I. INTRODUCTION

Newton created classical mechanics almost single-handedly in less than three years, starting in the late summer of 1684 and completing his *Mathematical Principles of Natural Philosophy* (usually called the *Principia* from their Latin title) in the early summer of 1687. The creation of quantum mechanics took a little longer, starting with de Broglie’s wave nature of the electron in 1924 and ending with the general quantization of wave fields by Heisenberg and Pauli in 1929. The intervening five years present a unique flowering of new ideas that form the conceptual base for all of modern physics.

Quantum mechanics describes physics on the atomic scale. The word atom was chosen to apply to an object that cannot be cut into smaller pieces. Therefore, its mathematical description requires concepts that contradict to some extent the foundations of classical mechanics. Since our intuition is based on our experience with objects on the macroscopic scale, however, we have to deal with the contrast and take advantage of any connections between the apparent opposites when we try to solve particular problems. The alternative would be a purely logical application of abstract rules, such as one could expect a computing machine to handle all by itself. The results would still have to be interpreted just as any data from a survey or a controlled experiment. Thus, in order to understand modern physics, we have to be aware of the interplay between classical and quantum mechanics.

The same situation occurs in the relations between geometric (ray) optics and wave optics, although the contrast between the macroscopic and the microscopic scale is not as extreme and the special applications may be quite different. The time interval for working out the conceptual difficulties, however, was much longer since the diffraction of light is first described in the work of Grimaldi, which was published in 1665. The resulting controversy was only settled by Young and Fresnel 150 years later. Another century passed before the electromagnetic theory of light got its final triumph when the diffraction of x rays by crystals was discovered by Laue in 1912. This happened after Einstein had revalidated the corpuscular nature of light in his work on the light quanta.

Wave phenomena could be observed during all this time most easily on the surface of water. The mathematical theory of hydrodynamics and acoustics was developed without much controversy. In the *Principia*, Newton discusses the
motion of a fluid through the opening of a screen, complete with the standard picture of a wave that is diffracted as it passes a narrow opening in a wall. The interplay between classical and quantum mechanics has a close analog both in the electromagnetic theory of light, and its wave phenomena themselves can be visualized most directly in the propagation of sound and water waves.

These analogies played a crucial role in the historical development of physics on the atomic scale, and they are an important part of the introductory teaching in modern physics. But they are usually hidden in the textbooks and monographs, and they are rarely acknowledged in the articles of the main scientific journals. Apparently, most authors nowadays feel completely comfortable with applying quantum mechanics to their special area of research. They are concerned exclusively with proving that their results are in total agreement with the mathematical predictions of quantum mechanics. Consequently, they tend to hide any approximate and intuitive idea that could be used (and probably was used) as a simple preliminary check, and provide some motivation to the reader. Many theoretical physicists seem to imitate the mathematicians in eliminating any crude approximation from their publications because it could be misleading to the readers.

This Resource Letter is meant to provide some relevant information for three distinct, but related purposes: (i) to cover the areas of physics where the interplay between classical and quantum mechanics may help in explaining the basic ingredients; (ii) to list the most easily accessible sources, primary as well as secondary, that deal with the historic transition from classical to quantum mechanics; and (iii) to provide a rough idea of the recent applications where the semiclassical approach to quantum mechanics has been used with some success.

II. THE HISTORICAL APPROACH

The main purpose of this Resource Letter might be best served if the transition from classical to quantum mechanics is studied in the chronological order of events. Such a program is hard to carry out, however, because many crucial problems were solved quite unexpectedly and completely in a very short time. It was as if a mighty fortress had been besieged for decades, and then taken by storm after the troops had been able to penetrate through a small opening in the walls, and overwhelm all further resistance. The exact process of conquest is of less interest than the stubborn resistance that preceded the capture.

The development from classical to quantum mechanics will be divided into four periods. Each period is defined as a time interval between some definite years; these limits were chosen to some extent in order to have simple dates that are related to our year of 1996.

The Classical Period—from 1596 to 1895. This period starts with the year of Kepler’s first publication, Mysterium Cosmographicum (The Secret of the Universe), which is in rough coincidence with Galileo’s first works, De Motu (On Motion) of 1590 and Le Meccaniche (On Mechanics) of 1600. Preceding both is a marvelous book by Simon Stevin of Brugge in Flanders of 1586, De Beginseilen der Weeghconst (The Beginnings of the Art of Weighing).

These three centuries cover several basic developments in classical physics: the progress of mechanics from its first formulation by Isaac Newton to the sophisticated formalism of Lagrange, Hamilton, and Jacobi; the advance of optics from the geometry of light rays to the wave phenomena of diffraction and interference; and the development of field theories to cope with hydrodynamics, acoustics, elastic vibrations, and particularly, electricity and magnetism.

The nineteenth century also produced the laws of thermodynamics and the beginning of statistical mechanics; the same century brought us spectroscopy and its many applications in gases, liquids, and solids; finally, there is the spectacular development of chemistry that led to the structure of many molecules and the periodic table of elements. All of these areas had a profound effect on the birth of quantum mechanics, and were in turn deeply affected by it.

The Transition Period—from 1896 to 1925. Some of the world-shaking discoveries that mark its beginning include: x rays by Wilhelm Conrad Roentgen in 1895, radioactivity by Antoine-Henri Becquerel in 1896, and the electron by Joseph John Thomson in 1897. This period covers the early work by the great pioneers Max Planck, Albert Einstein, and Niels Bohr, who found the first explanations for the simplest problems in the interaction between isolated atoms like hydrogen and helium, and the electromagnetic field. They were the masters in stretching the validity of classical mechanics way beyond its natural domain with the help of purely empirical rules.

The Modern Period—from 1926 to 1965. In the first six months of this period the four monumental papers on wave mechanics by Erwin Schrödinger were produced. Physics once more looked like an open book where the answers to all questions can be reduced to very few elementary principles that are expressed in precise mathematical language. There no longer seemed any need to fall back on the magical inventions of the transition period, because the explanation for every concrete experiment seemed reducible to a tractable, mathematical problem. No discrepancy was ever found in the comparison between the results of the laboratory and of the quantum-mechanical theory.

The Post-Modern Period—from 1966 until now. This period is characterized by the realization, on one hand, that the basic issues concerning nuclear and elementary-particle physics are still far from resolved, and on the other hand, that most of the important phenomena on the atomic scale and above cannot be readily deduced from the accepted foundations in these fields.

In spite of the tremendous increase of scientific activity in these last 30 years, the simple-minded optimism of the Modern Period for an ultimate understanding of the whole universe is no longer justified. Physics has acquired many of the attributes of engineering where the known fundamental principles are used to think of inventions and novel applications, and complete explanations are a luxury to be left for later generations.

Since the problems to be tackled have become much harder, there is more emphasis on intuitive insight, and approximate methods are appreciated because they provide a better understanding. Therefore, classical pictures and models are again called upon, as they were in the period of transition. But they are now used in the full knowledge of what an exact treatment would demand in problems from the atomic scale on up, or of the large territory still to be discovered in the nuclear and elementary-particle domain. The date of 1966 was chosen by the author because it marks the beginning of his work on the connection between classical and quantum mechanics, which was the first to take into account the possibility of chaotic behavior in classical mechanics.
Each of these four periods has contributed in its own way to the interplay between classical and quantum mechanics. It seemed, therefore, most natural to divide up the vast number of individual connections according to their origin in one of these four time intervals. The bridges from one to the other will be pointed out if they appear to be particularly significant. But within each period the various items will be grouped together according to their technical areas.

