Self-Organized Criticality (SOC)

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Biological Computation

Agenda

- Introduction
- Background material
- Self-Organized Criticality Defined
- Examples in Nature
- Experiments
- Conclusion

SOC in a Nutshell

- Is the attempt to explain the occurrence of complex phenomena

Background Material

What is a System?

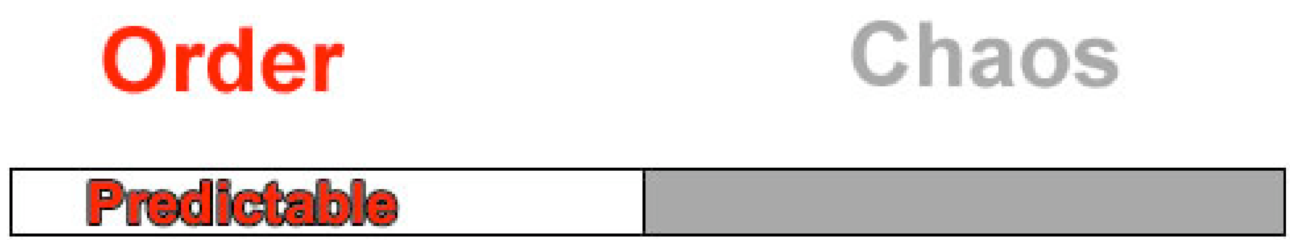
- A group of components functioning as a whole

Obey the Law!

- Single components in a system are governed by rules that dictate how the component interacts with others

System in Balance

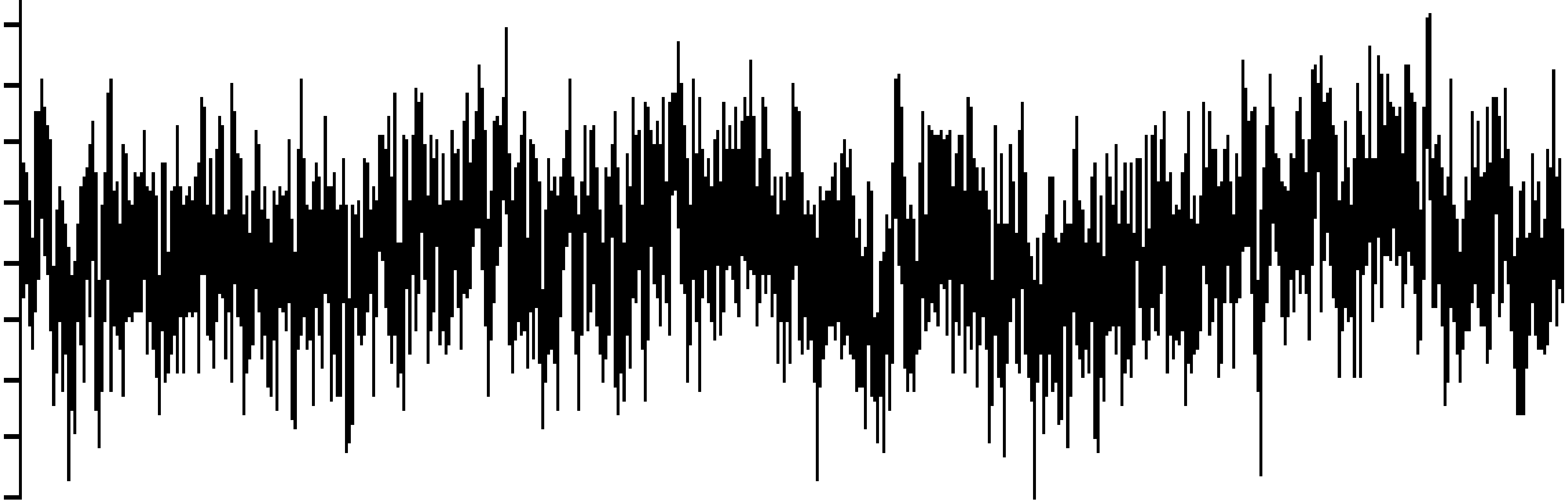
- Predictable
- States of equilibrium
  - Stable, small disturbances in system have only local impact



