# Evolution without evolution: Dynamics described by stationary observables

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**Abstract**

Because the time parameter in the Schrödinger equation is not observable, energy apparently obeys a superselection rule in the same sense that charge does. That is, observabies must all commute with the Hamiltonian and hence be stationary. This means that it is consistent with all observations to assume that any closed system such as the Universe is in a stationary state. We show how the observed dynamic evolution of a system can be described entirely in terms of stationary observables as a dependence upon internal clock readings.

# 1. UNOBSERVABILITY OF COORDINATE TIME

It is widely believed that there is a charge superselection rule,[1-5] so that superpositions of states of different charge are unobservable. Indeed it has been proved in local relativistic quantum field theory that the long-range Coulomb field causes the total charge operator to commute with all quasilocal observables.[6] Because energy is coupled to a long- range gravitational field, there should be an analogous superselection rule for it.[6] This would say that only operators that commute with the Hamiltonian can be observables. But such operators are stationary, so how is it that we observe time dependence in the world?

We shall argue that the temporal behavior we observe is actually a dependence on some internal clock time, not on an external coordinate time. It is perfectly consistent with our observations to assume that any closed system is in an eigenstate of energy and thus stationary with respect to coordinate time, since coordinate time translations are unobservable. Such a state can be decomposed into states of definite clock time. The dependence of these component states upon the clock time labeling them can then represent the observed temporal behavior of the system. A formula for this dependence in terms of conditional probabilities will be given below.

The circumvention of the energy superselection rule is precisely analogous to what Aharonov and Susskind[7], Mirman[8], and Lubkin[9] have shown about the charge[1-4] and univalence[1,10] superselection rules. These authors demonstrate that any additively conserved quantity may be considered to have a superselection rule in the sense that all subsystems at all time have density matrices which commute with the conserved quantity, if the density matrix of the cornposite system commutes with that quantity. Nevertheless, if the conserved quantity is coherently shared between subsystems, the state of one subsystem relative to a particular state of another can show interference between its eigenstates of the con- served quantity. Indeed, the univalence superselection rule has been experimentally circumvented by the observation of a relative rotation of $2\pi$ for one half of a split beam of neutrons.[11,12]

Our analysis applies to any closed system that may be completely described quantum mechanically and has a well-defined Hamiltonian. Since a closed system by definition interacts only with itself, any observations must be done entirely within the system, so the system must include all its observers. By virtue of the long-range gravitational and electromagnetic interactions, the smallest closed system we observe appears to be astronomically large and is generally known as the Universe. (Of course we cannot rule out other quantum systems which do not interact with ours. ) In order that our Universe will have a well-defined Hamiltonian, we assume that spacetime is asymptotically flat in some appropriate sense.[13,14] This is inconsistent with the cosmological principle of large-scale homogeneity and isotropy if the average energy density is nonzero, but it is not inconsistent with observations: one can imagine that the energy density drops off beyond the maximum distance we can now see, so that the total energy of the Universe is finite. We do not claim that this model is necessarily correct but only that it is consistent.

Alternatively, one may ignore long-range interactions and consider hypothetical closed systems within a single spacetime universe. However, then one must restrict attention to the interactions within each system and not invoke the neglected interactions between them in discussing the observed temporal behavior of a closed system. That is, an observer cannot be allowed to read clocks outside his own closed system.

Although a rigorous demonstration is lacking, one would expect the Strocchi-Wightman proof[6] of the charge supers election rule in quantum electrodynamics to apply to energy in quantum gravity. That is, one would expect the energy, as determined by the gravitational field at spacelike infinity, to commute with all quasilocal observables, which de- pend upon fields interior to infinity. But even if we ignore gravity and consider quantum field theory in

Minkowski spacetime, the unobservability of a global time translation of the entire Universe would seem to imply that energy obeys a superselection rule. (This is not a trivial consequence of the Wigner-Araki-Yanase theorem,[15-17] because we are not assuming precise von Neumann-type measurements.) In other words, a shift in the time coordinate $t$, which changes the relative phase between two different energy eigenstates, is an unobservable coordinate transformation. Thus only operators which have no explicit or implicit dependence on $t$ can be observables. These operators commute with the Hamiltonian and are stationary, so how can they show any dynamical behavior? In other words, how can the energy superselection rule be reconciled with the fact that we do observe interference effects between states of different energy whenever we see anything change?

