Superposition Continued

Let us remind ourselves of the interference experiments we discussed earlier.

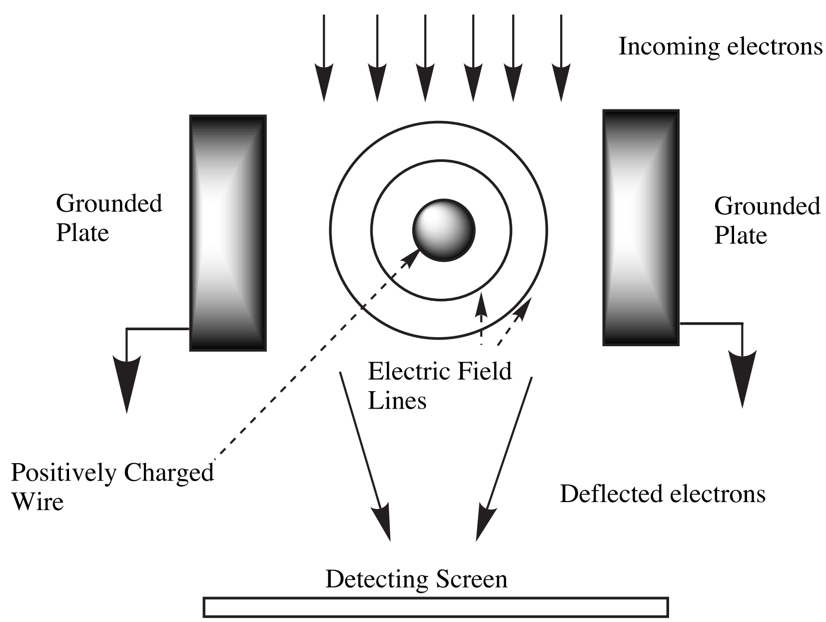
Interference among Electrons

Another example is one we have already discussed but it is useful to look at it again in this new context.

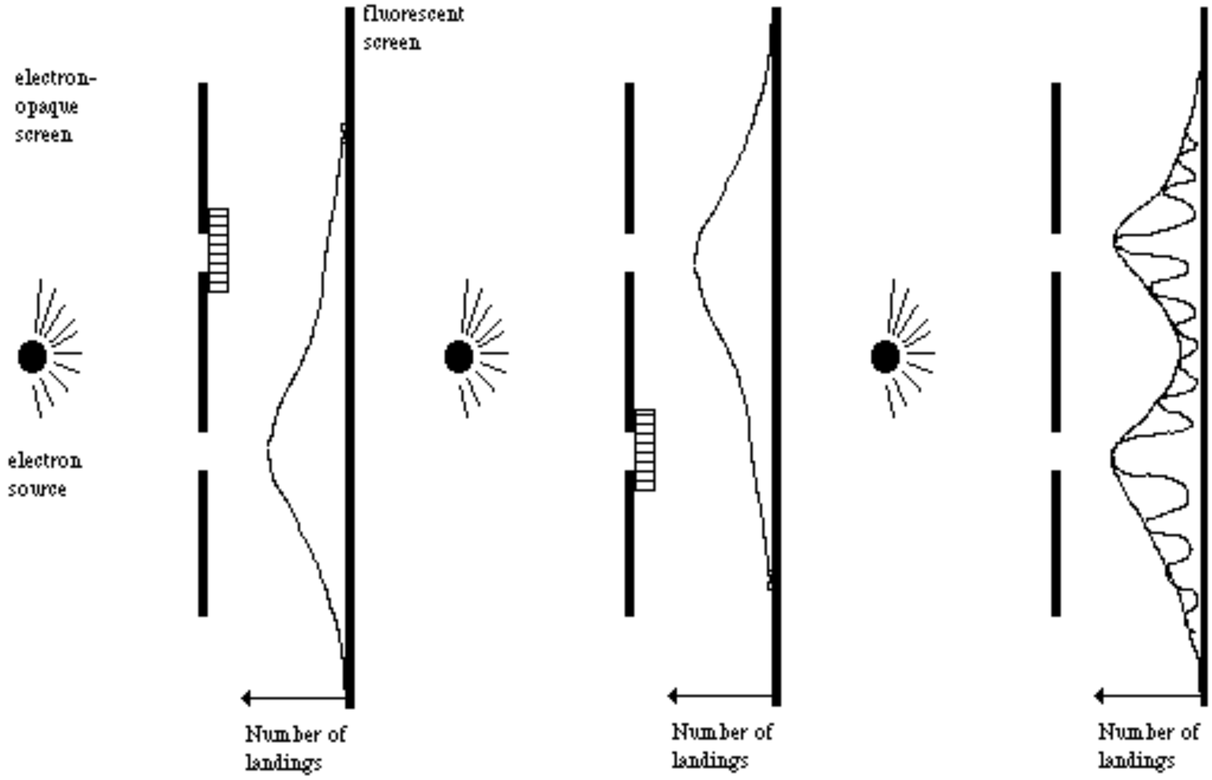
We have an electron source which gives off electrons uniformly in all directions (like a light bulb with photons). We then have a screen, which electrons cannot pass through, with two holes in it. Still further on is a fluorescent screen, much like a television screen, which lights up, at the point of impact, whenever it is struck by an electron, which implies that the fluorescent screen is a **measuring device** for the **POSITION** of the electrons.