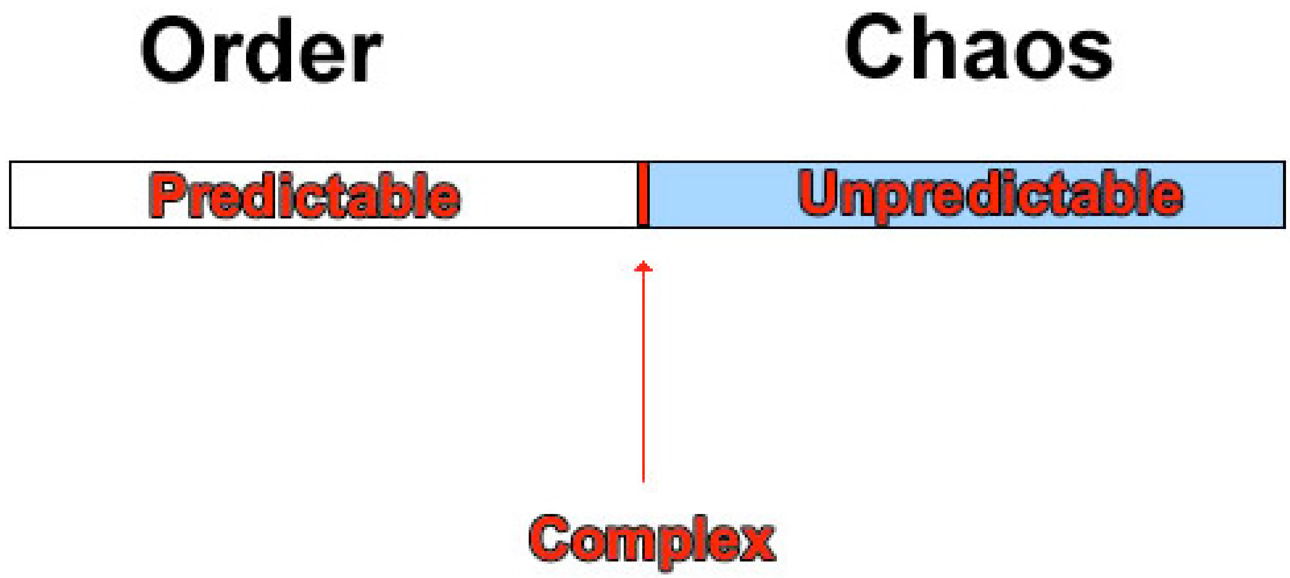
Systems in Chaos

- Unpredictable
- Boring

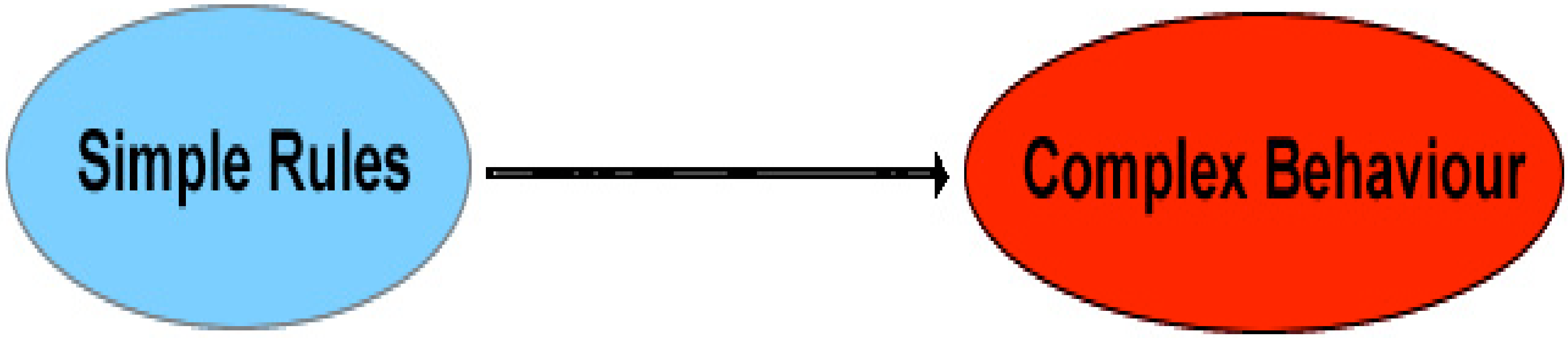
Example Chaos: White Noise



Edge of Chaos



Emergent Complexity



Self-Organized Criticality

Self-Organized Criticality: Defined

- Self-Organized Criticality can be considered as a characteristic state of criticality which is formed by self-organization in a long transient period at the border of stability and chaos

Characteristics

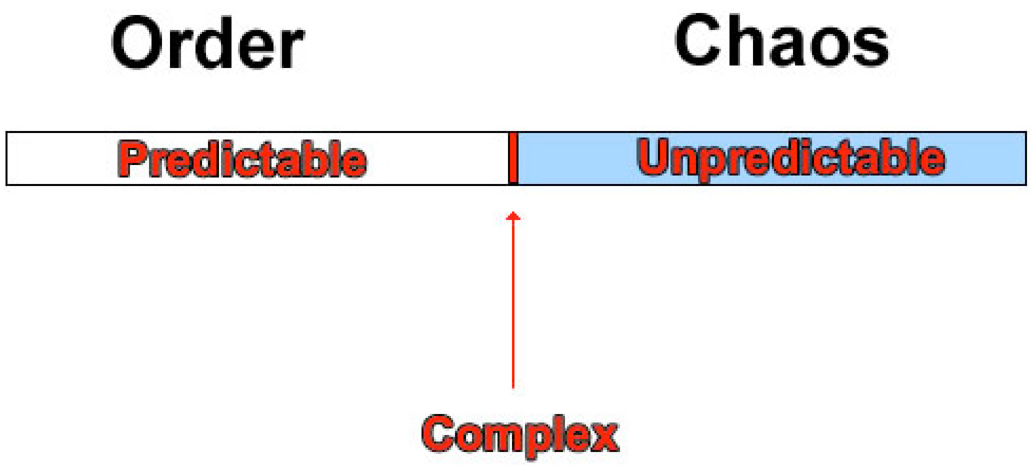
- Open dissipative systems
- The components in the system are governed by simple rules

Characteristics (continued)

- Thresholds exists within the system
- Pressure builds in the system until it exceeds threshold

Characteristics (Continued)

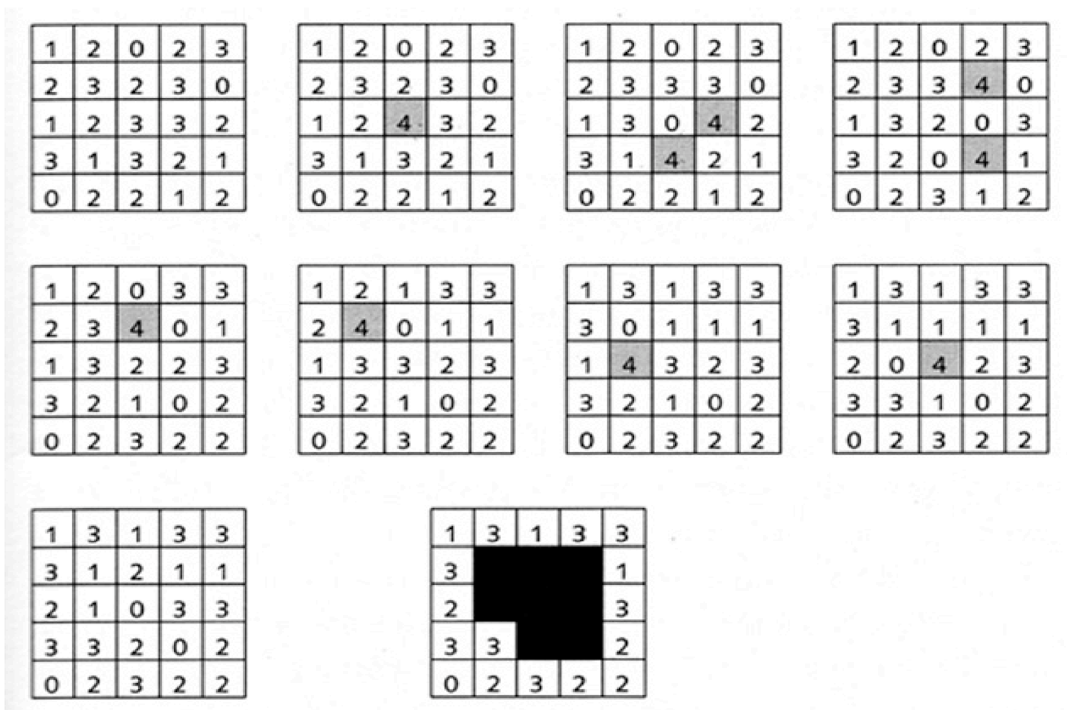
- Naturally Progresses towards critical state
  - Small agitations in system can lead to system effects called avalanches
  - This happens regardless of the initial state of the system