Each of the four consecutive periods will be covered in this Resource Letter. The text dealing with each period will be subdivided into sections that are devoted to the main topics of investigation during that period, as far as they are related to the basic issue of this Resource Letter. Inside each section, we will try to adhere to the usual format with subsections for each of the various modes of publication.

III. SOME REMARKS CONCERNING THE CHOICE OF THE REFERENCES

Sections V–VII contain mostly the titles of textbooks and monographs, whereas Sec. VIII contains mostly the titles of conference proceedings and collections, plus a large selection of articles in contemporary scientific journals. Many institutional libraries have recently adopted a policy where books are kept in the accessible stacks, but bound journals older than ten years are relegated to some warehouse where they can be recovered on special request; browsing in old journals is essentially discouraged.

Most journals are actively trying to limit the number and length of the articles that are published. Unfortunately, the authors do not seem to cooperate, and the result is an exponential growth at a rapid rate of the number of pages published in almost every journal. At the same time, either the area covered in any one article gets smaller in order to ensure rapid dissemination of new information, or the same information gets spread out with minor variations and in different combinations, particularly in conference proceedings.

Nevertheless, there is a lot of very good work coming out all the time. But the monthly tables of content demand more patience and concentration if a potential reader wants to find the items that are most interesting in any special field. General classifications are hard to establish because any article is likely to touch more than one area. Section VIII is subdivided into many more categories than Secs. V–VII to help the reader through the available literature. These sections are not meant to propose a systematic and exhaustive scheme, but only a practical way to distinguish among the existing publications.

The original intention of the author was to put together a list of references that could make some claims to completeness, at least for the purpose of an introduction concerning the interplay between classical and quantum mechanics. This topic, however, not only covers all of the twentieth century and pervades a large part of the active branches in physics, but it has also become ever more active in the last 30 years, with many applications coming into view all the time. Therefore, the present list is hardly more than a personal collection coming from somebody who has been an active participant in the recent revival. Some helpful suggestions by Doug Stone and Turgay Uzer are gratefully acknowledged.

IV. JOURNALS

American Journal of Physics
Annals of Physics (New York)

CHAOS
Chemical Physics Letters
International Journal of Bifurcation and Chaos
Journal of Chemical Physics
Journal of Mathematical Physics
Journal of Physics A and B (occasionally G)
Nature
Nonlinearity
Nuclear Physics A
Physica D (occasionally A)
Physical Review (before being split into different parts)
Physical Review A and E (occasionally B and D)
Physical Review Letters
Physics Reports
Physics Today
Physics World
Proceedings of the Royal Society of London, Series A
Progress of Theoretical Physics
Reports of Progress in Physics
Reviews of Modern Physics
Science
Science News
Scientific American
SIAM (Society for Industrial and Applied Mathematics)
Review
Zeitschrift für Physik B and D

V. THE CLASSICAL PERIOD

Although some of the original texts from this period may be hard to obtain, the author has found many of them languishing in university libraries, completely unappreciated for their critical role in the development of physics. The author would like to encourage the reader to take at least a casual look at them, even if they are written in an unfamiliar language, nothing worse than Latin in any case, and often available in English translation. The density of new information is usually orders of magnitude above the standard publication in our time.

A. Some historical documents of classical mechanics

1. The Art of Weighing, Simon Stevin (De Beghinselen der Weeghconst beschreven duer Simon Stevin van Brugghe, tot Leyden Inde Druck- eye van Christoffel Plantijn, By Francoys van Raphelighen, 1586). Reprinted with English translation in Principia Works of Simon Stevin, edited by E. J. Dijkerhuis (C. V. Swets & Zeitlinger, Amsterdam, 1955), Vol. 1, General Introduction, Mechanics. A marvelous exposition of the statics, including hydrostatics, shortly before the beginning of the seventeenth century when dynamics became the focus of scientific activity. This text is full of illustrations and practical examples, and its title page is decorated with a sketch of the experimental evidence for equilibrium on an inclined plane. (E)

2. On Motion and On Mechanics, Galileo Galilei, Comprising De Motu (~ 1590) translated with Introduction and Notes by J. E. Drabkin and Le Meccaniche (~ 1600), translated with Introduction and Notes by Stillman Drake (The University of Wisconsin Press, Madison, 1960). In contrast to Stevin’s work these texts are relatively short and much more philosophical; they are also accompanied by extended historical comments. Obviously, Galileo is still bound to the scientific discussions of antiquity and middle ages, and his skill in problems of statics hardly matches Stevin’s. Yet, we are less than 100 years away from Newton’s breakthrough in the Principia. (I)

4. Sir Isaac Newton’s Mathematical Principles of Natural Philosophical and His System of the World, translated into English by Andrew Motte in 1729, revised by Florian Cajori (California U.P., Berkeley, 1934), reprinted in Great Books of the Western World (Encyclopædia Britannica, Chicago, 1952), Vol. 34 as well as in several other recent translations, e.g., by I. B. Cohen and Anne Whitman (to appear in the University of California Press, Los Angeles, 1997). For any person interested in physics, there is nothing at all comparable to reading this fundamental text, which is the foundation of everything that has happened since its first publication in 1687. The technical requirements are no more than high-school geometry, and an appreciation for the great leap forward that occurred with this book. (A)

5. The Key to Newton’s Dynamics—The Kepler Problem and the Principia, J. Bruce Brackenridge with English translation from the Latin by May Ann Rossi (University of California Press, Berkeley, 1995). A mostly technical presentation of the general reasoning and the detailed arguments of Newton to prove Kepler’s laws from the assumption of the gravitational force between two masses. The text follows the original draft On Motion and the relevant sections of the Principia; a high-school student should be able to understand everything with the help of all the figures. Learned historical comments are not always well separated, and may discourage some people’s interest. (I)


B. The history of (classical) mechanics

7. The Mechanization of the World Picture (Pythagoras to Newton), E. J. Dijksterhuis, translated by C. Dikshoorn (Oxford U.P., London, 1961; reprinted by Princeton U.P., Princeton, NJ, 1986). One of the best and most complete accounts of the historical process that led from the Greek views of science through late antiquity, the Middle Ages, and the Renaissance to Newton’s synthesis. The world runs like a mechanical construction on the basis of some well-defined mathematical relations that allow precise predictions. (I)

8. The Great Physicists from Galileo to Einstein, George Gamow, originally published as Biography of Physicists (Harper Brothers, New York, 1961; reprinted by Dover, New York, 1988). Most enjoyable discussion of physics in its historical development, requiring some mathematical literacy, but lightened with many of the author’s drawings. Special effort is devoted to the discussion of wave phenomena in various contexts. (E)

C. Ray and wave optics

9. Traité de la Lumière, Christiaan Huygens, Leiden 1690 (original written in French, translated by S. P. Thompson in volume 34 of Great Books of the Western World, Encyclopedia Britannica, Chicago 1952, pages 545–619). Generally recognized as the first discussion of wave optics, and of Huygens’s principle where each disturbance spreads in a spherical wave, and every point in turn serves as the origin of the secondary wave. This idea leads directly to Feynman’s path integral in quantum mechanics. It is important to emphasize, however, that Huygens’s principle deals with a single pulse traveling at the speed of light, and not with a sinusoidal wave as in most optical experiments (such as Grimaldi’s). (E)