How is this actually done?

In particular, let us look at a spectacular experiment that clearly demonstrates the interference of electrons. The experiment was carried out in 1989 in Tokyo by Tonomura, et al at the Hitachi Advanced Research Lab. In the experimental set-up, electrons are generated on a hot wire with a well determined energy (velocity) and then a beam is created using electrostatic lenses. The monoenergetic (single wavelength) electron beam is then sent through an electron biprism to simulate a double slit geometry. A schematic of the biprism is shown below:



Now back to our experiment. We close up one hole. Electrons emerge one by one from the source, some get through the open hole and end up somewhere on the fluorescent screen. We get one pattern when the top hole is closed and another similar but shifted one when the bottom hole is closed. What pattern should we expect when both holes are open? The results for all cases are shown below:



All electrons reaching the screen have either passed through the top hole or the bottom hole or both or neither(**oh no....deja vu sets in....**) so we guess(classical reasoning) the solid line pattern shown, which is the **sum** of the other two patterns.

But as before, the experiment result is completely different and totally unexplainable by any classical reasoning (like the places where no particles arrive even though they did when one or the other slit was open).

So once again we are forced to say that the pattern is not formed because the electron passed through the top hole or passed through the bottom hole or passed through both holes or passed through no holes. We must say that the pattern forms because the electrons are in superpositions of passing through the upper hole and passing through the lower one (**again without really knowing what that means**).

The appearance of an interference fringe pattern is not in itself extraordinary. If we say that the electrons can exhibit wave properties(if we allow a wave-particle duality to exist), then we have recreated an optical double slit system and the resulting fringes make sense and can be explained using classical wave theory.

On the other hand, suppose we reduce the electron beam intensity such that the mean interval between successive electrons is many orders of magnitude greater than the size of the apparatus or, in other words, we arrange the experiment so that only one electron is in the apparatus at any given time (**the other electrons have not even been generated at the source as yet!**).

We are now sending a **single quantum system** through the apparatus. It is clearly difficult, in this case, to propose that we are seeing any cooperative interactions (wave phenomena) among the electrons of the beam. This is the same effect as in the color/hardness device and will require a probabilistic explanation.

This property of the experiment will be the center of our discussions about **what is really happening** in quantum experiments.

How does all this stuff about superpositions connect to the randomizing interactions between color and hardness properties?

Electrons emerge from the hard aperture of a hardness box **if and only if** they are hard electrons when the enter the box (**that was how we defined the hardness box**). Similarly for soft electrons.

However, when a magenta electron is sent into a hardness box it emerges neither through the hard aperture nor through the soft one nor through both nor through neither. So it follows that the magenta electron can't be a hard electron or a soft electron or somehow both or neither.