Domino Effect: System wide events

- The same perturbation may lead to small avalanches up to system wide avalanches

Example: Domino Effect



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Characteristics (continued)

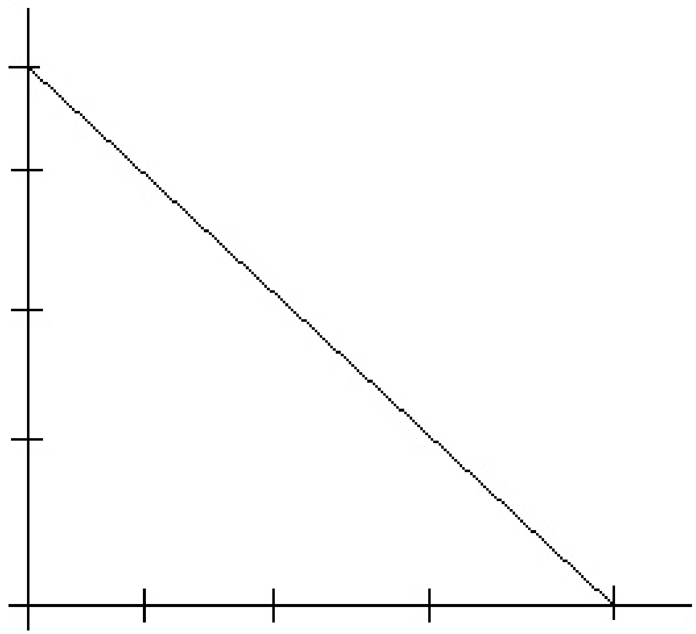
- Power Law
  - Events in the system follow a simple power law

$$N(s) = s^{-t}$$

Power Law: graphed

$N(s) = s^{-t}$ i)

$\log N(s) = -t \log s$ ii)



Characteristics (continued)

- Most changes occurs through catastrophic event rather than a gradual change
    - Punctuations, large catastrophic events that effect the entire system

How did they come up with this?

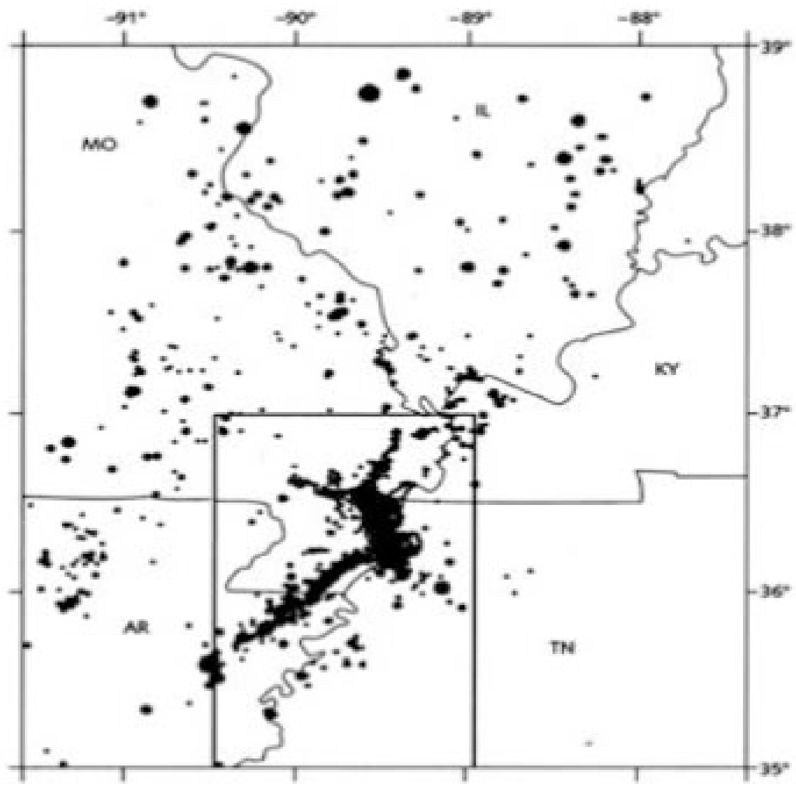
Nature can be viewed as a system

- It has many individual components working together
- Each component is governed by laws
  - e.g, basic laws of physics

Nature is full of complexity

- Gutenberg-Richter Law
- Fractals
- 1-over-f noise

Earthquake distribution



1974-1983

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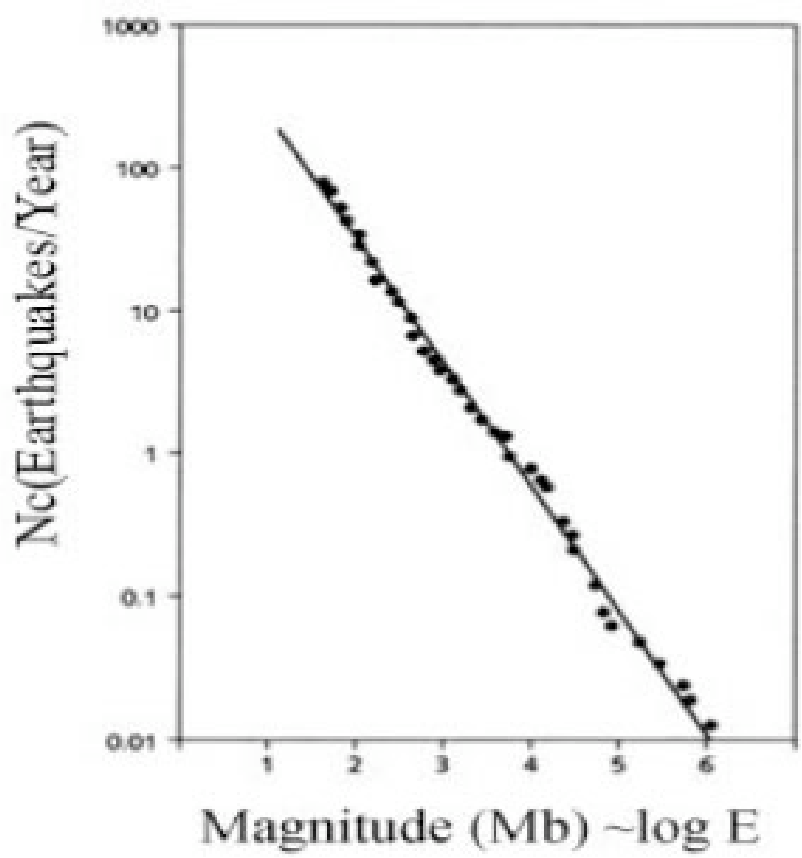
Gutenberg-Richter Law



