Interpreting the macroscopic pointer via the elements of reality of a Schrödinger cat

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Abstract

We examine Einstein-Podolsky-Rosen’s asymmetric form of local realism for “Schrodinger cat” states in which the “cat” is entangled with a second system. Specific “elements of reality” exist for the “cat” based on the EPR assumption which precludes action-at-a-distance on the cat by measurements performed on the second system. We construct elements of reality for two classic cases thus developing a signature for a Schrodinger cat by negating that the “cat” is probabilistically in one of two macroscopically distinct states. The negation requires the elements of reality to be microscopically specified. This elucidates the interpretation that the cat is “both dead and alive” and suggests that many Schrodinger cat paradoxes can be explained by microscopic nonlocality.

The original arguments of Einstein-Podolsky-Rosen (EPR) and Bell dealt only with small symmetrical systems: two particles or two spins [1,2,16]. The arguments are based on EPR’s notion of local realism (LR) - put simply, that there can be no “spooky action-at-a-distance” [3] on one system as a result of measurements made on the other. In revealing inconsistencies between the predictions of quantum mechanics and the premise of local realism (LR), these arguments have had profound implications for physics [4,16]. Schrodinger was quick to point out that the consequences of such paradoxes would be most significant for larger systems, where any loss of reality manifests at a macroscopic level [5]. Schrodinger analyzed a gedanken experiment whereby, according to QM, a macroscopic system (a “cat”) becomes entangled with a microscopic spin 1/2 system, the final state being

\[ |\psi\rangle = \frac{1}{\sqrt{2}}(|A\rangle_{\text{cat}}|\uparrow\rangle + |D\rangle_{\text{cat}}|\downarrow\rangle) \]  

Here, the cat is in a superposition of states \(|A\rangle_{\text{cat}}\) (“alive”) and \(|D\rangle_{\text{cat}}\) (“dead”), correlated with the “up” and “down” spin eigenstates \(|\uparrow\rangle, |\downarrow\rangle\). The spin system and the cat can in principle be spatially separated.

While Schrodinger pointed out the natural interpretation of this state - that the cat cannot be viewed as either dead or alive until measured - he did not construct an EPR or Bell-type gedanken experiment that would demonstrate such failure of reality. While some such signatures have since been developed [6-10], experimental work has mainly focused on providing evidence (such as
a fidelity measure or negative Wigner distribution) for the cat-state within quantum theory [11-15]. Yet, understanding the precise nature of the quantum “unreality” of a Schrodinger cat is of topical interest: Many well-known proposals have been put forward so that the paradoxical situation in which a cat is ‘both alive and dead’ can be better understood or remedied [16-24].

The objective of this Letter is to give a connection between the EPR and Schrodinger cat paradoxes, in a way that probes the asymmetrical nature of the entangled superposition. Given that an entangled state of type (1) is predicted when a measurement is made on a microscopic spin, we will see that this approach may be useful in elucidating some of the paradoxes of quantum measurement theory. Measurement paradoxes come about because the “cat” acts as a meter for the state of the spin, and the interpretation of the cat being alive and dead is then that the “pointer” of the measurement device is at two positions of a dial at once. While decoherence mechanisms preclude such a result, an issue has been how to understand the entangled state without decoherence and in particular to address if and when a collapse into a state of “one position or the other” occurs. The results of this paper suggest an alternative hybrid picture of the pointer, that immediately after interaction with the microscopic system, it is indeed located at one place or the other but is subject to nonlocal effects constrained in size by the quantum uncertainty relation. An important feature of our analysis is that EPR’s local realism is defined asymmetrically [1,25,26], so one may consider either “spooky” action on the cat system by measurements on the spin system $S \rightarrow C$, or vice versa. Our interest is to probe the “quantumness” of the cat, rather than the microscopic spin system. This information is not given by the mere observation of entanglement [27]. Therefore we focus on the first case $S \rightarrow C$ where the LR premise implies specific “elements of reality” for the “cat” system. These elements of reality, as EPR called them, are hidden variables that predetermine results for (future) specific measurements on the cat, regardless of an observer.