To say that the electron is magenta **MUST** be just the same as saying that it is in a **SUPERPOSITION of being hard and soft.** Remember that we **could not say that the color of this electron is now such-and-such and its hardness is now such-and-such.**

It **isn't** that our color and hardness boxes are not built well enough. It **isn't AT ALL** a matter of our being unable to **SIMULTANEOUSLY KNOW** what the color and the hardness of a certain electron is (**that is....it isn't a matter of ignorance**).

It is much deeper than that.

It **is** that any electron's even **HAVING** any definite color apparently **means** that it is neither hard nor soft nor both nor neither, and that any electron's even **HAVING** any definite hardness apparently **means** that it is neither green nor magenta nor both nor neither.
As we said, it turns out also that the rules for predicting the outcome of a measurement of (say) hardness of a magenta electron must be probabilistic rather than deterministic.

The reasoning goes like this:

If it could ever be said of a magenta electron that a measurement of its hardness will with certainty produce the outcome(say) **soft** or if it could ever be said of a magenta electron that a measurement of its hardness will with certainty produce the outcome(say) **hard**, then that would apparently be inconsistent with what we **KNOW** to be the case, which is that such an electron(a magenta one) is in a **SUPERPOSITION (or none of the above)** of being hard and soft.

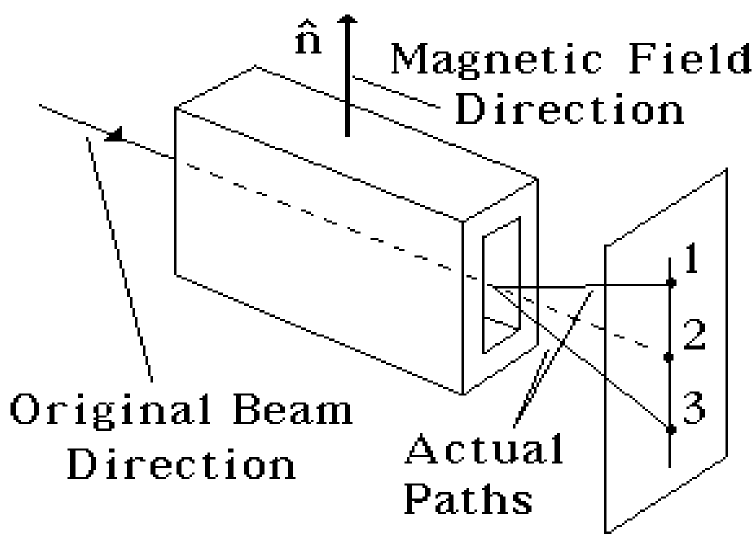
But we also know that **every measurement** whatsoever either comes out **hard** or comes out **soft. Thus, the outcome of a hardness measurement on a magenta electron has got to be a matter of probability.**

The World of Stern-Gerlach

A Stern-Gerlach apparatus is a real-world device. This will be more difficult to understand, but if we go slowly, I think we can understand what is happening. It will convince us we can really build color and hardness boxes. I will choose **Stern-Gerlach devices** although we could do the same with **Polaroids.** **We** will deal with Polaroids later.

Stern-Gerlach Experiments

A schematic diagram of a Stern-Gerlach apparatus is shown below:



A **Stern-Gerlach(SG)** apparatus acts on an incoming beam of charged particles with a particle property called **angular momentum.** There are two types of angular momentum. The first type, is called orbital angular momentum and is associated with the ordinary 3-space motion of the particle(like orbital motion in Bohr atom or a planet around the Sun) and the second is called spin angular momentum. It is associated with internal degrees of freedom of the particle and has no classical analogue(it is not the same as a spinning planet).