10. Opticks or a Treatise of the Reflexions, Refractions, Inflexions and Colours of Light, Isaac Newton (London, 1704). Published many times thereafter; available with a foreword by A. Einstein in Dover Publications, New York, 1979, as well as in Great Books of the Western World, Encyclopedia Britannica, Chicago 1952, pp. 373–544), Vol. 34. Although Newton’s many discoveries in optics were made 30 years earlier, this work is his definitive explanation. While the bulk deals with ray optics and particularly with the refraction of white light by a prism, the last part deals with “inflexion,” Newton’s word for diffraction that has not survived, and for a good reason. He recognizes Grimaldi’s work in the first sentence, but he has an explanation in terms of particle trajectories that are bent in the neighborhood of a sharp edge. (I)

11. The Wave Theory of Light—Memoirs of Huygens, Young, and Fresnel, edited by Henry Crew (American, New York, 1900). A short history of the most important developments in the nineteenth century precedes the crucial papers by Huygens, Young, and Fresnel, each of which is followed by a short biographical sketch. A very useful collection. (E)

12. The Nature of Light—A Historical Survey, V. Ronchi, translated by V. Barocas (Harvard U.P., Cambridge, MA, 1970). A readable account of the long history that starts in antiquity and the middle ages, and leads to the great debate between ray and wave optics at the beginning of the nineteenth century. This story is important in our context because the relations between classical and quantum mechanics are clearly foreshadowed in the optical phenomena. They have directly inspired a lot of the crucial experiments that established quantum mechanics as the ultimate theory on the atomic scale. (E)


15. The Feynman Lectures on Physics, Volume I: Mainly Mechanics, Radiation, and Heat, Richard P. Feynman, Robert B. Leighton, and Matthew Sands (Addison–Wesley, Reading, MA, 1963). Among the many elementary textbooks on classical physics, this one is mentioned because of its five chapters, 26–30, which cover various parts of optics, from Fermat’s principle to diffraction, with a minimum of mathematical formalism and a maximum of intuition. A person with only a beginner’s training in mathematics should be able to learn about the most important ideas. (E)

16. Almost All About Waves, John R. Pierce (MIT, Cambridge, MA, 1974). A very general introduction to vibration and wave phenomena requiring only relatively simple mathematics. (E)


D. Classical field theories


E. Thermodynamics and statistical mechanics

20. Elements of Classical Thermodynamics for Advanced Students of Physics, A. B. Pippard (Cambridge U.P., Cambridge, 1957). In spite of its title the demands on mathematical sophistication are quite limited, but the discussions are simple and clear. (E)
VI. TRANSITION

Since the Transition Period came to an end 70 years ago, it is receding into the distant past. Its eminent representatives died a generation ago, and their work is no longer studied for its own merit, but only as a stepping stone to a more complete and definitive theory. Nevertheless, their struggles with the inherent difficulties of quantum mechanics, which still is the first to realize that even for this form to be applied, the system cannot be allowed to be chaotic (to use the modern term). The seminal importance of this deceptively simple paper was not recognized for another 40 years, and not fully understood for another 20 years. (I)

A. Historical surveys

1. Textbooks and monographs

22. Planck’s Original Papers in Quantum Physics, German and English edition, annotated by Hans Kangro, translated by D. ter Haar and Stephen Brush (Wiley, New York, 1972). This small paperback contains the two short papers that Planck read at the meetings of October 19 and December 14, 1900, to interpret the recent measurements of the blackbody radiation. The arguments in the second paper are quite simple, and the constant $h$ is defined, with a numerical value too small by only 1%. (E)

23. Thirty Years that Shook Physics—The Story of the Quantum Theory, George Gamow (Doubleday, New York, 1966; reprinted by Dover, New York, 1985). A highly entertaining introduction for the layperson who is able to read simple mathematical formulas, and enjoy the author’s amusing drawings as well as the photographs of all the participants. The Appendix contains the English translation (from the original German) of the famous Copenhagen “Faust,” a physicist’s spoof of Goethe’s drama. (E)

24. Sources of Quantum Mechanics, edited with historical introduction by B. L. van der Waerden (North-Holland, Amsterdam, 1967; reprinted by Dover, New York, 1968). A selection of the essential papers including some letters from the short, but crucial period when modern quantum mechanics was developed first by Bohr and his school with the help of the correspondence principle, and then by Heisenberg, Born, Dirac, and Pauli. All in English translation with illuminating comments. (I)


27. The History of Quantum Theory, Friedrich Hund, translated by Gordon Reed (Harper & Row, New York, 1974). A rather detailed historical account in fairly simple language, but requiring a good knowledge of the underlying physics. Mathematical formulas are used freely, and many references to the original works are provided. A very useful introduction into a more thorough study of the whole development. (E)

28. Niels Bohr’s Times (in Physics, Philosophy, and Polity), Abraham Pais (Clarendon, Oxford, 1991). Perhaps the best biography of Niels Bohr as the great innovator, skillful organizer, and fearless leader in the transition from classical to quantum mechanics. It is written for the scientist who is already familiar with the most important developments in the first half of this century, and contains a great many details about the individual events and their main actors, but without giving any of the formal mathematical arguments. (I)

29. Order, Chaos, Order—Transition from Classical to Quantum Physics, Philip Stehle (Oxford U.P., New York, 1994). A well-researched history of the 30 years from 1895 to 1925, which is written for the beginning student in physics. Some of the crucial experiments are described, and a few theoretical developments are explained in separate boxes. The reader will gain appreciation for the difficulties and occasional detours that led to the ultimately satisfying outcome. (E)

2. Articles in scientific journals

30. “On the Quantization Condition of Sommerfeld and Epstein.” Albert Einstein, translation by Charles Jaffe of the article “Zum Quantensatz von Sommerfeld und Epstein” from A. Einstein in Verhandlungen der Deutschen Physikalischen Gesellschaft 19, 82–92 (1917); Joint Institute for Laboratory Astrophysics (JILA) Report No. 116 (1980). The quantization condition is rewritten in a form that does not refer to any special coordinate system. But Einstein (with a reference to Poincaré) is the first to realize that even for this form to be applied, the system cannot be allowed to be chaotic (to use the modern term). The seminal importance of this deceptively simple paper was not recognized for another 40 years, and not fully understood for another 20 years. (I)


B. Photons

1. Textbooks and monographs

32. Black-Body Theory and the Quantum Discontinuity 1894–1912, T. S. Kuhn, second edition with a New Afterword (The University of Chicago Press, Chicago, 1987; first Edition, Oxford U.P., New York, 1978). A historical account of the thermodynamics, and then statistical mechanics, concerned with the blackbody radiation. The central role of Max Planck in this development is studied with great care, with the help of many details and numerous notes at the end as well as an extensive bibliography. There have been some vocal objections, however, both to Kuhn’s interpretation of science history in terms of revolutions, and to his view of the developments in Planck’s views; see R. Jost (1995) Ref. 34. (I)

33. A History of Theories of Aether and Electricity, Volume II: The Modern Theories 1900–1926, Sir Edmund Whittaker (first published by Nelson and Sons, London, 1933; reprinted by Humanities, New York, 1973). This second volume covers the transition period again very thoroughly (cf. the mention of the first volume in the preceding chapter), including special relativity, the old quantum theory, gravitation, and the discovery of the new quantum mechanics. It has acquired a special reputation because it assigns most of the credit for the discovery of special relativity to Poincaré and Lorentz. (I)


C. Atoms

1. Textbooks and monographs

35. Atomic Structure and Spectral Lines, Arnold Sommerfeld, translated from the third edition of Atombau und Spektrallinien by H. L. Brouse (Methuen, London, 1923). The whole generation of physicists who started the modern period was raised on the strength of this basic text. It still is the best introduction to the atomic physics of the transition period, an ideal combination of simple qualitative arguments with a minimum of elementary mathematics, covering the whole area including experiments. (E)