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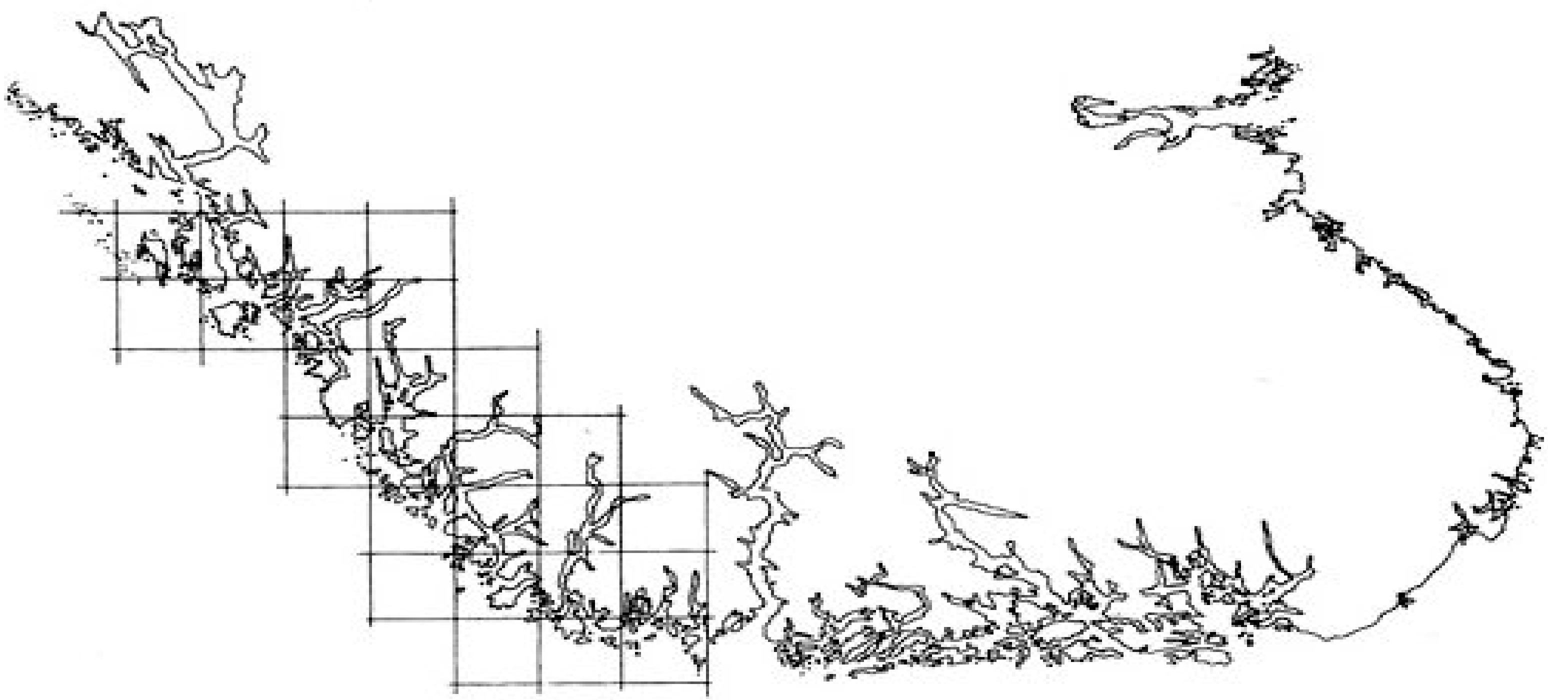
Fractals:

- Geometric structures with features of all length scales (e.g. scale free)
    - Ubiquitous in nature
        - Snowflakes
        - Coast lines

Fractal: Coast of Norway



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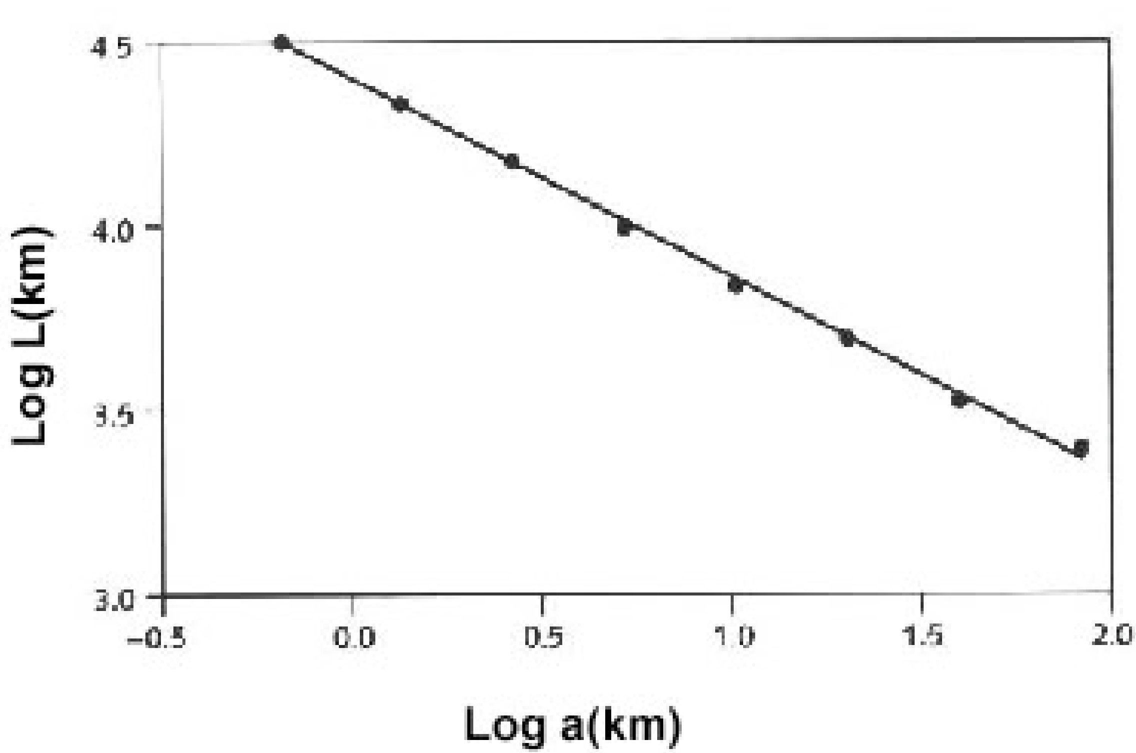
Log (Length) Vs. Log (box size)