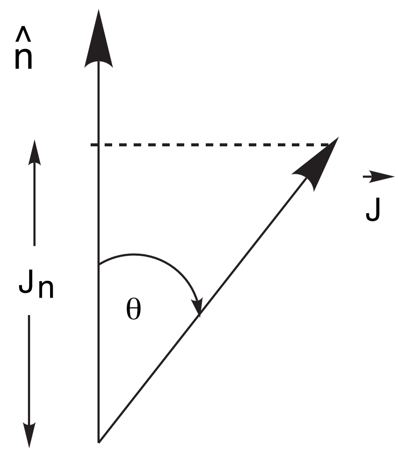
Angular momentum is a vector quantity $\hbar\vec{J}$. The component of $\hbar\vec{J}$ in the direction of the magnetic field $\hat{n}$ (a unit vector) is given by the **scalar product**

$$\hbar J_n = \hbar\vec{J}\cdot\hat{n} = \hbar J\cos\theta \qquad 0 \le \theta \le \pi$$

where this operation is defined by

$$\vec{A}\cdot\vec{B} = (\text{magnitude of }\vec{A})(\text{magnitude of }\vec{B})\cos(\text{angle between }\vec{A}\ \&\ \vec{B})$$

as shown in the figure.



Classically, the component of $\hbar\vec{J}$ in the direction of the magnetic field $\hat{n}$ can take on **all** possible values(**continuously**) from $-\hbar J$ to $+\hbar J$, where $\hbar J$ = length of the angular momentum vector. This is an infinite number of allowed values and $\hbar J_n$ can have any of these values classically.

Quantum mechanically, however, the component of $\hbar\vec{J}$ in the direction of the magnetic field $\hat{n}$ can take on only a **discrete** number of values for any system(it is quantized). The number of discrete values is $2J+1$ and they are

$$\hbar J_n = -\hbar J, \hbar(-J+1), \hbar(-J+2), \ldots\ldots\ldots, \hbar(J-1), \hbar J$$

where the length of the angular momentum vector is $\hbar\sqrt{J(J+1)}$.

Quantum mechanics also tells us that J can only take on integer or a half-integer values.

The SG device (a non-uniform magnetic field) deflects the particles in the beam through an angle that depends on the component of the angular momentum in the direction of the magnetic field.

If this were a classical system, then we would see a continuous stripe on the screen corresponding to J_n taking on all possible values between $-J$ and $+J$ continuously(the vertical line in the screen).

If this were a quantum mechanical system, then the original beam splits into $2J+1$ beams and we see $2J+1$ distinct spots on a screen (the spots labelled 1, 2 and 3 on the screen, for example).

Observation confirms the existence of discrete spots agreeing with the quantum prediction and interpretation.

In the diagram, spot #2(undeflected beam) is where the beam would have hit the screen if no magnetic field were present in the apparatus. Spots #1 and #3 are an example of what the $2J+1=2$ spots we would observe in an experiment when $J=\frac{1}{2}$ (an electron).

The important features of the apparatus for our theoretical discussion are as follows:

(1) The breakup into a finite number of discrete beams (we will assume we are working with $J=\frac{1}{2}$ particles and thus have 2 beams exiting the apparatus). These are called spin-1/2 particles.

(2) The beams are separated in 3-dimensional space and each contains 1/2 of the original particles entering the device.

(3) The possible values of $\vec{J}\cdot\hat{n}$ for $J=\frac{1}{2}$ are $\pm\frac{1}{2}$ (the angular momentum mvalues are $\pm\frac{\hbar}{2}$

(4) One exiting beam contains **only** particles with $\vec{J}\cdot\hat{n}=+\frac{1}{2}$ (called spin **up** in the $\hat{n}$ direction) and the other beam contains **only** particles with $\vec{J}\cdot\hat{n}=-\frac{1}{2}$ (called spin **down** in the $\hat{n}$ direction).

(5) These same results occur for all beams no matter what the direction of the unit vector $\hat{n}$

Thus, the SG device with spin-1/2 particles behaves exactly like the color and hardness boxes. In fact, if we say a color box is an SG device with $\hat{n}$ in the z-direction (say green = spin up along z and magenta = spin down along z), then a hardness box is an SG device with $\hat{n}$ in the x- or y-direction (say hard = spin up along x(or y) and soft = spin down along x(or y)).