# 2. OBSERVABILITY OF CLOCK TIME BY STATIONARY OPERATORS

To illustrate how we see change through the use of observables which are necessarily stationary, consider a system of $n$ spin-$j$ particles at fixed positions but precessing around the $z$ axis with frequency $\omega$. For simplicity, we assume there is a surrounding laboratory with respect to which relative positions, orientations, and velocities are defined; such a reference system is necessary since the momentum and angular momentum superselection rules prevent the observation of any absolute positions, orientations, or velocities.[7-9] However, we assume that in the relative laboratory frame the laboratory is stationary, so that all the time dependence is given by the particle precession; i.e., there is no clock built into the laboratory. The Hamiltonian is taken to be

$$H = \hbar\omega J_z = \sum_{m_1=-j}^{j} \cdots \sum_{m_n=-j}^{j} (m_1 + \cdots + m_n)\hbar\omega \left|m_1\right\rangle \cdots \left|m_n\right\rangle \left\langle m_1\right| \cdots \left\langle m_n\right| \tag{1}$$

where $m_i$ labels the $z$ component of the angular momentum of the $i$th particle. That is,

$$J_{iz} \left|m_i\right\rangle = m_i \left|m_i\right\rangle \quad , \quad i = 1, ...., n \tag{2}$$

Suppose we consider the product state in which each particle has angular momentum $j$ in the $x$ direction at $t = 0$. The transformation[18,19] to eigenstates of $J_z$ allows us to write this as

$$\begin{aligned} |\psi\rangle &= \prod_{i=1}^{n} |\psi_i\rangle \\ &= \prod_{i=1}^{n} 2^{-j} \sum_{m_i=-j}^{j} \binom{2j}{j+m_i}^{1/2} |m_i\rangle \end{aligned} \tag{3}$$

$$\binom{n}{m} \equiv \frac{n!}{m!(n-m)!} \tag{4}$$

$$J_{ix}(t=0)\,|\psi_i(t=0)\rangle = j\,|\psi_i(t=0)\rangle \tag{5}$$

This state is nonstationary with respect to $t$, since

$$\langle\psi|\, J_{ix} \pm iJ_{iy}\,|\psi\rangle = je^{\pm i\omega t} \tag{6}$$

but this $t$ dependence cannot be observed. There is no way to select a particular $t$ at which a measurement of $J_{ix}$ or $J_{iy}$ is to be performed, so these nonstationary operators are not observables. The time average of a nonstationary operator is certainly stationary and may therefore be observable, but for $J_{ix}$ and $J_{iy}$ the averages are zero.

The time dependence that is actually observable is the dependence of the dynamical variables upon each other, in particular, upon any variable that represents a clock reading. The measurement of time by quantum clocks has been discussed Peres. Here we wish to emphasize how the dependence of a system upon a clock reading is deter- mined entirely by stationary observables.

Consider how the precessing orientation of particle 2 depends upon the orientation of particle 1, which is viewed as a clock. For example, we may ask what value $J_{2x}$ has when particle 1 reads 12 o'clock, by which we mean that $J_{1x}$ is measured to give angular momentum $j$ in the $x$ direction. In the state (3) this is certain to occur if $t = 0$, but it can occur with lower probability at almost any other $t$, so $t$ is not precisely determined by the clock reading. If the laboratory apparatus is triggered to measure $J_{2x}$ when $J_{1x}$ gives $j$, the measurement does not necessarily occur at $t = 0$, so $J_{2x}(t = 0)$ cannot be measured. Instead, what is measurable is the value of $J_{2x}$ given a particular value of $J_{1x}$. The value measured is governed by the conditional probabilities of the various eigenstates of $J_{2x}$ with respect to a particular eigenstate of $J_{1x}$.

The relative frequen- cy distribution of values measured on an ensemble of identical systems gives a statistical estimate of these conditional probabilities, so we say these are observable.

