be done in seminar.

Hartle Problems

1. Hartle 2.05 Area of a circle on a spherical surface.
2. Hartle 2.06 Angles on a spherical surface.
3. Hartle 2.07 A new set of coordinates
4. Hartle 2.08 Surface of an egg.
5. Hartle 2.11 Conical projections
6. Hartle 3.03 Gravitational potential for solid sphere

7. Hartle 3.04 Least action.
8. Hartle 3.05 Gravitational self-energy

Boccio Extra Problems

1. **Two Giant Frogs**

   Two giant frogs are captured, imprisoned in a large metal cylinder, and placed on an airplane. While in flight, the storage doors accidentally open and the cylinder containing the frogs falls out. Sensing something amiss, the frogs decide to try to break out. Centering themselves in the cylinder, they push off from each other and slam simultaneously into the ends of the cylinder. They instantly push off from the ends and shoot across the cylinder past each other into the opposite ends. This continues until the cylinder hits the ground. Consider how this looks from some other inertial frame, falling at another speed. In this frame, the frogs do not hit the ends hit the ends of the cylinder simultaneously, so the cylinder jerks back and forth about its mean speed. The cylinder, however, was at rest in one inertial frame. Does this mean that one inertial frame can jerk back and forth with respect to another?

2. **Newtonian Gravity**

   Poissons formulation of Newtonian gravity is

   $$\nabla^2\varphi = 4\pi\rho \quad , \quad \vec{g} = -\nabla\varphi$$

   where ρ is the matter density, φ is the gravitational potential and $\vec{g}$ is the acceleration due to gravity. Show that this gives the usual Newtonian formula for a point-like source.

3. **Lagrange Equations for Kepler Orbits**

   Use Lagrange equations to solve the problem of Kepler planetary orbits in a gravitational field. Work in 3 dimensions in spherical coordinates. Determine the orbital equation $r(\theta)$.

Part 3: LaTeX Writeups

Random choice of writers announced at end of seminar.

Turn in by email to instructor BEFORE next seminar.

Part 4: Grading

All grades based on weekly seminars work (as in old-style seminars).
No exams.