In this paper, we calculate elements of reality of the cat for two classic realizations of (1) involving coherent states and Greenberger-Horne-Zeilinger (GHZ) spin states. In both cases, measurable contradictions are revealed that become the signature of the Schrodinger cat. These contradictions falsify that the cat can be in any classical mixture of dead and alive states, in a way that is consistent with LR between the cat and the spin. We discuss
how they also negate the directional EPR premise of LR $S \rightarrow C$. A second feature that we use to probe the superposition (1) is that the LR premise $S \rightarrow C$ can be quantified. This is because a macroscopic system might have different amounts of nonlocal change made to it. We define: For $\delta$-scopic LR $S \rightarrow C$, it is assumed that the measurement of the microscopic spin does not affect the (value of measurement on the) macroscopic cat by more than an amount $\delta$.

In analyzing the “elements of reality” of the “cat”, it then becomes apparent there are two types: either macroscopic or microscopic, depending on the resolution of their prediction i.e. whether the predetermined results for the measurements on the cat are specified microscopically or macroscopically. For the realizations of Schrodinger cat (1), we find that the signatures rely on the negation of microscopic elements of reality i.e., to signify the Schrodinger cat, it is necessary to assume there can be no microscopic change to the cat, based on measurements performed elsewhere. This is consistent with known results and is conjectured true for all signatures of (1). It would then be apparent that in the context of the superposition (1) (which is fundamental to interpretation of measurement), the macroscopic elements of reality cannot be negated. We discuss how this leaves open the interpretation that the cat is always dead or alive, but that the microscopic details of its whole “state” are subject to “spooky” nonlocal events.

**First classic cat:** We begin by considering the following classic prototype for the Schrödinger cat (1):

$$|\psi_{coh}\rangle = \frac{1}{\sqrt{2}} (e^{-i\pi/4}|\alpha\rangle|\uparrow\rangle_Z + e^{-i\pi/4}|-\alpha\rangle|\downarrow\rangle_Z)$$

(2)

Her $|\alpha\rangle$ is a coherent state for a quantum harmonic oscillator “cat” system and $|\uparrow\rangle_Z, |\downarrow\rangle_Z$ are the spin-1/2 eigenstates for the Pauli spin operator $\sigma_Z$. We take $\alpha$ to be real and large. This state for moderate $\alpha > 1$ has been realized experimentally [11]. Let us apply the argument of EPR. Observers Alice and Bob can make measurements on the spin and cat systems respectively. We consider that the two systems have become spatially separated after the interaction that created the entanglement. If Alice measures $\sigma_Z$ and the result is 1, then the state of Bob’s system is $|\alpha\rangle$. Similarly, if the result is $-1$, the state of Bob’s system is $|-\alpha\rangle$. Thus, Alice can predict the statistics for Bob’s measurements, conditional on her outcome. Suppose Bob makes
a measurement of either the position or momentum quadrature defined (in a rotating frame) by 

\[ X = \frac{1}{\sqrt{2}}(a^\dagger + a) \]

and

\[ P = \frac{i}{\sqrt{2}}(a - a^\dagger) \]

Here \(a^\dagger, a\) are the creation, destruction operators for the “cat” system. If Alice’s outcome is ±1, the conditional probability distribution \(P(x)\) for Bob’s measurement \(X\) in each case is the Gaussian hill

\[ P_\pm(x) = \frac{1}{\sqrt{\pi}} \exp\left\{ -\left( x \mp \sqrt{2}\alpha \right)^2 \right\} \]  

(3)

centered at \(\pm \sqrt{2}\alpha\) respectively. The results are distinguished as “alive” and “dead”. The distributions have variances \((\Delta x)^2 = 1/2\) as required for a coherent state.

**Result (1a):** EPR’s local realism LR states that measurement by one observer makes no difference to the system of the other observer. Therefore the argument leads to the conclusion that regardless of Alice’s measurement, the “cat” system is consistent with being either in a state with the distribution for \(X\) given by \(P_+(x)\) (“alive”), or in a state with statistics given by \(P_-(x)\) (“dead”). We denote the two different predetermined states by the variable \(\lambda_Z\), where the variable takes the value \(\lambda_Z = +1\) or \(-1\) respectively. EPR used the term “elements of reality” to describe the predetermination.