(To determine the conditional probabilities precisely, one would need an infinite ensemble. This is an idealization not realized in nature, so no predictions of quantum mechanics can ever be completely verified by quantum-mechanical observers within the Universe, for whom the theory can only make statistical predictions. One can only verify that the predictions are consistent with observations to some confidence level. Since the theory does predict that all but a relatively small set of conditional probabilities are very small, its consistency with observation is nontrivial. It is only in this consistency sense that the conditional probabilities we shall discuss are observable.)

To determine the quantum-mechanical predictions for the conditional probabilities of $J_{2x}$ when $J_{1x}$ has the value $j$, we need the projection operator onto the subspace of states with $J_{1x} = j$ and the one onto the subspace with $J_{1x} = j$ and $J_{2x} = j - k$:

$$P_j = |J_{1x} = j\rangle \langle J_{1x} = j| \otimes \prod_{i=2}^{n} I_i \tag{7}$$

$$P_{j,j-k} = |J_{1x} = j\rangle \langle J_{1x} = j| \otimes |J_{2x} = j-k\rangle \langle J_{2x} = j-k| \prod_{i=3}^{n} I_i \tag{8}$$

where,[18,19] in the Heisenberg picture,

$$|J_{1x} = j\rangle = 2^{-j} \sum_{m_1=-j}^{j} \binom{2j}{j+m_1}^{\frac{1}{2}} e^{-im_1\omega t} |m_1\rangle \tag{9}$$

$$|J_{2x} = j-k\rangle = \sum_{m_2=-j}^{j} \binom{2j}{k}^{\frac{1}{2}} \binom{2j}{j+m_2}^{-\frac{1}{2}} \sum_{l=0}^{k} \binom{k}{l} \binom{2j-k}{j+m_2-l} \frac{1}{2^j} (-1)^l e^{-im_2\omega t} |m_2\rangle \tag{10}$$

$$I_i = \sum_{m_i=-j}^{j} |m_i\rangle \langle m_i| \tag{11}$$

These projection operators depend upon the undetermined $t$ and hence are not observables, but their time averages, $\bar{P}_j$ and $\bar{P}_{j,j-k}$, may be (though we do not claim to have found what the *sufficient* conditions are for an operator to be an observable):

$$\bar{P}_j = 2^{-2j} = \sum_{m_l=-j}^{j} \binom{2j}{j+m_1} |m_1\rangle \langle m_1| \otimes \prod_{i=2}^{n} I_i \tag{12}$$

and $\bar{P}_{j,j-k}$ is a more complicated stationary observable which shows correlations between the two particles (i.e., the time averaging breaks the product structure of $P_{j,j-k}$). Then the conditional probability for $J_{2x} = j-k$ given $J_{1x} = j$ is

$$P(J_{2x} = j-k | J_{1x} = j) = \frac{\langle \bar{P}_{j,j-k} \rangle}{\langle \bar{P}_j \rangle} \tag{13}$$

In the product state (3), the conditional probability (13) turns out to be

$$P(J_{2x} = j-k | J_{1x} = j) = 2^{-4j} \binom{4j}{2j}^{-1} \binom{8j}{4j} \binom{8j}{2k}^{-1} \binom{2j}{k} \binom{4j}{k} \tag{14}$$

The conditional probability for $J_{2x} = j$ given $J_{1x} = j$ is then

$$P(J_{2x} = j | J_{1x} = j) = 2^{-4j} \frac{(8j-1)!!}{[(4j-1)!!]^2} = \prod_{l=1}^{2j} \left[ 1 - \frac{1}{(4l-2)^2} \right] \underset{j\to\infty}{\longrightarrow} \frac{1}{\sqrt{2}} \tag{15}$$

so there is always at least a 70% probability that the second particle will have all its spin in the $x$ direction if the first one does. Indeed, for large $j$

$$P_\psi(J_{2x} = j-k | J_{1x} = j) \approx 2^{-1/2-3k} \binom{2k}{k} \tag{16}$$

which is very strongly concentrated around $k = 0$. Another value of interest is the conditional expectation value of $J_{2x}$ given $J_{1x} = j$. This is

$$E_\psi(J_{2x} | J_{1x} = j) = \frac{\langle \psi | \overline{J_{2x} P_j} | \psi \rangle}{\langle \psi | \bar{J_{P_j}} | \psi \rangle} = \frac{2j^2}{2j+1} \tag{17}$$

which again shows the strong agreement between the two spins for large $j$.