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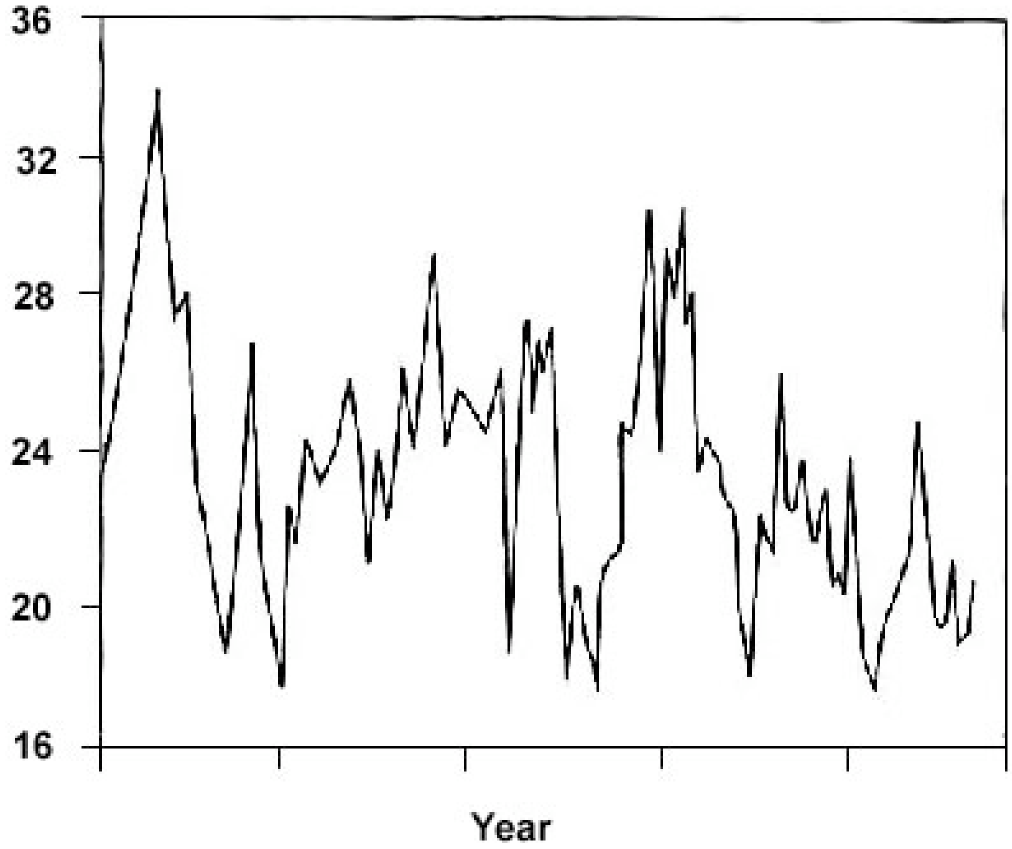
1/F Noise



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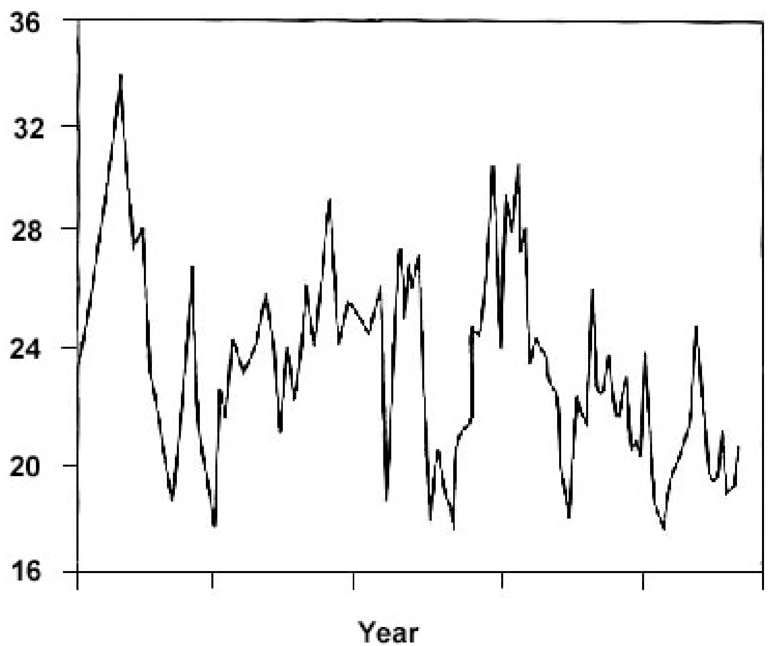
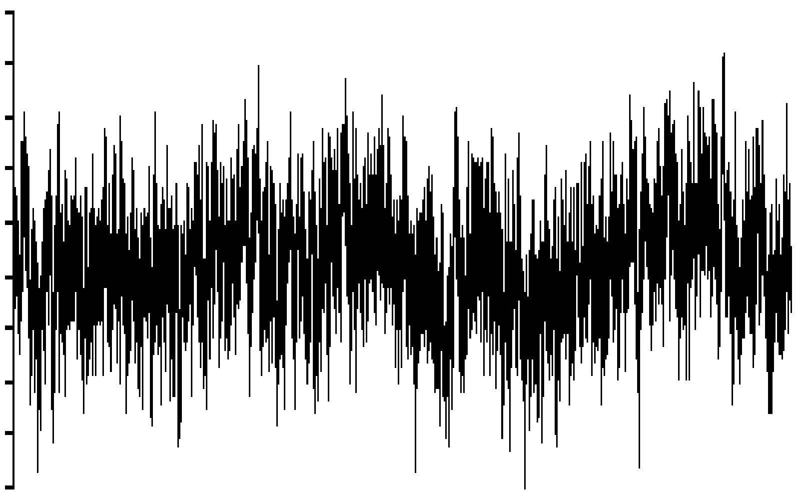
1/f noise has interesting patterns



