Physics 50 **Assignment #9** **Spring 2010**

Readings: Riley, Hobson and Bence - Chapter 16
Boccio - 011_ODEs, 012_ODEs and 013_ODEs

Problems:

16.01 Series Solution

EP-5 For the ODE

$$36x^2 \frac{d^2y}{dx^2} + (5 - 9x^2)y = 0$$

use the method of **Frobenius** to find <u>both</u> homogeneous solutions.

EP-6 Solve the ODE for a general solution using the power-series method (non-singular equations).

(b) $(x^2 - 1)u'' - 4u = 0$

(c) $(4 - x^2)u'' + 2u = 0 \quad , \quad u(0) = 0, u'(0) = 1$

EP-7 Solve the ODE for a general solution using the power-series method (singular equations).

(a) $2x(1 - x)u'' + u' - u = 0$

(c) $2x^2u'' + x^2u' + (4/9)u = 0$

EP-8 Roots Differing by an Integer - Series + Wronskian Method
Solve

$$xu'' + (1 - x)u' - u = 0$$

by finding the first solution using the series method and then finding the second solution using the Wronskian method.

EP-9 Bessel's Equation (use solutions in text)
Determine the general solution for

$$xu'' + u' + (x - 1/9x)u = 0$$

EP-10 Properties of Bessel Functions
Find an expression in terms of $J_1(x)$ and $J_0(x)$ for each integral below:

(a) $\int \frac{J_4(x)}{x} dx$ (b) $\int x^3 J_1(x) dx$

Answers(not in textbook) for Assignment 9

EP-5 $$y(x) = Ax^{1/6}\left[1 + \frac{3}{32}x^2 + \frac{9}{5120}x^4 + ...\right] + Bx^{5/6}\left[1 + \frac{3}{64}x^2 + \frac{9}{14336}x^4 + ...\right]$$

EP-6 (b) $$u(x) = b_0\left[1 - 2x^2 + \frac{1}{3}x^4 - ....\right] + b_1\left[x - \frac{2}{3}x^3 - \frac{1}{15}x^5 + ...\right]$$

(c) $$u(x) = x - \frac{1}{12}x^3 - \frac{1}{240}x^5 - \frac{1}{2240}x^7 + ...$$

EP-7 (a)

$$u_1(x) = 1 + x + \frac{1}{6}x^2 + \frac{1}{18}x^3 + ...$$

$$u_2(x) = x^{1/2}\left(1 + \frac{1}{6}x + \frac{1}{24}x^2 + \frac{17}{1008}x^3 + ...\right)$$

(c)

$$u_1(x) = x^{2/3}\left(1 - \frac{1}{4}x + \frac{5}{112}x^2 + ...\right)$$

$$u_2(x) = x^{1/3}\left(1 - \frac{1}{4}x + \frac{1}{20}x^2 + ...\right)$$

EP-8

$$u_1(x) = e^x$$

$$u_2(x) = e^x\left(\ln x - x + \frac{1}{4}x^2 - \frac{1}{18}x^3 + ...\right)$$

EP-9 Bessel's equation: $u(x) = c_1 J_{1/3}(x) + c_2 J_{-1/3}(x)$

EP-10 (a) $$\left(\frac{1}{x} - \frac{12}{x^3}\right)J_1(x) + \frac{6}{x^2}J_0(x) + C$$

(b) $$3x^2 J_1(x) - \left(x^3 - 3x\right)J_0(x) - 3\int J_0(x)dx$$

Computational Work:

Non-linear Oscillator using Runge-Kutta Method

Write a MATLAB code to do solve this ODE system numerically. See MATLAB Lab_3 notes.

Put the MATLAB code in your homework solutions and include graphs for cases a=1.07 and a=1.50 (see below).

Problem #20: Finally, let's do a nonlinear oscillator; one that exhibits chaotic dynamics. There is no analytical solution - numerical solution is the only way to go here. The equation of motion is:

$$\ddot{\theta} = a\cos(\omega_0 t) - \sin(\theta) - q\dot{\theta}$$
$$\theta(0) = \dot{\theta}(0) = 0$$

Some explanation of the terms on the right is necessary:

(1) The second term is what we'd write down for any pendulum - it's the general case rather than the small-angle limit
(2) The third is a damping term, with "damping strength" q.
(3) The first is a driving term, with driving amplitude a and frequency ω_0.

Some suggested parameter sets are below. Take h = 0.05 throughout.

q = 1/2 , ω_0 = 2/3

a=0.9	; periodic
a=1.07	; period doubling
a=1.15	; chaos
a=1.35	; periodic
a=1.45	; period doubling
a=1.47	; period doubling
a=1.50	; chaos

Use Runge-Kutta to calculate the motion of the pendulum for several cycles. Create a phase-space plot, where you plot $\dot{\theta}$ as a function of θ. Restrict the θ motion to $-\pi \le \theta \le \pi$.