One may also consider conditional probabilities given that particle 1 has angular momentum $j$ in any other direction. The agreement between the particle angular momenta in any direction in the $x-y$ plane in which the spins are all precessing will be as good as it is in the $x$ direction. If one considers particle 1 to be a clock, so that different directions of its spin correspond to different clock readings, then the dependence of the angular momenta of the other particles on the clock readings constitutes the observable time behavior of the system. A sequence of clock readings along with the corresponding states of the other particles forms an evolution of the system, with no reference to the dependence on the unobservable coordinate time $t$.

As a technical point, since angular momenta in different directions do not commute, making more than one clock reading on the same particle will alter its precession and make the dependence of the other particles on it more complicated. Therefore it may be preferable to form a clock out of many particles in identical spin states and make each reading on a different one. Then the projection operators for all clock readings will commute, so the dependence of the rest of the system on each reading will be the same as if only that one reading were taken. Analogously, checking the clock-time dependence of the rest of the system without destroying what is to be checked could be done by making many copies and measuring a different copy at each different clock reading. Of course, this procedure requires an ability to make copies, a discussion of which would take us too far afield for this paper.

Since the observable conditional probabilities depend only upon stationary operators (the time- averaged projection operators), they are not at all sensitive to terms in the density operator of the system that connect different energy eigenstates. That is, the results are the same for all density operators that are identical in their matrix elements between states of equal definite energy. In particular, the probabilities are the same for the corresponding block-diagonal density matrix in which all terms connecting states of different energy are set equal to zero. The expectation values of all operators have no dependence on $t$ for this density matrix, so it represents a stationary state. As a result, all of the observable clock-time dependence of any state, stationary or not, is precisely the same as that of the corresponding stationary statistical state. Thus there is no way to tell whether or not the Universe is in a stationary state.

In general, the stationary state that gives the correct conditional probabilities will be a mixed state, with nonzero probabilities for different energies. The presence of more than one energy in a system would be detectable to an observer who had access to a large number of copies of the system so that he could determine the conditional probabilities. But if the system is closed and therefore contains the observer, he will not be able to determine the precise conditional probabilities. Since the conditional probabilities for his measurement results are an average over the energy eigenstates, weighted by the diagonal elements of the density matrix, his observations are always consistent with at least one energy eigenstate. Indeed, for the purpose of describing the observed dynamics inside the closed system there is no loss of generality in assuming the system is in a pure state. Thus the Universe may not only be stationary, it may be in an energy eigenstate. The two-particle conditional probabilities given above for the nonstationary state (3) would be the same in any state that agrees with the state (3) in its probability distribution for $m = m_1 + m_2$ and has the same density matrix over $m_1$ and $m_2$ within each two-particle energy eigenspace of fixed $m$. The corresponding two-particle stationary density matrix which gives the same conditional probabilities is

$$\rho_{(12)} = \sum_{m=-2j}^{2j} P(m) \, |2j, m\rangle \, \langle 2j, m| \tag{18}$$

a mixed state composed of pure states

$$|2j, m\rangle = \binom{4j}{2j+m}^{-1/2} \sum_{m_1=-j}^{j} \binom{2j}{j+m_1}^{1/2} \binom{2j}{j+m-m_1}^{1/2} |m_1\rangle \, |m_2 = m - m_1\rangle \tag{19}$$

weighted with probabilities

$$P(m) = 2^{-4j} \binom{4j}{2j+m} \tag{20}$$

The individual result of a complete measurement of the two-particle system in this state would be consistent with at least one of the eigenstates (19) of the density matrix (18), so a measurement within the system could not distinguish the mixed state from an energy eigenstate of the system. But even if one contemplated making measurements on an ensemble of identical systems and wished to maintain the precise values of the conditional probabilities, one could do this by combining each two-particle system with another system having the reversed energy spectrum. One simply takes the pure energy eigenstate of the composite system which has the appropriate probability distribution for the energy of the two-particle subsystem (whose energy is now precisely anticorrelated with the energy of the other subsystem, so that each subsystem is in a mixed state). Since the energy spectrum of the two-particle subsystem is symmetric about zero, it may simply be combined with another two-particle subsystem. Suppose the latter has spin-$2j$ states