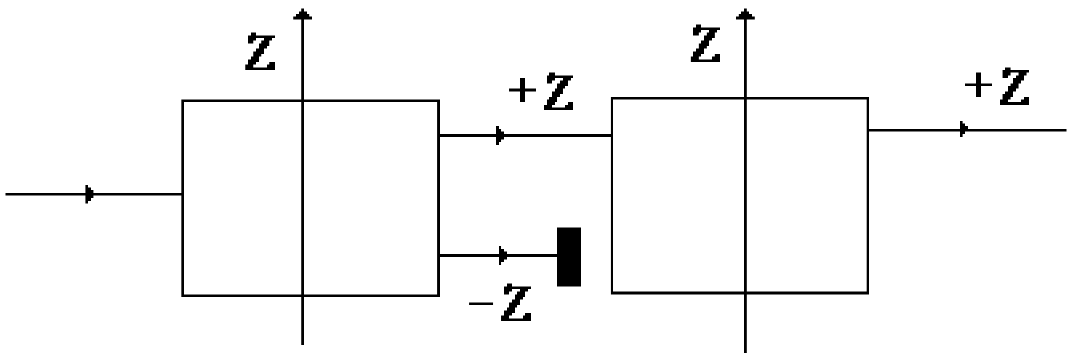
We will represent the Stern-Gerlach apparatus with a magnetic field in the $\hat{n}$-direction by the symbol $SG\hat{n}$.

We now report the results of some **actual experiments** with SG devices:

Experiment #1

We send N particles in a superposition of the states spin up and spin down (any direction) into an $SG\hat{z}$ device ... like sending a hard electron beam into a color box.
The beam splits into spin up($\hat{S}\cdot\hat{z} = +\frac{1}{2}$) and spin down($\hat{S}\cdot\hat{z} = -\frac{1}{2}$) beams.

We select out the beam where the particles are in the spin up state ($\hat{S}\cdot\hat{z} = +\frac{1}{2}$) (put in a wall in the other beam). It contains $N/2$ particles. We then send this second beam into another $SG\hat{z}$ device. We find that all $N/2$ exit in the spin up ($\hat{S}\cdot\hat{z} = +\frac{1}{2}$) state, i.e., there is only one exit beam as shown below:



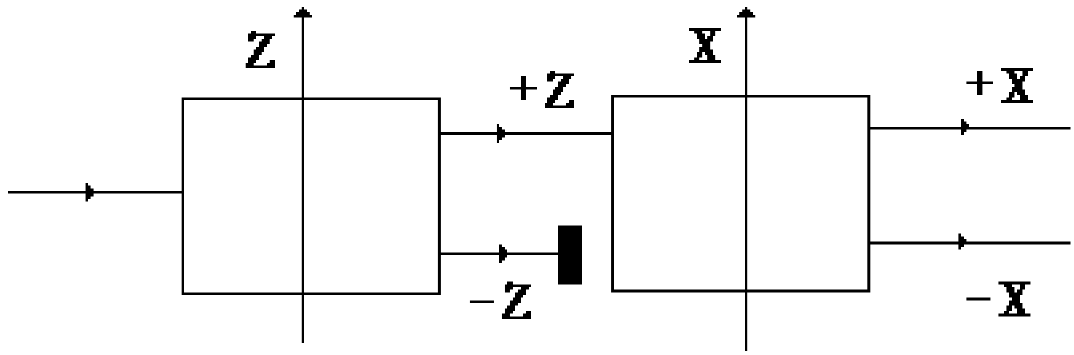
This says that when we make a measurement, say $\hat{S}\cdot\hat{z}$ and then we immediately make another measurement of the same quantity, we get the same result as first measurement.

The is just the repeatability assumption we made earlier.

The first measurement seems to have caused the system to change from a superposition of states into a definite state so that the next measurement has a definite value.

Experiment #2

We send N particles in a superposition state into an $SG\hat{z}$ device and select out the beam (put in a wall) where the particles are in the spin up state ($\hat{S}\cdot\hat{z} = +\frac{1}{2}$). It contains $N/2$ particles. We then send the selected beam into an $SG\hat{x}$ device. We find that $N/4$ exit in the spin up state($\hat{S}\cdot\hat{x} = +\frac{1}{2} \rightarrow$ now up in the x-direction) and $N/4$ exit in the spin down state($\vec{S}\cdot\hat{x} = -\frac{1}{2}$). There are now two exit beams as shown.