36. The Mechanics of the Atom, Max Born, translated by J. W. Fisher and D. R. Hartree (Bell, London, 1927; Unger, New York, 1960). The original edition was completed in November 1924, just before the great breakthrough to the new quantum mechanics. The classical mechanics of multiply periodic motion is discussed in general terms, as the basis for the spectra and the interaction with the electromagnetic field. Examples include the Zeeman and Stark effects, and the single electron in the potential of two fixed nuclei; perturbation theory is used for a first attempt at the helium atom. (I)

37. The Quantum Theory of the Atom, George Birtwistle (Cambridge U.P., Cambridge, 1926). A contemporary textbook for the student who...
is familiar with basic physics and wants to learn the best explanations then available, starting with radiation theory and classical mechanics to deal with the spectral features of atoms and molecules. An excellent account. (I)

38. Quantum Principles and Line Spectra, J. H. Van Vleck, Bulletin of the National Research Council, Vol. 10 part 4, published by the National Academy of Sciences, Washington DC, 1926. A very complete presentation of the ‘‘old’’ quantum theory, its explanations and results, with special emphasis on the correspondence principle of Bohr for the interpretation of the spectral intensities. Its unusually large set of references to contemporary work also includes the first reference on page 44 to Einstein’s 1917 paper on the quantization conditions in a non-separable system; I have found only one more reference (by Lanczos 1949 in Sec. VII A; Ref. 49) until Keller’s 1958 paper in Sec. VII C; Ref. 75. (A)

39. Zeemaneffekt und Multiplettstruktur der Spektrallinien, E. Back und A. Landé (Springer-Verlag, Berlin, 1925). A status report by the experts on both experiments and theory concerning atomic spectroscopy, on the eve of the big breakthrough. The detailed knowledge of the facts is impressive, and is reminiscent of the present condition in nuclear and particle physics where a consistent theory is also missing right now. (I)

2. Articles in scientific journals


D. Molecules

42. ‘‘Über das Modell des Wasserstoffmolekül,’’ W. Pauli, Jr., Ann. Phys. (Vierte Folge) 68, 177–240 (1922). This published version of Pauli’s Ph.D. dissertation with Sommerfeld is remarkable because it fails to predict the simplest case of molecular binding in spite of its impressive mastery of the old quantum theory. The reason is the erroneous handling of the angular momentum quantization, cf. Strand and Reinhardt in Sec. 4.11; Ref. 282. (I)

E. Thermodynamics

I. Textbooks and monographs

43. The New Heat Theorem—Its Foundation in Theory and Experiment, W. Nernst, translated from the second edition by Gay Barr (Methuen, London, 1926; reprinted by Dover, New York, 1969). The definitive account and textbook of the so-called third law of thermodynamics, the vanishing of the entropy at the absolute zero of the temperature scale, which turned out to have its explanation in quantum mechanics. (I)

F. Statistical mechanics


I. Articles in scientific journals


VII. MODERN PERIOD

A. Classical mechanics with a modern viewpoint

The emphasis in classical mechanics has changed greatly since Newton, and the arrival of quantum mechanics in the 1920s can be seen in the choice of the methods and the examples that are discussed. But the new ideas from the work of Poincaré have not penetrated into the awareness of most physicists. The solutions of problems in classical mechanics, and quantum mechanics as well, are still presented as a matter of finding the right transformation of coordinates.

1. Textbooks and monographs

48. Classical Mechanics, Herbert Goldstein (Addison–Wesley, Reading, MA, 1950). A generation of physicists has been brought up with this textbook. It emphasizes subjects that are close to the problems of quantum mechanics such as central forces, rigid-body rotation, small oscillations, special relativity, the Hamiltonian formalism, and some simple field theory. There are many references with short comments to related monographs. (I)

49. The Variational Principles of Mechanics, Cornelius Lanczos (University of Toronto Press, Toronto, 1949; reprinted by Dover, New York, 1986). A very useful, well organized, and quite explicit presentation of classical mechanics for physicists rather than mathematicians, starting from the variational principle in its different garbs, as well as the resulting canonical formalism, and some beginner’s field theory. It contains the second reference to Einstein’s 1917 paper on quantization conditions, Ref. 30. (I)

50. A Treatise on Analytical Dynamics, L. A. Pars (Heminen, London, 1965). A valuable textbook where traditional mechanics is developed systematically from general principles, emphasizing direct insight rather than strict mathematical analysis, but with many applications at each stage. There is still no awareness of the unavoidable irregularities in the typical motions; all the methods and examples allow explicit solutions in terms of relatively simple formulas. (I)

51. Variational Principles in Dynamics and Quantum Theory (3rd ed.), W. Yourgrau and S. Mandelstam (Saunders, Philadelphia, 1968). A fine historical survey starting with Fermat and his successors in classical mechanics, going on to the ‘‘old’’ and ‘‘new’’ quantum mechanics including Feynman and Schwinger. By way of contrast even hydrodynamics is covered; there is a general emphasis on some of the philosophical meaning. (A)

52. Mechanics: Classical and Quantum, T. T. Taylor (Pergamon, Oxford, 1976). Although this textbook was written in the ‘‘Post-Modern’’ Period, the author decided to adopt the views of the earlier ‘‘Modern’’ Period. The connection between classical and quantum mechanics is found in the similarities of the mathematical formalisms rather than the phenomena. (I)

B. The new wave mechanics

No serious student in physics can do without any knowledge of wave mechanics as it can be found in a vast number of textbooks. The intended audience for these books covers a wide range, from pure mathematicians who are interested in proving general theorems, to engineers and chemists who want to concentrate on limited and well-defined applications. We will mention only the relatively few books where the transition from classical to quantum mechanics gets more than just a perfunctory treatment. A number of monographs can be considered classics in QC because of their independent approach and depth.
Finally, some authors have tried to present the subject without the help of its almost inevitable mathematical apparatus, but rather by appealing to the reader’s intuition with the help of ingenious illustrations. This kind of effort is of great importance for physics, however, because it helps in making the origin and the practice of quantum mechanics more understandable to the lay people. A teacher might find some inspiration to get away from a purely formal discussion of the important concepts.

1. Qualitative discussions for the general reader
a. Textbooks and monographs

53. The Strange Story of the Quantum, Banesh Hoffmann (Harper, 1947, Dover, New York, 1959). “An account for the general reader of the growth of ideas underlying our present atomic knowledge.” Written in a colloquial style, with a minimum of mathematics and few pictures, for a reader who needs constant encouragement and enjoys the little devices of live conversation. (E)

54. Are Quanta Real? A Galilean Dialogue, J. M. Jauch (Indiana U.P., Bloomington/London, 1973). The main issues concerning classical and quantum mechanics are discussed in a fictional dialogue between the three persons of Galileo’s Dialogue on the Two Major Systems of the World of 1632 and his Discourses and Demonstrations Concerning Two New Sciences of 1638. Salvati represents the modern scientist, Sagredo is the intelligent layman, and Simplicio the defender of the conventional wisdom. A clever imitation of Galileo’s learned and polemic language. (E)

55. In Quest of the Quantum, Leonid Ponomarev, translated from the Russian by Nicholas Weinstein (Mir, Moscow, 1973). A very entertaining and well-written introduction with hardly any mathematical formulas, but numerous whimsical drawings to emphasize some of the simple arguments. Short biographies of the main actors are found throughout. (E)

56. Taking the Quantum Leap; the New Physics for Non-Scientists, Fred Alan Wolf (Harper & Row, San Francisco, 1981). A very attractive, general introduction for the layperson into the difficulties that haunt quantum mechanics. The questions concerning the wave nature of a particle are emphasized with the help of many sketches, and occasional works of art, as well as pictures and biographical notes on the main actors. (E)

57. The Conceptual Development of Quantum Mechanics (2nd ed.), Max Jammer (American Institute of Physics, New York, 1989). An understandable and extremely well documented history that explains the origins of quantum mechanics starting with Planck’s work on blackbody radiation and going into some of the controversies in the interpretation of wave mechanics. The author is remarkably even-handed and stays away from unwarranted generalizations and formalistic arguments. A unique source of information.