$$|2j, m'\rangle = \binom{4j}{2j+m'}^{-1/2} \sum_{m_3=-j}^{j} \binom{2j}{j+m_3}^{1/2} \binom{2j}{j+m'-m_3}^{1/2} |m_3\rangle\, |m_4 = m' - m_3\rangle \tag{21}$$

Then the pure zero-energy eigenstate

$$2^{-2j} \sum_{m=-2j}^{2j} \binom{4j}{2j+m}^{1/2} |2j, m\rangle\, |2j, m' = -m\rangle \tag{22}$$

has the first two-particle subsystem in the stationary mixed state (18), so for measurements confined to that subsystem the conditional probabilities are exactly the same as they are for the nonstationary state (3). This example illustrates that even if one knows that a system is not in an energy eigenstate, it may be part of a larger system which is.

It might be objected that if one knows that a system is in a pure Hilbert-space superposition of different energy eigenstates, it cannot possibly be part of a larger system which is in an energy eigenstate. This is because the joint state must be a product state in order that the first system be in a pure state, and such a product state containing a superposition of different energies cannot be an energy eigenstate. (Compare Refs. 9 and 5, which show that if an additive quantity commutes with the density matrix of a joint system, it also commutes with the density matrix of each subsystem obtained by Landau tracing. ) The rebuttal is that one cannot know that any system is in a pure superposition state of different energies, since the $t$ dependence that would show this is completely unobservable. What one often naively calls an observable $t$ dependence is actually a dependence on a clock reading, and we have argued that this can always be expressed in terms of stationary operators which cannot distinguish energy superpositions from mixtures.

Although the mathematical details might be more cumbersome, we could apply our analysis to any other system that contains some dynamical variable that could serve as a clock. If the projection operator onto a clock reading $\tau$ is $P_\tau$, then the dependence upon $\tau$ of some other dynamical variable $A$ is given by its conditional expectation value

$$E(A|\tau) = \frac{\langle\overline{P_\tau A P_\tau}\rangle}{\langle\overline{P_\tau}\rangle} = \frac{\mathrm{Tr}(\overline{P_\tau A P_\tau}\rho)}{\mathrm{Tr}(\overline{P_\tau}\rho)} \tag{23}$$

where the overbar denotes the averaging over the unobservable $t$ needed to make $\overline{P_\tau}$ and $\overline{P_\tau A P_\tau}$ (which is not, in general, the same as $\overline{P_\tau A P_\tau}$) stationary in order that they might be observable, and $\rho$ is the density operator representing the state of the system. If $A$ is a projection operator onto some eigenspace labeled by $\alpha$, $A = P_\alpha$, then

$$E(P_\alpha|\tau) = P(\alpha|\tau) \tag{24}$$

the conditional probability of the eigenspace $\alpha$ given the clock reading $\tau$.

Note that the conditional expectation values would be the same if $\rho$ were replaced by $\bar{\rho}$, the time average of $e^{-iHt}\rho e^{iHt}$ (rather than of $\rho$ itself, which is constant in the Heisenberg picture we are using). This $\bar{\rho}$ is the stationary state corresponding to $\rho$, with no nonzero terms connecting states of different energy. Thus the observable conditional expectation values give no way to tell whether or not a system is in a stationary state.

# 3. EVOLUTION IN A STATIONARY UNIVERSE

If there is no observable difference between a stationary state of the Universe and a nonstationary state, is it really necessary to have a law of evolution? In the above example the spin direction of each particle precesses with angular frequency $\omega$ ac- cording to clock time, clock time being defined by the state of a particular particle which has been chosen as the clock. Thus the clock-time evolution is the same as what one would normally predict by applying the equation of motion to the spin of each particle. But as we have shown, the state of the whole system, including the clock, could with no observable differences be chosen to be a stationary state, and for such a state the evolution is trivial. In this stationary state, therefore, the clock-time evolution of the system is not being dictated by the usual law of evolution, but rather is determined by correlations between the clock and the rest of the system. This raises the question whether this sort of evolution (evolution by correlations) always imitates evolution as obtained from the equation of motion.