1/f Noise



White Noise

Can SOC be the common link?

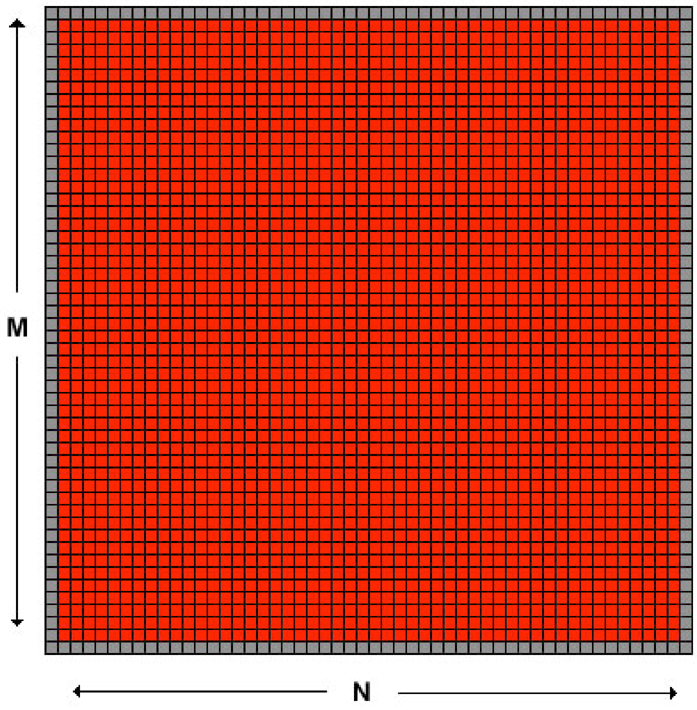
- Ubiquitous phenomena
  - No self-tuning
  - Must be self-organized
- Is there some underlying link

Experimental Models

Sand Pile Model

- An MxN grid Z
- Energy enters the model by randomly adding sand to the model
- We want to measure the avalanches caused by adding sand to the model

Example Sand pile grid

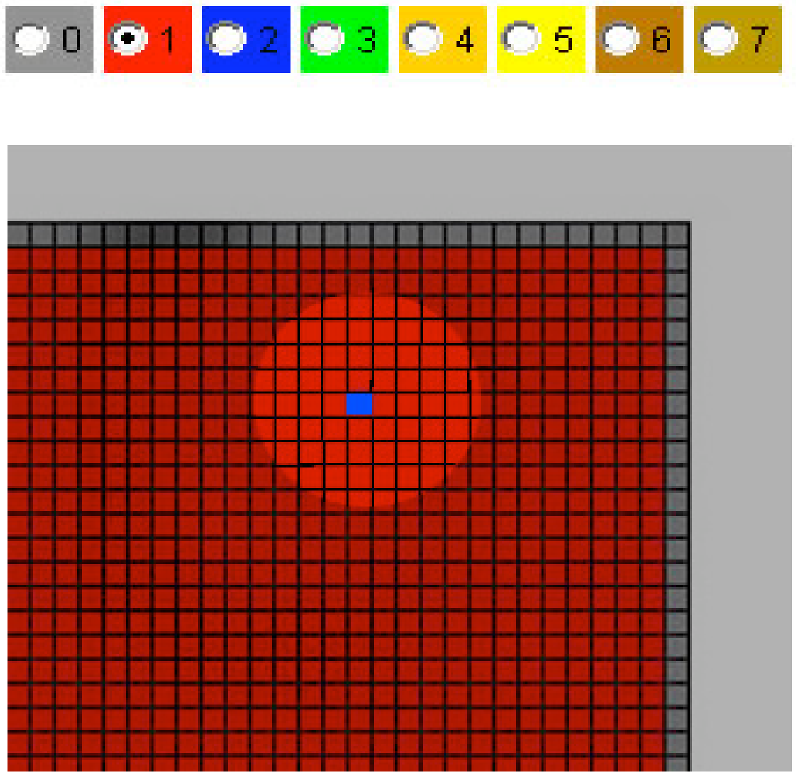


- Grey border represents the edge of the pile
- Each cell, represents a column of sand

Model Rules

- Drop a single grain of sand at a random location on the grid
  - Random (x,y)
  - Update model at that point: $Z(x,y) \rightarrow Z(x,y)+1$
- If $Z(x,y) >$ Threshold, spark an avalanche
  - Threshold = 3