Result (1a) applies to any of the conditional distributions, and is a generalization of EPR’s original argument for which the conditional predictions were precise values. The proof is given in the Supplemental Materials [28] where it is shown that Result (1a) follows as a consequence of Bell’s description of LR which assumes a local hidden variable model (LHV). There we also discuss how the LHV model (being applied to the asymmetrical scenario where Bob’s measurements though spacelike separated from Alice’s are in the future) involves asymmetry in the locality assumption. We call this premise LR \(S \to C\).

To realize an EPR paradox for “cat” (2), we follow [29] and consider that Alice measures \(\sigma_X\). Alice is able to predict the probability distribution for Bob’s measurement \(P\) on the “cat” system, conditional on her outcome. The distribution \(P(p)\) for \(P\) given Alice’s outcome ±1 is

\[ P_\pm(p) = \frac{1}{\sqrt{\pi}} \exp\left\{ (-p^2)(1 \pm \sin(2\sqrt{2}\alpha p)) \right\} \]  

(4)
The distribution exhibits interference fringes and has a variance \((\Delta p)^2 = \frac{1}{2} - 2\alpha^2 e^{-4\alpha^4}\) which is reduced below that of the coherent state, for which \((\Delta p)^2 = \frac{1}{2}\). The EPR argument as generalized by Result (1a) leads to the conclusion the cat is predetermined to be in one or other of two states, that correspond to the distributions \(P_+(p)\) and \(P_-(p)\) respectively. We denote the two elements of reality states by the variable \(\lambda_X\), which assumes the value +1 and −1 in each case. Continuing the EPR argument, for consistency with the LR premise, the state of the cat would simultaneously be described by both variables: \(\lambda_Z\) and \(\lambda_X\). We can represent such an element of reality state by the ordered pair \((\lambda_Z, \lambda_X)\). That this simultaneous description is indeed a consequence of LR (Result 1b) is proved in the Supplemental Materials.

Now we note the inconsistency that gives an EPR paradox. There are four possible element of reality states of the cat, as depicted in Figure 1: each \((\lambda_Z, \lambda_X)\) has predictions for \(X\) and \(P\) given by \(P_{\lambda_X}(x)\) and \(P_{\lambda_Z}(p)\) respectively.

![Figure 1: Predictions \(P(x)\) and \(P(p)\) for the “element of reality” states \((\lambda_Z, \lambda_X)\) of the Schrodinger cat (2) with \(\alpha = 2\). Top: The “alive cat” with \(1, -1\) or \(1, 1\). Lower: The “dead cat” with \(-1, -1\) or \(-1, 1\).](image)

We see that for each of these “element of reality” states

\[
\Delta X \Delta P = \frac{1}{2} (1 - 4\alpha^2 e^{-4\alpha^4})^{1/2} < \frac{1}{2}
\]

which contradicts the Heisenberg uncertainty relation \(\Delta X \Delta P \geq \frac{1}{2}\). Thus, the element of reality states cannot be quantum states: The realization of (5) is therefore an EPR (steering) paradox [25, 26, 29]. That this is so is apparent.
from the logic of the argument: The LR premise leads to the conclusion that the cat system is predetermined to be in one of the “element of reality” states. Yet, these states are not describable as quantum states, and hence an inconsistency (paradox). The inequality (5) signifies a Schrödinger cat, in that it is negated that the cat is in a mixture of any dead or alive quantum states in a way that is consistent with LR.