58. The Historical Development of Quantum Mechanics (5 volumes), J. Mehra and H. Rechenberg (Springer-Verlag, New York, 1982 to 1987). A report that reads more like a story for the general public with many anecdotal details, but contains a lot of information not generally found, e.g., the discovery of QM (Vol. 2), the formulation of matrix mechanics and its modifications (Vol. 3), Erwin Schrödinger and the rise of wave mechanics (Vol. 5). (I)

2. General discussion with a philosophical bent
a. Textbooks and monographs

59. The New Science—Where is Science going? The Universe in the Light of Modern Physics, The Philosophy of Physics, Max Planck, with a preface by A. Einstein and an introduction by James Franck (Meridian, 1959). This volume 3 of Planck’s Complete Works contains many of the popular talks that its illustrious author gave throughout his life, after his original discovery had completely transformed classical physics, and his personal life ended in tragedy during and after World War II. (E)

60. The Physicist’s Conception of Nature (Symposium in honor of the seventieth birthday of P. A. M. Dirac), edited by Jagdish Mehra (Reidel, Dordrecht, 1973). A large and remarkable collection of articles by many of the most outstanding participants in the development of QM. Hardly any special problems in physics are discussed, although the contrast between CM and QM is often mentioned in many different contexts, such as the classical electron, indeterminacy, symmetry, irreversibility, the measurement process. (A)

3. Fundamental treatises for the beginning expert
a. Textbooks and monographs

61. Problems of Atomic Dynamics, Max Born (MIT, Cambridge, 1926). Reprinted as a paperback by MIT Press in 1970. These 30 lectures were given at MIT between November 14, 1925 and January 22, 1926; the first 20 lectures are concerned with the structure of atoms, and the last 10 with the lattice theory of solids. This is the first account of the operator version of quantum mechanics as discovered by Heisenberg, and elaborated by Born, Jordan, and Pauli, including the latter’s theory of the hydrogen spectrum; its preface is dated January 22, 1926, so that this text predates Schrödinger’s first paper. The arguments that led from the ‘‘old’’ to the ‘‘new’’ quantum theory, i.e., from the reliance on classical mechanics to the use of matrices, are discussed in a way that is no longer found in the textbooks, unfortunately. (I)

62. The Principles of Quantum Mechanics, P. A. M. Dirac (Oxford U.P., London, 1930, 1935, 1947). This monograph is the first comprehensive text of the new quantum mechanics, and each new edition presents some important new view of the author. It is in a class by itself because it records the progress in his systematic development that starts from his insights into classical mechanics. The second edition presents the first approach to the path integral, and the third is phrased in the formalism of bra’s (and ket’s) whose juxtaposition forms the bracket (|⟩ or scalar product. (A)

63. Lectures on Quantum Mechanics, Paul A. M. Dirac (Belfer Graduate School of Science, Yeshiva University, New York, 1964). A slender volume of four lectures where Dirac exposes his views on the process that leads from the classical Hamiltonian mechanics through the study of Poisson brackets to the quantization in both curved and flat spaces. Although the ideas are general and abstract, the formalism is simple in the Dirac fashion. (I)

64. The Feynman Lectures on Physics, III. Quantum Mechanics, Richard P. Feynman, Robert B. Leighton, and Matthew Sands (Addison–Wesley, Reading, MA, 1965). This textbook on QM is unique because it tries to motivate everything for an audience that is not familiar with the use of mathematical language. There is an abundance of simple sketches with explanations based on intuition, and that very often means pictures from classical mechanics. Special attention is devoted to the effects of symmetry, which are usually hard to grasp except through formal arguments. (E)

b. Articles in scientific journals


C. WKB wave functions and path integrals

1. Textbooks and monographs

66. An Introduction to Phase-Integral Methods, J. Heading (Wiley, New York, 1962). A short and very useful introduction to the problems of the transition from CM to QM for the motion in one dimension. After a good historical survey, the WKBJ solutions and the Stokes phenomenon are discussed with the help of the functions in the complex plane. The behavior near transition points (from classically allowed to forbidden regions) as well as various applications including waves in the ionosphere are treated. (I)

67. Quantum Mechanics and Path Integrals, R. P. Feynman and A. R. Hibbs (McGraw–Hill, New York, 1965). Introductory textbook that treats many examples very explicitly, but only when they are amenable to complete solutions, i.e., integrable. (I)

68. The Phase Approximation to the Theory, Nanny Fryman and Per Olof Fryman (North-Holland, Amsterdam, 1965). A short but systematic treatise where this approximation to Schrödinger’s equation in one dimension is discussed in the complex plane with some mathematical sophistication, including higher-order terms in powers of Planck’s constant. (A)
69. Semi-Classical Approximations in Quantum Mechanics, V. P. Maslov and M. V. Fedoriuk (Reidel, Boston, 1981). This is the first English version of a book in Russian by the first author from the early 1960s that was translated into French in 1972. Although the phase-integral solution of Schrödinger’s equation is treated in more than one dimension, and many clever models are discussed, the emphasis lies on the mathematical justification rather than on the physical ideas. The main challenge of dealing with the classically chaotic motions is not taken up. Maslov’s name seems forever tied to the phase losses at caustics. (A)

70. Classical Dynamics and its Quantum Analogues, David Park (Springer, New York, 1979). Although less than 20 years old, these lecture notes give the standard version of the classical-quantal connection, with all the familiar examples worked out in detail. The explanations and the mathematics are kept at a level for undergraduates, and include discussions of Hamiltonian dynamics, perturbation theory, rigid body, and simple continuous systems like strings. (E)

2. Articles in scientific journals

71. “Space-Time Approach to Non-Relativistic Quantum Mechanics,” R. P. Feynman, Rev. Mod. Phys. 20, 367–387 (1948). [Reprinted in Selected Papers on Quantum Electrodynamics, edited by Julian Schwinger (Dover, New York, 1958).] Dirac’s idea is carried a step further by interpreting the propagator (or probability amplitude) in quantum mechanics as an integral over all the different paths between the given end points where the integrand is the exponential of the classical action integral. Classical mechanics comes out immediately because the path of stationary action with respect to small variations is the one to suffer least from the destructive interference among the paths. Feynman’s view has become the starting point for most of the more refined models in QM. (I)

72. “On the Definition and Approximation of Feynman’s Path Integral,” C. Morette, Phys. Rev. 81, 848–852 (1951). The first attempt of a mathematical proof to show that the path integral is a solution of Schrödinger’s equation for sufficiently short times. (A)


74. “Reflections in One-Dimensional Wave Mechanics,” R. Landauer, Phys. Rev. 82, 80–83 (1951). Correction to the WKBJ approximation by including multiple reflections in one dimension. (E)

75. “Corrected Bohr–Sommerfeld Quantum Condition for Nonseparable Systems,” J. B. Keller, Ann. Phys. (N.Y.) 4, 180–188 (1958). The seminal paper of Einstein from 1917 is interpreted quantum mechanically: Each invariant torus in phase space is covered by a simple wave function which is then projected onto position space, and leads to the quantization conditions, including the effect of caustics. (I)