To answer this question let us consider a (not necessarily pure) stationary state $\rho$ of the closed system ($[\rho, H] = 0$), and assume that the closed system consists of two parts: a clock with Hamiltonian $H_c$, and the rest, which has Hamiltonian $H_r$. We assume the clock does not interact dynamically with the rest of the system, so that $H = H_c \otimes I_r + I_c \otimes H_r$, $I_c$ and $I_r$ being the identity operators in their respective spaces. We now define clock time as follows. Choose a special state $|\psi_c(0)\rangle$ of the clock to mark the zero of clock time (an optimal choice for this state may be very difficult, but the optimization is inessential for our discussion), and then with each of the states

$$e^{-iH_c\tau} |\psi_c(0)\rangle \equiv |\psi_c(\tau)\rangle$$

associate the value $\tau$ of clock time. Other states of the clock are assumed not to be associated with any definite value of $\tau$. For any stationary observable $A = I_c \otimes A_r$ f the rest of the system, the conditional expectation value of $A$ given that the clock reads $\tau$ is [cf. Eq. (23) with $A$ and $P_\tau$ stationary and commuting]

$$E(A|\tau) = \frac{\mathrm{Tr}(AP_\tau\rho)}{\mathrm{Tr}(P_\tau\rho)} \tag{25}$$

where

$$P_\tau = |\psi_c(\tau)\rangle \langle\psi_c(\tau)| \otimes I_r$$

We wish to show that this expectation value evolves in clock time in accordance with the Heisenberg equation of motion for the rest of the system.

Using the fact that $[H, \rho] = 0$, we have

$$\begin{aligned}
\mathrm{Tr}(AP_\tau\rho) &= \mathrm{Tr}\left[Ae^{-i(H_c\otimes I_r)\tau} P_0 e^{i(H_c\otimes I_r)\tau}\rho\right] \\
&= \mathrm{Tr}\left[AP_0 e^{-i(H_c\otimes I_r)\tau}\rho e^{i(H_c\otimes I_r)\tau}\right] \\
&= \mathrm{Tr}\left[AP_0 e^{-i(I_c\otimes H_r)\tau}\rho e^{i(I_c\otimes H_r)\tau}\right] \\
&= \mathrm{Tr}\left[e^{i(I_c\otimes H_r)\tau} A e^{-i(I_c\otimes H_r)\tau} P_0\rho\right]
\end{aligned} \tag{26}$$

In the special case where $A$ is the identity operator, this equation tells us that $\mathrm{Tr}(P_\tau\rho) = \mathrm{Tr}(P_0\rho)$, a constant. From these results and from Eq. (25) it follows finally that

$$E(A|\tau)\mathrm{Tr}_r[A_r(\tau)\rho_r] \tag{27}$$

where $A_r(\tau)$ is the Heisenberg-evolved operator

$$A_r(\tau) = e^{iH_r\tau} A_r e^{-iH_r\tau} \tag{28}$$

and $\rho_r$ is the relative density matrix

$$\rho_r = \frac{\mathrm{Tr}_c(P_0\rho)}{\mathrm{Tr}(P_0\rho)} \tag{29}$$

Thus it is not necessary to assume, *a priori*, any equation of motion for the rest of the closed system with respect to coordinate time. If one instead requires that the density matrix of the entire system commute with the Hamiltonian, then it follows automatically that expectation values can be computed from operators obeying the Heisenberg equations of motion with respect to clock time.