Next we examine the second classic realization of the Schrödinger cat state (1). The GHZ state $|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle^{\otimes N} - |\downarrow\rangle^{\otimes N})$ is formed from $N$ spin-1/2 particles: Here $|\uparrow\rangle^{\otimes N} = \Pi_{k=1}^N |\uparrow\rangle^{(k)}$ and $|\downarrow\rangle^{(k)}$ is the spin eigenstate for $\sigma_{z}^{(k)}$, the $\sigma_{z}$ observable for the $k$-th particle. This sort of Schrödinger cat state has been generated in ion trap and optical experiments [13, 14]. If we separate the $N$-th spin from the remaining $N-1$ spins, the GHZ state is a microscopic spin entangled with a macroscopic system similar to (1):

$$|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle^{\otimes N-1} |\uparrow\rangle_{N} - |\downarrow\rangle^{\otimes N-1} |\downarrow\rangle_{N}) \quad (6)$$

Alice makes a measurement on the single spin, while Bob measures a larger “cat” system associated with the $N-1$ particles. We identify the collective spin for Bob’s cat: $\sigma_{z}^{B} = \sum_{k=1}^{N-1} \sigma_{z}^{(k)}$. The measurement of $\sigma_{z}^{(N)}$ by Alice will reduce Bob’s system into the “alive” state $|\uparrow\rangle^{\otimes N-1}$ if her result is $+1$, or to the “dead” state $|\downarrow\rangle^{\otimes N-1}$ if her result is $-1$. Using the EPR Result1 based on LR $S \rightarrow C$, we see the cat is then deduced to be “alive” or “dead” i.e., always in one or the other of two element of reality states that correspond to the dead and alive outcomes $\pm(N-1)/2$ for $\sigma_{z}^{B}$ respectively. We denote these respective states by a hidden variable $\lambda_{X}$ with values $\pm 1$.

To realize an EPR paradox, we analyze Bob’s system, given Alice’s measurement of $\sigma_{X}^{A} = \sigma_{X}^{(N)}$. For odd $N$ the GHZ state is an eigenstate of $\sigma_{X}^{(1)} \Pi_{k=2}^{N} \sigma_{y}^{(k)}$ and all permutations including $(\Pi_{k=1}^{N-1} \sigma_{y}^{(k)}) \sigma_{X}^{(N)}$ with eigenvalue $(-1)^{(N+1)/2}$; and is also an eigenstate of $\Pi_{k=1}^{N} \sigma_{X}^{(k)}$ with eigenvalue $-1$ [30]. Take for simplicity $N = 3, 7, \ldots$ A measurement of $\sigma_{X}^{A}$ will give the result 1 or $-1$ and this predicts precise outcomes for $Pr_{Y}^{B} = \prod_{k=1}^{N-1} \sigma_{y}^{(k)}$. Assuming LR $S \rightarrow C$, Results 1 imply system $B$ to be in one of the element of reality states specified by a hidden variable $\lambda_{X}$, where the value $\lambda_{X} = \pm 1$ corresponds to respective outcomes for $Pr_{Y}^{B}$ being $\pm 1$. Suppose Alice measures $\sigma_{y}^{A}$. According to LR $S \rightarrow C$, the system $B$ is also in an element of reality state denoted by a
second hidden variable $\lambda_Y$ where the value $\lambda_Y = 1$ (or $-1$) corresponds to outcomes for all products $Pr^B_Y(J) = \sigma^{(j)}_X \prod_{k=1, k\neq j}^{N-1} \sigma^{(k)}_Y$, $J = 1,..,N-1$ being 1 (or $-1$). Results 1a,b thus imply the system $B$ to be in one of the simultaneous element of reality states $(\lambda_Z, \lambda_X, \lambda_Y)$ in which the outcomes for $\sigma^B_Z, Pr^B_Y, Pr^B_Y(J)$ are each predetermined with no uncertainty. Yet, the observables $\sigma^B_Z, Pr^B_Y, \sum_{j=1}^{N-1} Pr^B_Y(J)$ satisfy the Heisenberg uncertainty relation

$$\Delta(\sigma^B_Z)\Delta(Pr^B_Y) \geq \frac{|(\sum_{j=1}^{N-1} Pr^B_Y(J))|}{2}$$  \hspace{0.5cm} (7)

Immediately we see that the states $(\lambda_Z, \lambda_X, \lambda_Y)$ (which specify predetermined nonzero values for each of these observables) contradict (7). As with (5) this contradiction signifies an EPR paradox and a Schrodinger cat, by negating any mixture of quantum “dead” and “alive” states consistent with LR $S \rightarrow C$.