The same thing happens if we stop the ($\vec{S}\cdot\hat{z} = +\frac{1}{2}$) spin up beam and let the ($\vec{S}\cdot\hat{z} = -\frac{1}{2}$) spin down beam into the $SG\hat{x}$ device. So an $SG\hat{x}$ device takes a beam with a definite value of $\vec{S}\cdot\hat{z}$ and **randomizes** it, i.e., we once again have two exiting beams with equal numbers of particles with spin up/down in the x-direction.

Again this is identical to what happened with the color and hardness boxes.

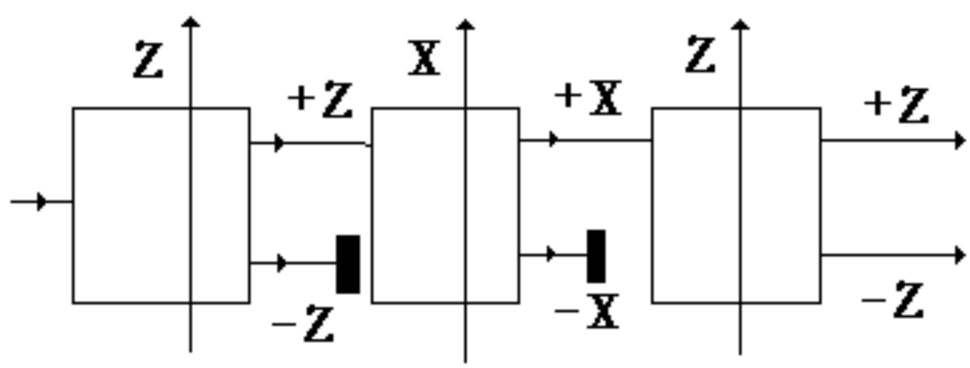
Experiment #3

We now add a third SG device to experiment #2.

It is an $SG\hat{z}$ device. This is the same as our earlier experiment with

$$\rightarrow COLOR \rightarrow HARDNESS \rightarrow COLOR \rightarrow$$

We also block the $\vec{S}\cdot\hat{x} = -\frac{1}{2}$ exiting spin down beam as shown.



We found that $N/4$ exited in the spin up state $\vec{S}\cdot\hat{x} = +\frac{1}{2}$ from the $SG\hat{x}$ device. After the third device we find that $N/8$ exit in the spin up state $\vec{S} \cdot \hat{z} = +\frac{1}{2}$ and N/8 exit in the spin down state $\vec{S} \cdot \hat{z} = -\frac{1}{2}$.

What has happened?

It seems that making a measurement of $\vec{S} \cdot \hat{x}$ on a beam with definite $\vec{S} \cdot \hat{z}$ modifies the system rather **dramatically**(same as with color/hardness).

We did two successive measurements on these particles. Since we isolated the + beam in each case we might be led to think that the beam entering the last (because of our selections) has

$$\vec{S} \cdot \hat{z} = +\frac{1}{2} \quad \text{AND} \quad \vec{S} \cdot \hat{x} = +\frac{1}{2}$$

spin up(z) AND spin up(x)   (like magenta AND soft)

But the experiment says this cannot is not so, since 50% of the particles exiting the last device have $\vec{S} \cdot \hat{z} = -\frac{1}{2}$ or spin down.

We are forced to say that the $SG\hat{z}$ device takes a definite value of $\vec{S} \cdot \hat{x}$ and **randomizes** it so that we end up with two exiting beams with equal numbers of particles. Why?

Like color and hardness, $\vec{S} \cdot \hat{x}$ and $\vec{S} \cdot \hat{z}$ must be **incompatible** which means that we cannot simultaneously measure them. Our two successive measurements **DOES NO**T produce definite values for both quantities.

Each measurement **only** produces a definite value for the quantity it is measuring and **randomizes** the incompatible quantity (**actually randomizes all other incompatible quantities**)!

Another way to think about this is to say the following:

An $SG\hat{n}$ device is a measurement of the angular momentum in the $\hat{n}$-direction. Any such measurement then randomizes the next measurement of any other incompatible quantity.

So we can find real-world devices that behave like color and hardness boxes (there analysis is more complicated).