Examples of dynamical variables that can serve as clock times are the size or expansion rate of the ob- served part of the Universe, the cosmic radiation density, the relative abundances of radioactive elements, the orientation of the Earth-Sun combination or of the Earth itself relative to distant stars, the vertical angle of a pendulum, the orientation of the hands of a watch, etc. In each case one may consider the conditional expectation values of other variables with respect to these. These realistic clocks do interact with the rest of the Universe so the dependence on clock time will not be so simple as in the noninteracting case analyzed above. Indeed, such an interaction is necessary if the clock is to be read. The determination of a very precise time resolution entails a large energy exchange,[20] so if the total energy available is 'finite, there is a limit to the precision of any clock.[21] The theoretical limit is usually so short that it is ignored. For a good clock the interaction can thus have a practically negligible effect. One may idealize the behavior of actual clocks to form the abstract notion of a preferred coordinate time. ("Time is defined so that motion looks simple."[14]) However, this abstraction, useful though it is, is not directly observable. Only the dependence on an internal clock time can be observable. No one can tell whether something happens at $t = 12$, but one can tell whether it happens at 12 o'clock.

# 4. DISCUSSION

We have shown that although the dependence of a system upon coordinate time is completely unobservable, one can observe a dependence upon an internal clock time. Such a description of dynamics is given entirely in terms of stationary operators, which indeed are the only operators that can be observables. We now consider briefly a number of consequences and extensions of this idea.

The observed arrow of time described by the second law of thermodynamics must be formulated in terms of the dependence of some observable representing entropy upon certain clock times. It is completely unobservable whether or not entropy increases with coordinate time. There is the addition- al question of what observables can be used to define entropy. Most functionals of the density operator are not observable, even if they are invariant under unitary transformations such as those representing time translations. For example, the usual quantum-mechanical formula for the entropy of a system, $-\mathrm{Tr}(\rho \ln \rho)$, is an invariant but depends upon the off-diagonal elements connecting eigenstates of dif- ferent charge or energy, which are completely unob- servable. However, one should be able to formulate an entropy based on coarse-grained properties that one actually observes. Such a quantity need not be zero even for pure states.[21]

In an asymptotically flat spacetime, our analysis for energy would apply equally well for momentum and angular momentum. There should be superselection rules for all of these quantities,[6] either because they are determined by the asymptotic gravitational field which should commute with all quasilocal observables, or because asymptotically defined Poincarè transformations are unobservable: there is no way to determine the absolute position, orientation, or velocity of any system. We thus expect only Poincarè-invariant operators to be observable.

Of course, not all components of the four-momentum $P_\mu$ and the angular momentum tensor $J_{\mu\nu}$ commute, so the Universe cannot be in an eigenstate of all of them. However, it would be consistent with observations to assume the Universe is in an eigenstate of any combination that do commute, though which combination is chosen is completely arbitrary. For example, one might assume for simplicity that the Universe has a definite energy, zero spatial momentum, zero spatial angular momentum $J_{ij}$, but completely undetermined time-space angular momentum components $J_{0k}$ (which represent the energy multiplied by the position of the center of mass at $t = 0$). Another .choice would be to assume all $J_{\mu\nu}$ are zero but that only the magnitude $P_\mu P^\mu$ of the four-momentum is definite. Then the Universe would have a definite rest mass and zero spin, and its center of mass would pass through the coordinate origin at $t = 0$ with a completely undetermined velocity. No observations could contradict either assignment or any other, but no observations could confirm any choice either.

If one goes to curvilinear coordinates, only generally covariant operators can be observable. In flat spacetime the Minkowski metric reduces the symmetry to the Poincarè group, but in quantum gravity in which the metric is treated as a dynamical variable, the spacetime intervals that in flat spactime are invariant under the Poincarè group would have to be defined relative to intrinsic geometrical variables. For example, the distribution of fields over a spatial hypersurface cannot be uniquely specified by some functional dependence on an arbitrary choice of coordinates but can only be unambiguously defined relative to some three-geometry of the surface which gives invariant distance relations between the points.

Many of the ideas of this paper have been ex- pressed before, particularly in the context of quantum gravity where after much initial confusion it has now long been recognized that observable time can only be defined intrinsically.[21-29] However, it has not generally been recognized that the same concepts also apply in quantum field theory in flat spacetime or even in nonrelativistic quantum mechanics. For example, the axiomatic approach to quantum field theory in terms of the algebra of operators in definite regions of Minkowski space[30] should be reformulated in terms of Poincarè-invariant observables. No operator that depends explicitly or implicitly upon coordinate time is observable.

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