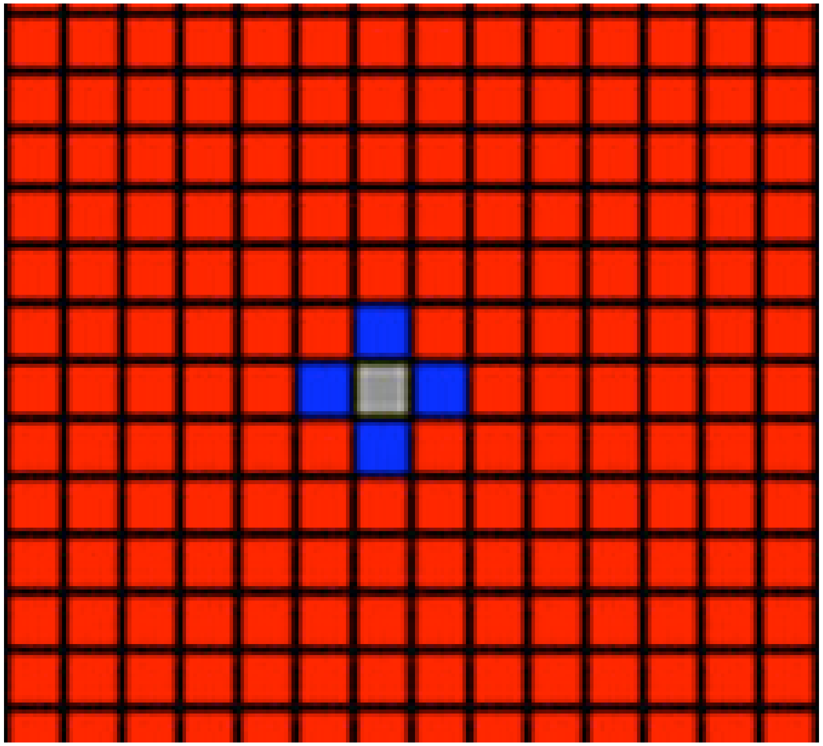
Adding Sand to pile



By: Maslov [6]

- Chose Random (x,y) position on grid
- Increment that cell
  - Z(x,y) → Z(x,y)+1
- Number of sand grains indicated by colour code

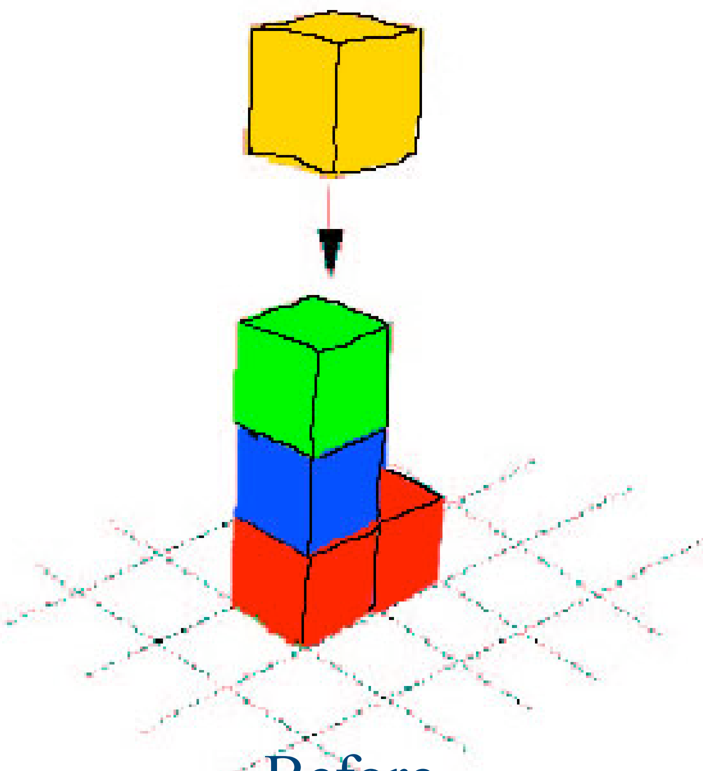
Avalanches



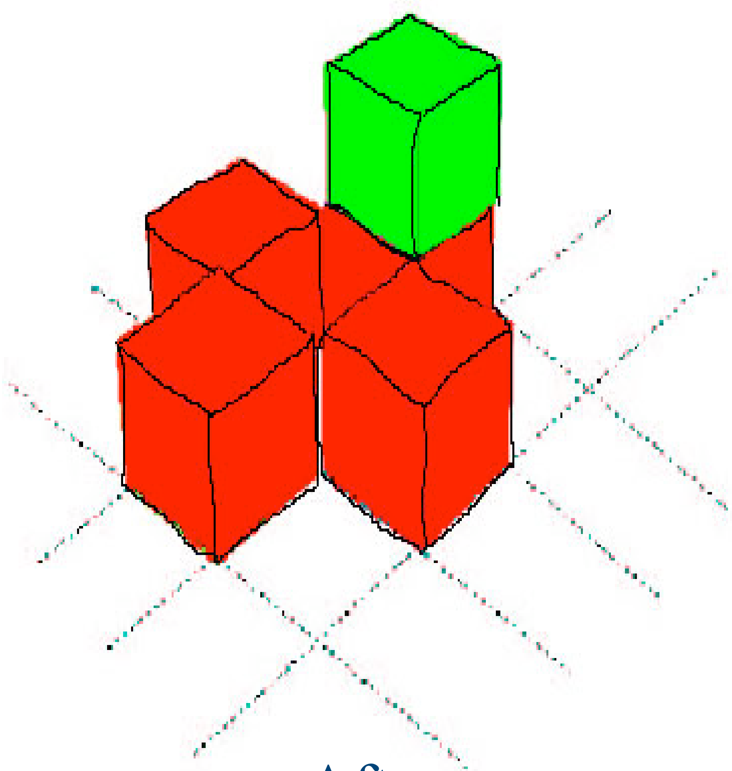
By: Maslov [6]

- When threshold has been exceeded, an avalanche occurs
- If $Z(x,y) > 3$
  - $Z(x,y) \rightarrow Z(x,y) - 4$
  - $Z(x+-1,y) \rightarrow Z(x+-1,y) + 1$
  - $Z(x,y) \rightarrow Z(x,y+-1) + 1$

Before and After



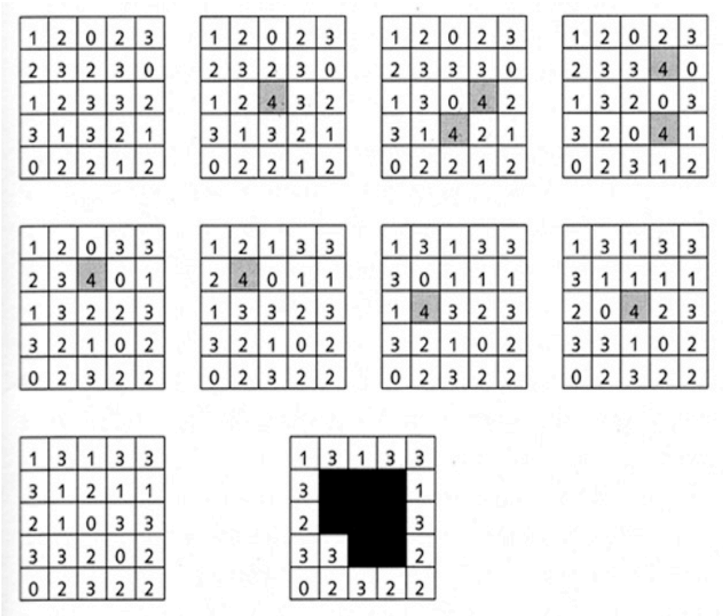
Before



After

Domino Effect

- Avalanches may propagate



By: Bak [1]