76. “Asymptotic Solutions for Eigenvalue Problems,” J. B. Keller and S. I. Rubinow, Ann. Phys. (N.Y.) 9, 24–75 (1960). The method of Keller’s paper above is used to treat a whole collection of integrable, but not easily separable, systems with two degrees of freedom: The possibility of classical chaos is not discussed at all. (A)

D. Atomic spectra

1. Textbooks and monographs

77. “Quantum Mechanics of One- and Two-Electron Atoms”, Hans A. Bethe and Edwin E. Salpeter (Springer-Verlag, Berlin, 1957). Second enlarged edition of Bethe’s article of the same title in the Handbuch der Physik, Vol. XXIV in 1933. It is meant as a reference work pertaining to the calculations and their comparison with experiment, and is intended for ‘graduate students who wish to learn ‘applied quantum mechanics.’ ‘giving’ ‘low-brow’ explicit derivations.’ ‘Specific application to atomic systems of general field-theoretic results is described in detail.’ A unique source of information in an area where semiclassical methods have recently been used with some success. (I)

78. Atomic Spectra and Atomic Structure, Gerhard Herzberg, translated from Atomspektren und Atomstruktur by J. W. T. Spinks (Prentice–Hall, New York, 1937). The introductory volume to a series of monographs on spectroscopy for the specialist in the field. The author discusses the experimental techniques as well as the results in a systematic manner using the most direct theoretical approach, including classical arguments. (E)

79. The Spectrum of the Atomic Hydrogen, G. W. Series (Oxford U.P., New York, 1957). A short technical history of this central subject, from the earliest results in the late nineteenth century to the latest triumphs of the quantum electrodynamics. The newest high-precision data are quoted and discussed, but not fully explained. (I)

80. Atomic Spectra, H. G. Kuhn (Longmans, London, 1962). An elementary introduction that uses the classical pictures whenever useful, and includes a discussion of the most important experimental evidence. (E)

81. Structure of Matter. W. Finkelburg, translation from German by O. Mattosi-Riechemier (Springer-Verlag, New York, 1964). A very successful introductory text into nuclear and atomic, as well as some molecular and condensed matter physics, on the basis of simplified quantum pictures. (E)


E. Spin and statistics

83. “Exclusion Principle and Spin,” B. L. Van der Waerden in Theoretical Physics in the Twentieth Century, A Memorial Volume to Wolfgang Pauli, edited by M. Fierz and V. F. Weisskopf (Inter- science, New York, 1960), p. 199. Gives the best account of the difficult process of getting Pauli to accept the electron spin in the critical year of 1925; his objections were exactly the quasiclassical picture that was used by the discoverers, Kronig as well as Goudsmit and Uhlenbeck. (I) [cf. the remarks by A. Herrmann, co-editor of Pauli’s Correspondence, Volume 1. 1919–1929 (Springer-Verlag, Berlin, 1979), p. XV]

F. Scattering

84. The Theory of Atomic Collisions, N. F. Mott and H. S. W. Massey (Oxford U.P., London, 1933) (first edition, with a second edition concentrating more on nuclear problems in 1949). The first systematic introduction into the practical solution of problems in the scattering of particles. Starting with electrons in a Coulomb field, including spin, proceeding to whole atoms hit by electrons elastically and inelastically, ending with massive particles in gas and solids, as well as nuclear phenomena, including relativistic two-body problems and radiation. (A)

G. Molecular bond

85. Band Structure and Molecular Structure, R. de L Kronig (Cambridge U.P., London, 1930). A primer on the analysis of molecular structure with the help of rotational and vibrational spectra, on the basis of quantum mechanics. The previously semiempirical results can now be explained quite systematically and completely. (E)

86. Introduction to Quantum Mechanics with Applications to Chemistry, Linus Pauling and E. Bright Wilson (McGraw–Hill, New York, 1935). The textbook for a generation of chemists trying to learn an important subject with an unexpectedly heavy use of mathematics. Short chapters on classical mechanics and the old quantum theory are followed by the regular fare and occasional excursions into the semi-classical regime. Subjects of interest to chemists such as the molecular bond, the rotation, vibration, as well as excited states of molecules, and the Franck–Condon principle are emphasized. (I)

87. Molecular Spectra and Molecular Structure—I. Spectra of the Diatomic Molecules, II. Infrared and Raman Spectra of Polyatomic Molecules, Gerhard Herzberg (Van Nostrand, Princeton, NJ, 1939 for first volume; second volume and enlarged edition of the first, 1950). The basic and, at its time, complete monograph on spectra of diatomic molecules. It can fairly be proclaimed as the book that proved quantum mechanics to be the foundation of molecular physics, and therefore of all chemistry. But the experimental results are way ahead of any detailed calculations, so that explanations have to remain qualitative, using simple, sometimes classical arguments. Both volumes are an impressive demonstration of the amount of knowledge in this special field that was already known, and understood in a qualitative way, some 50 years ago. (A)
I. Second quantization and field theory

1. Textbooks and monographs

88. The Theory of the Properties of Metals and Alloys, N. F. Mott and H. Jones (Clarendon, Oxford, 1936; reprinted by Dover, New York, 1958). The conduction of electrons in a solid is always approached classically to the extent that external fields, both electric and magnetic, are applied, and a Boltzmann-like transport equation is solved. (I)

89. Quantum Theory of Solids, R. E. Peierls (Clarendon, Oxford, 1955). Introductory text of unusual clarity and simplicity of mathematical tools by one of the original contributors in this field. (I)

90. Solid State Physics, N. W. Ashcroft and N. D. Mermin (Holt, Rinehart and Winston, Philadelphia, 1976). A comprehensive textbook for graduate students in physics that starts out with three valuable sections on the old Drude and Sommerfeld theories of metals, as well as their failures. The remainder of this impressive work gives frequent discussions of the limits for many of the standard methods in the field, as well as extensive experimental data. (I)

I. Second quantization and field theory

1. Textbooks and monographs

91. The Quantum Theory of Radiation, W. Heitler (Oxford U.P., London, 1936). The first monograph on this topic, and the reference work until the renewal of the radiation theory in the late 1940s by Schwinger, Tomonaga, Feynman, and Dyson. The classical theory is first treated in its Hamiltonian form, which is then quantized first in vacuo and then in interaction with matter. This represents the prelude to the great explosion of quantum field theory that now dominates high-energy physics as well as other areas like superconductivity, magnetism, and statistical mechanics. All these developments are included in the much larger third edition which never became as important as this first one. (A)

92. Quantum Electrodynamics—34 selected papers, edited by Julian Schwinger (Dover, New York, 1958). A collection of the important papers that led to the development of the modern theory in the late 1940s including the radiative corrections in atomic physics. (A)


J. Quantum statistics

1. Textbooks and monographs


96. Thermodynamics and Statistical Mechanics, A. H. Wilson (Cambridge U.P., London, 1957). A very useful monograph that covers wide areas and discusses many applications in simple mathematical language without much fuss about the philosophical foundations. Classical as well as quantum mechanical aspects of statistical mechanics are treated in some detail, including the third law of thermodynamics, electric and magnetic phenomena. (I)

97. Ergodic Theory in Statistical Mechanics, I. E. Farquhar (Interscience, London, 1964). Survey of both the classical and quantum ergodic theory in the form that was originally proposed by Boltzmann, i.e., without the sophisticated techniques of functional analysis, emphasizing coarse graining, classical phase-space theorems, averaging over initial states. (I)

