Physics 130 General Relativity Seminar

Assignment 9 March 25, 2013

General topic: **Curved Spacetimes in General Relativity**

Part 1: Readings

Hartle: Ch 15 Rotating Black Holes

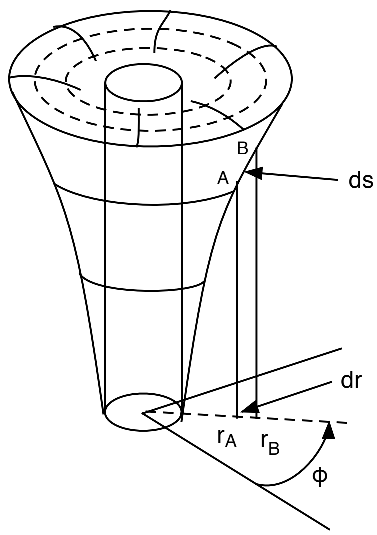
Part 2: Problems Hartle Problems

1. Hartle 15.03 Get rid of singularities
2. Hartle 15.04 Where do future-directed light rays go?
3. Hartle 15.05 Stay on the horizon?
4. Hartle 15.07 Compare distances
5. Hartle 15.10 Cross and never return!
6. Hartle 15.12 Angular velocity in circular orbit
7. Hartle 15.13 Which orbit is bigger?
8. Hartle 15.14 Range of angular velocities in the ergosphere(discuss)

Boccio Extra Problems

1. **Meter Stick Near Black Hole**

   A standard meter stick lies on the surface shown below(AB). The surface is the two-dimensional riemannian surface defined by the Schwarzschild metric with two coordinates held constant (t = constant, $\theta = \pi/2$ = constant) as viewed(embedded) in three-dimensional euclidean space. The meter stick is oriented in the radial direction. It is then slid inward toward the symmetry axis. The location of its two ends at a given instant in t are reported to a record keeper who plots the the two points shown in the (r, ϕ) plane.



(a) What does the record keeper actually see?

(b) How is the record keeper able to keep track of what is happening physically?

2. **Strange Metric**

   Write down the geodesic equations for the metric

$$ds^2 = dudv + \log(x^2 + y^2)du^2 - dx^2 - dy^2$$

   $(0 < x^2 + y^2 < 1)$. Show that $K = x\dot{y} - y\dot{x}$ is a constant of the motion.

   By considering an equivalent problem in Newtonian mechanics, show that no geodesic on which $K \neq 0$ can reach $x^2 + y^2 = 0$.

3. **Rotating Frames**

   The line element of flat spacetime in a frame (t, x, y, z) that is rotating with an angular velocity Ω about the $z-$axis of an inertial frame is

$$ds^2 = -[1 - \Omega^2(x^2 + y^2)]dt^2 + 2\Omega(ydx - xdy)dt + dx^2 + dy^2 + dz^2$$

   (a) Find the geodesic equations for x, y, and z in the rotating frame.

(b) Show that in the non-relativistic limit these reduce to the usual equations of Newtonian mechanics for a free particle in a rotating frame exhibiting the centrifugal force and the Coriolis force.

4. **Negative Mass**

Negative mass does not occur in nature. But just as an exercise analyze the behavior of radial light rays in a Schwarzschild geometry with a negative value of mass M. Sketch the Eddington-Finkelstein diagram showing these light rays. Is the negative mass Schwarzschild geometry a black hole?