DEMO: By Sergei Maslov

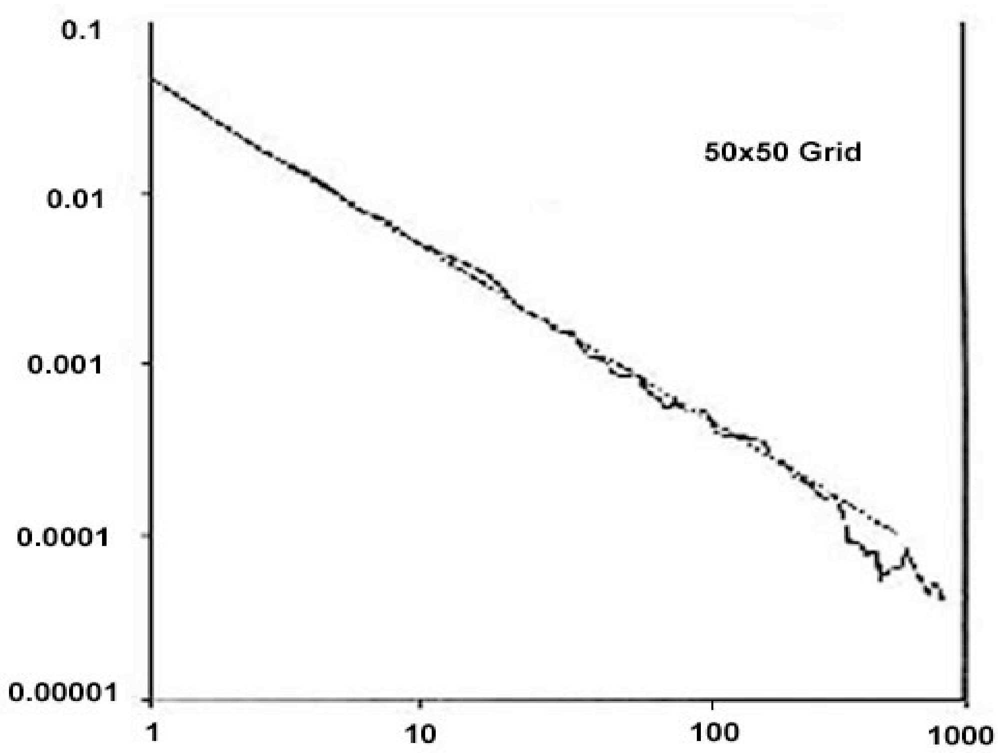
Sandpile Applet

http://cmth.phy.bnl.gov/~maslov/Sandpile.htm

Observances

- Transient/stable phase
- Progresses towards Critical phase
  - At which avalanches of all sizes and durations
- Critical state was robust
  - Various initial states. Random, not random
- Measured events follow the desired Power Law

Size Distribution of Avalanches



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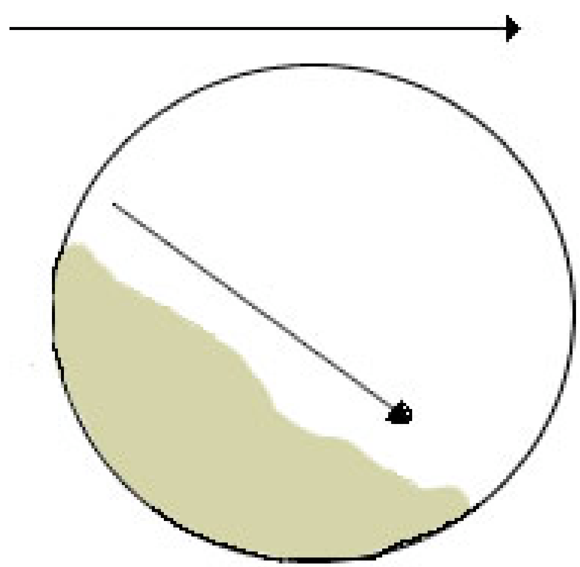
Sandpile: Model Variations



Rotating Drum

- Rotating Drum
- Done by Heinrich Jaeger
- Sand pile forms along the outside of the drum

Other applications

- Evolution
- Mass Extinction
- Stock Market Prices
- The Brain

Conclusion

- Shortfalls
  - Does not explain why or how things self-organize into the critical state
  - Cannot mathematically prove that systems follow the power law
- Benefits
  - Gives us a new way of looking at old problems

References:

- [1] P. Bak, How Nature Works. Springer -Verlag, NY, 1986.
- [2] H.J.Jensen. Self-Organized Criticality – Emergent Complex Behavior in Physical and Biological Systems. Cambridge University Press, NY, 1998.
- [3] T. Krink, R. Tomsen. Self-Organized Criticality and Mass Extinction in Evolutionary Algorithms. Proc. IEEE int. Conf, on Evolutionary Computing 2001: 1155-1161.
- [4] P.Bak, C. Tang, K. WiesenFeld. Self-Organized Criticality: An Explanation of 1/f Noise. Physical Review Letters. Volume 59, Number 4, July 1987.

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- [5] P.Bak. C. Tang. Kurt Wiesenfeld. Self-Organized Criticality. A Physical Review. Volume 38, Number 1. July 1988.
- [6]S. Maslov. Simple Model of a limit order-driven market. Physica A. Volume 278, pg 571-578. 2000.
- [7] P.Bak. Website: http://cmth.phy.bnl.gov/~maslov/Sandpile.htm. Downloaded on March 15th 2003.
- [8] Website: http://platon.ee.duth.gr/~soeist7t/Lessons/lessons4.htm. Downloaded March 3rd 2003.

Questions ?