98. The Principles of Statistical Mechanics, Richard C. Tolman (Oxford U.P., London, 1938). A broadly based effort to "give a reasonably clear and complete picture," "from a modern point of view," of "the more powerful methods of Gibbs" both in the classical and quantum domain. Only relatively simple examples are treated as illustrations of the fundamental ideas, which are presented in verbal rather than abstract mathematical terms. (A)

99. Statistical Mechanics, Kerson Huang (Wiley, New York, 1963). Standard text for the beginning student in this area, before the more formal methods from field theory became dominant. (A)

2. Articles in scientific journals

100. "On the Quantum Correction for Thermodynamic Equilibrium," E. Wigner, Phys. Rev. 40, 749–759 (1932). The "Wigner distribution function" for a quantum system is defined in this paper, and shown to satisfy an equation reminiscent of Boltzmann's classical distribution in phase space. Wigner's idea has inspired countless applications where the statistical features are inherent in quantum mechanics rather than coming from thermodynamics. (I)

VIII. POST-MODERN PERIOD

The last 30 years in almost every area of physics have seen an accumulation of results that far surpasses what any individual is able to appreciate. Whereas this growth seems exponential at a relatively steady rate in many well-established fields, it looks as if a sleeping giant had been awakened in the special area of this Resource Letter.

The sudden explosion is intimately connected with the growing awareness of what is now called chaos, for the lack of both a better word and of any description, let alone explanation, of this general phenomenon. The various quarters where quantum mechanics is the fundamental tool have responded quite differently. Nuclear, atomic, and molecular physicists have been in the forefront of trying out new ideas for making the connection with classical mechanics. Condensed-matter physicists are jumping on the band wagon with a vengeance right now, but the main practitioners of high-energy and even of statistical physics have as yet to take notice.

Some of the fields with a long history, going back more than a century, experience this development differently because quantum mechanics is not part of their foundation. Therefore, fluid dynamics and acoustics will not be mentioned although they face similar problems in reconciling the ray and the wave picture. The same holds true for all of optics over its wide range of frequencies, with an additional complication. The electromagnetic field has to be "second-quantized" to allow for the photons as classical particles with Bose–Einstein statistics. The resulting behavior of many nonlinear devices in quantum optics is again chaotic, but it is described by the same theory as the chaos in classical mechanics, and is hardly mentioned in this Resource Letter.

The vast literature has been organized somewhat artificially into separate groups, but many textbooks, monographs, and conference proceedings spread out into more than one of these classifications. More serious, however, is a large amount of duplication where the same author contributes very similar papers to different conference proceedings. Many summary descriptions may sound alike, so that one collection of contributed articles may be just as good as another. The reader suffers no great loss if only one such collection is available at the local library. Yet many of these proceedings provide a good starting point to find out what is happening and who is active in any of the specialties.

It would have been impossible to give even a cursory survey of the papers that appear in the archival journals, because so many people are working hard and publish their results as soon as they have something to say. I have made a choice of articles that I consider particularly interesting be-
cause of some new result or novel idea. Such an article may not always provide the most efficient route for the newcomer, compared with some later discussion of the same topic, but it may offer a better insight into the discovery process at the price of some extra effort. But on the whole, the list of publications below is no more than a sampling from a vast set, with some inevitable personal bias.

A. The new classical mechanics

1. Textbooks and monographs

101. Dynamical Systems, George D. Birkhoff (American Mathematical Society Colloquium Publications, volume IX, published by the AMS, Providence, RI, 1927, with a new edition in 1966). "It represents essentially a continuation of Poincaré’s profound and extensive work on Celestial mechanics... the style may appear less formal and rigorous than is now (1966) customary. But just the informal and lively manner of writing has been inspiring to many mathematicians." (I)


103. Introduction to Dynamics, I. Percival and D. Richards (Cambridge U.P., Cambridge, 1982). A very useful short textbook to get an undergraduate student acquainted with the modern ideas and simple examples (and many problems) in classical mechanics that will lead eventually to (post-modern) quantum mechanics. (E)


108. Newton’s Clock—Chaos in the Solar System, I. Peterson (Freeman, New York, 1993). A very entertaining and highly instructive introduction to chaotic phenomena for the general reader. The history of coping with the astronomical observations of the motions in the solar system is told with many interesting sidetracks, as well as illustrations, diagrams, and explicit data. (E)

2. Conference proceedings


110. "Order in Chaos," Proceedings of a Conference at the Center for Nonlinear Studies in Los Alamos, New Mexico 1982, edited by D. Campbell and H. Rose [Physica D 7 (1983)]. One of the first get-togethers on chaos covering many aspects from experimental observations, mathematical properties and model systems, fractal structures, transition to chaos in circle maps, fluids and vortices, with a special section (for the first time) on quantum chaos. (I)


112. Hamiltonian Dynamical Systems, a reprint selection compiled and introduced by R. S. MacKay and J. D. Meiss (Adam Hilger, Bristol, 1987). A substantial collection containing articles from various scientific journals, about equilibria and periodic orbits, quasiperiodic orbits and the breakup of invariant tori, chaotic behaviors and many applications. (A)

3. Articles in scientific journals

113. "The Applicability of the Third Integral of Motion: Some Numerical Experiments," M. Hénon and C. Heiles, Astron. J. 69, 73–79 (1964). Two harmonic oscillators are coupled by the simplest third-order coupling, which leads to the most popular model for the transition from integrability at low energies to classical chaos at higher energies. (I)


115. "A Universal Instability of Many—Dimensional Oscillator Systems," B. V. Chirikov, Phys. Rep. 52, 263–379 (1979). The nonlinear coupling between linear oscillators is studied in detail with many numerical examples in order to establish the instability due to resonance overlap, various mechanisms of effective diffusion in phase space, and the meaning of entropy. (A)


118. "Chaotic Scattering modelled by an Inclined Billiard," M. Hénon, Physica D 33, 132–156 (1988). A very simple model for a classical particle that bounces off a corrugated boundary, and where a complete symbolic dynamics can be defined. (E)

119. "A General Model for Motion bound to an Impurity in an anisotropic semiconductor," J. Casasayas, A. Nunes, and A. M. Ozorio de Almeida, Physica D 48, 311–321 (1991). The chaos that is found in the anisotropic Kepler problem is shown to be a very general feature for a whole class of motions in potentials of similar kind. (A)


B. Integrable versus chaotic features in quantum mechanics

1. Textbooks and monographs

121. Hamiltonian Systems—Chaos and Quantization, Alfredo M. Ozorio de Almeida (Cambridge U.P., Cambridge, 1988). An introduction for theoretical physicists starting with classical mechanics and its techniques in the nonintegrable cases, and continuing on to some of the novel methods in quantum mechanics, such as Wigner functions, random matrices, and periodic orbits. (I)


123. Chaos in Classical and Quantum Mechanics, Martin C. Gutzwiller (Springer-Verlag, New York, 1990). The first half is concerned with a modern approach to classical mechanics with few degrees of freedom, including different types of approximation methods, periodic orbits and surfaces of section, entropy, and the discussion of instructive examples that are usually left to the specialists. The second half deals with the quantum mechanics of integrable versus chaotic classical sys-
systems, in particular their wave functions and energy-level statistics, the connection to periodic orbits in the trace formula, and again some important examples in detail. (I)

124. Quantum Signatures of Chaos, Fritz Haake (Springer-Verlag, Berlin, 1991). The statistics of the energy levels in QM gets a careful and quite complete discussion on the basis of rather abstract and general ideas, but with only few concrete examples. This monograph uses simple and yet quite elaborate mathematics; it ends with a discussion of dissipative systems. (A)