So far, there is no inconsistency between the elements of reality and hidden variable states since hidden variable states need not satisfy the Heisenberg uncertainty relation. The aim is to demonstrate such an inconsistency, thus providing a Bell’s theorem that negates the “elements of reality” of the cat. The contradiction is immediate if we assume locality between the spins of the cat. However we desire to probe the quantum “unreality” of the cat as a whole macroscopic object without internal assumptions. To do this we use the results of Refs [6]. Consider the complex operator $\prod_M = \prod_{j=1}^{M} F_j, M \leq N$ where $F_j = \sigma^{(j)}_X + i\sigma^{(j)}_Y$ ($j \neq N$) at each of Bob’s sites where $F_N = \sigma^{(N)}_{\pi/4} + i\sigma^{(N)}_{3\pi/4}$ ($\sigma^{(j)}_\theta = \sigma^{(j)}_X + \sigma^{(j)}_Y \sin \theta$): Observables Re$\Pi_M$ and Im$\Pi_M$ are defined according to $\Pi_M = \text{Re}\Pi_M + \text{Im}\Pi_M$. We consider the elements of reality for the predictions Re$\Pi_{N-1}$ and Im$\Pi_{N-1}$ of Bob’s “cat” system based on the measurements $\sigma^{(N)}_{\pi/4}$ (and $\sigma^{(N)}_{3\pi/4}$) of Alice. That there cannot be a hidden variable set consistent with Bob’s system being in each of the element of reality states is proved using the fact that there are algebraic bounds (Re$\Pi_M, \text{Im}\Pi_M) \leq 2^{N-1}$ and (Re$\Pi_M) + (\text{Im}\Pi_M) \leq 2^M$ for any such underlying hidden variable state. This assumption leads to the Svetlichny-type inequality $\langle \text{Re}\Pi_N \rangle + \langle \text{Im}\Pi_N \rangle \leq 2^{N-1}$ proved in [6]. However here where we test LR $S \rightarrow C$ the moments are measured asymmetrically e.g., $\langle \sigma^{(1)}_X \sigma^{(2)}_X \sigma^{(3)}_{X'} \rangle$ is evaluated by measurement of Bob’s $\sigma^{(1)}_X \sigma^{(2)}_X$ conditional on the (past) outcomes for $\sigma^{(3)}_{X'}$ [28]. The prediction for the conditional moments is $\langle \text{Re}\Pi_N \rangle + \langle \text{Im}\Pi_N \rangle = 2^{N-1/2}$ thus contradicting the premise of LR $S \rightarrow C$. The contradiction implies failure of any hidden variable description in which the cat can be “dead” or “alive” in
a way that is consistent with the assumption of no nonlocality between the cat and the spin. By ruling out all such descriptions, it is a rigorous signature of a Schrödinger cat. It is explained in the Supplemental Materials how the contradiction also holds in the case where Alice’s system becomes itself macroscopic. We also explain how the inequalities of [9, 10] lead to similar results for the first classic cat.

Discussion: The signatures derived in this paper falsify that the “cat” system can be in a “dead” or “alive” state provided we assume no nonlocal effects (i.e. LR $S \rightarrow C$) between the cat and spin. The falsification manifests in the first cat at large $\alpha$ through very fine interference fringes in distributions for $P$. For the second cat, it is necessary to measure every spin of a large number of particles. This ultra-sensitivity is consistent with proven fundamental requirements for signifying quantum superpositions [23, 31]. A second result follows.

Result (2a): The falsification depends on the assumption of no microscopic spooky action: Suppose we assume $\delta$-scopic LR $S \rightarrow C$ (defined above). We can determine a value of $\delta$ such that if we allow that there may be some spooky action by an amount greater than $\delta$, then the Schrödinger cat is indistinguishable from a classical cat. We find $\delta$ is classifiable as microscopic.

We explain for the first cat (2). To deduce the element of reality $\lambda_x = \lambda_Z$ for $X$ we assume that the measurement by Alice does not induce a change $\delta_x$ to the result $X$ for the cat. Since the outcomes for $X$ are macroscopically distinct (by $\Delta$ say), this element of reality can be defined with a large macroscopic uncertainty and we only require the assumption of $\Delta$-scopic LR to deduce that the cat is predetermined dead or alive ($\delta_x \rightarrow \Delta$). By contrast, in deducing the element of reality $\lambda_P = \lambda_X$ for the measurement $P$ on the cat, it needs to be assumed that Alice’s measurement does not induce even a small change $\delta_P$ to the result $P$. This is clear because that element of reality requires microscopic fringe detail if we are to distinguish the Schrödinger cat from its classical counterpart. The argument assumes both $\delta_P$-scopic LR and $\Delta$-scopic LR, but the first is a stronger assumption. Consequently, the signature of the Schrödinger cat is a falsification of microscopic $\delta_P$-scopic LR (but not of macroscopic $\Delta$-scopic LR). In fact we see that in this context we cannot negate the macroscopic element of reality.
Moreover, as $\alpha \to \infty$ the signature requires an increasingly stricter form of LR i.e. the value $\delta_p$ becomes smaller. This supports an explanation (similar to Refs. [23]) for the sensitivity to decoherence as the size of the cat increases - the Schrodinger cat is more difficult to observe because the elements of reality (which give a pre-determination of the results of measurement) are closer to classically consistent values (Figure 2). We can quantify with the following:

**Result (2b):** Suppose the uncertainty relation for conjugate observables $X, P$ is $(\Delta X)(\Delta P) \geq c$ where $c$ is a constant, and that the meaning of macroscopic is defined relative to $c$ so that $\Delta \gg \sqrt{c}$ is macroscopic. Suppose we assume $\Delta$-scopic LR and $\delta$-scopic LR to deduce the elements of reality $\lambda_x$ and $\lambda_p$ respectively. If $\Delta \delta \geq c$, we cannot rigorously signify the Schrodinger cat based on $X, P$ measurements.

**Proof:** The proof (given in the Supplemental Materials [28]) uses that $\delta$-scopic LR implies a bounded indeterminacy in the elements of reality, along with the known fact that adding noise of size $\delta$ and $\Delta$ to the outcomes $P$ and $X$ respectively destroys the signature of the cat by providing a hidden variable theory that predetermines the cat to be dead or alive [23].

Thus for the first cat state we see that the Schrodinger cat signature is constrained so that the maximum order of LR falsifiable is $\sim c$-scopic. As the cat becomes more macroscopic, we can add more noise $\Delta \to \infty$ and not change the predictions of the macroscopic element of reality (deduced via $\Delta$-scopic LR) that predetermines the cat to be dead or alive. But then we need $\delta$ to become increasingly smaller [28, 32].

**Conclusion:** The elements of reality suggest an interpretation of the macroscopic entangled state (1). Since we cannot in fact falsify the macroscopic element of reality, the interpretation of the Schrodinger cat being “both dead and alive” becomes debatable in this context. This is clear on recalling that the macroscopic element of reality is defined as the variable that predetermines the outcome of the measurement that distinguishes the two states (dead and alive) of the cat. We can however falsify that the cat is predetermined to be in a dead or alive state, if that state has microscopic predictions independent of measurements that might be made on the second system. The interpretation is of the measuring device pointer actually being posi-
tioned at one place on the dial or the other, but with its position and/or momentum subject to microscopic nonlocal effects (which have been verified experimentally [4]). On the other hand, we qualify, by noting the existence of Schrödinger cat-like states where both $\Delta$ and $\delta$ can be “macroscopic” because of the amplification of the quantum noise $c$ [33] e.g., for high spins the uncertainty relation $\Delta J_X \Delta J_Y \geq \frac{|\langle J_Z \rangle|}{2}$ can possess macroscopic values of $|\langle J_Z \rangle|$ measurable as large particle numbers [34].

The cats we describe can be realized as a mechanical oscillator coupled to a two-level atom or optical system and as a microwave field mode coupled to Rydberg atoms [11,13]. Similar photonic cats have been reported with Svetlichny-Bell violations [12,14]. The “elements of reality” are thus likely measurable by experiment.

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References


[28] Refer to Supplemental Materials for details.