125. The Transition to Chaos (in Conservative Classical Systems: Quantum Manifestations), Linda E. Reichl (Springer-Verlag, New York, 1992). A very complete and exhaustive treatise that covers the whole development from classical mechanics in its conventional form including area-preserving maps and the diffusion of trajectories, all the way to spectral statistics, semiclassical theory, driven and stochastic systems in QM. A well-organized and largely self-contained reference work in these areas with many illustrations and examples. (A)

126. Semiclassical Physics, M. Brack and R. K. Bhaduri (Addison–Wesley, Reading, MA, 1997). This textbook starts with some simple examples of the relation between classical and quantum mechanics, and goes on to explain how the density of states as well as individual energy levels are obtained in nonintegrable systems, with the help of many practical illustrations. (I)

2. Conference proceedings and collections

127. “Stochastic Behavior in Classical and Quantum Hamiltonian Systems,” Volta Memorial Conference in Como, Italy, 1977, edited by G. Casati and J. Ford (Springer-Verlag, Berlin, 1979). The first conference in this area assembled some of the pioneers coming from mathematics, astronomy, and theoretical physics. This volume is a source for some of the original work, and presents the various viewpoints and motivations. In particular, the editors demonstrate the absence of quantum diffusion in the kicked-rotator model. (I)

128. “Chaotic Behavior in Quantum Systems—Theory and Applications,” Proceedings of the NATO Advanced Research Workshop in Como, 1983, edited by G. Casati (Plenum, New York 1985). This sequel to the first Como meeting covers a spread of topics that is more narrowly focused on the relations between classical and quantal systems, but it gives a good picture of the field at this, still early date, covering all the fundamental, theoretical and experimental, problems that have since become branches of their own. (I)


134. “Periodic Orbit Theory in Classical and Quantum Mechanics,” Focus issue edited by P. Cvitanovic, Chaos 2, 1–158 (1992). An outstanding collection of contributions from the NORDITA program on “Physics of Quantum Chaos and Measurement” and the NATO Advanced Research Workshop on “Quantum Chaos—Theory and Experiment” in Copenhagen. The special role of the classical periodic orbits in various mathematical problems and physical experiments is explored; the importance of this topic had been first recognized only 20 years earlier. (I)


138. Quantum Chaos—between Order and Disorder, edited by G. Casati and B. Chirikov (Cambridge U.P., Cambridge, 1995). The most up-to-date collection with a number of reviews rather than reports on recent results. (I)

3. Articles in scientific journals


142. “Semi-Classical Quantization on Adiabatically Generated Tori, or Einstein on the Brink,” W. P. Reinhardt and R. E. Gillilan, in Path Integrals from meV to MeV, edited by M. C. Gutzwiller et al. (World Scientific, Singapore, 1986). The well-known adiabatic principle of quantum mechanics is used to find the quantization in the chaotic region by starting in the regular region where the conventional quantization conditions hold. (E)


C. Path integrals

1. Textbooks and monographs

158. **Morse Theory**, J. Milnor, based on lecture notes by M. Spivak and R. Wells (Princeton U.P., Princeton, NJ, 1963). Marston Morse laid the foundations in the 1930s for a general theory to understand the global behavior of variational quantities, like the integral over the Lagrangian in mechanics. His discussion of the second variation for the extremal curves (Morse index) is essential for the phases in all the semiclassical approximations. (A)

159. **Techniques and Applications of Path Integration**, L. S. Schulman (Wiley-Interscience, New York, 1981). A very well-organized general introduction to the idea of path integrals in all its many manifestations. The developments are carried out very explicitly with explanations concerning the motivation of each step. The applications cover first of all quantum mechanics, and go on to optics, polarons, spins, and multiply connected spaces, statistical mechanics, droplets, plus some mention of more difficult subjects from these areas. (I)


2. Conference proceedings


164. **Path Integrals from meV to MeV.** Proceedings of the Fourth Meeting of Path Integrals from meV to MeV, held in Tutzing, Bavaria, 1992, edited by H. Grabert, A. Inomata, L. S. Schulman, and U. Weiss (World Scientific, Singapore, 1993). Emphasis on the relevance in a wide variety of different fields (e.g., polymers, solid-state physics, disordered background), including quantum chaos. The path integral is seen as an indispensable aid to our intuition. (I)

3. Articles in scientific journals

165. **“Gaussian Path Integrals,”** G. J. Papadopoulos, Phys. Rev. D 11, 2870–2875 (1975). Gives the simplest practical method to obtain the semiclassical approximation to Feynman’s path integral. (E)

166. **“Oscillatory Integrals and the Method of Stationary Phase in Infinitely Many Dimensions, with Application to the Classical Limit of Quantum Mechanics,”** S. Albeverio and R. Hoegh-Krohn, Invent. Math. 40, 59–106 (1977). One of the more successful schemes to put Feynman’s path integral on a firm mathematical basis. (A)

167. **“Solution of the Path Integral for the H-Atom,”** H. Duru and H. Kleinert, Phys. Lett. B 84, 185–188 (1979). With the help of a new time-like variable a formal nonlinear transformation reduces the path integral to harmonic oscillators, but this procedure is not shown to be legitimate. (I)

168. **“Exact-Path-Integral Treatment of the Hydrogen Atom,”** R. Ho and A. Inomata, Phys. Rev. Lett. 48, 231–234 (1982). The nonlinear transformation of the coordinates to avoid the singularity in the trajectory, due to Kustanheimo and Stiefel, is carried out inside the path integral at the price of some rescaling of time; this exact evaluation of the path integral is now justified. (I)

D. Negative curvature surfaces and Riemann zeta function

1. Textbooks and monographs

169. **Scattering Theory for Automorphic Functions**, P. D. Lax and R. S. Phillips (Princeton U.P., Princeton, NJ, 1976). The classical motion on surfaces of negative curvature (along geodesics) is ideally chaotic; this monograph discusses the scattering of waves under these conditions, and provides the mathematical foundations for their treatment. (A)

170. **“The Selberg Trace Formula for PSL(2,R),”** D. A. Hejhal, Lecture Notes in Mathematics 548 and 1001 (Springer-Verlag, Berlin, 1976). The definitive monograph on the Selberg trace formula and all its variations in the special case of the two-dimensional surfaces of constant negative curvature which is the ideal case of purely hyperbolic (“hard”) classical chaos. The mathematics is rigorous, but not excessively abstract. (I)
E. Energy levels and eigenfunctions

1. Articles in scientific journals

172. “Stochastic Behavior in Quantum Scattering,” M. C. Gutzwiller, Physica D 7, 341–355 (1983). The model of a two-dimensional box (torus) with an exceptional point at infinity is realized with the help of the modular region in Poincare’s hyperbolic plane. The scattering phase shift for a wave is given by Riemann’s zeta function on the line x=1, and represents the essence of quantum chaos: analytically smooth and yet effectively unpredictable. (E)


175. “A New Asymptotic Representation for zeta (1/2+i) and Quantum Spectral Invariants,” M. V. Berry and J. P. Keating, Proc. R. Soc. London, Ser. A 437, 151–173 (1992). The famous method of Riemann and Siegel for computing the zeta function is interpreted in the light of the trace formula, and this idea leads to a remarkable improvement that avoids some of the well-known difficulties in lowest order. (A)


F. Periodic orbits and the trace formula

1. Articles in scientific journals


194. “Periodic Orbits and Classical Quantization Conditions,” M. C. Gutzwiller, J. Math. Phys. 12, 343–358 (1971). The trace of Green’s quantum Sinai billiard is consistent with the predictions of the Gaussian orthogonal ensemble which is known from the level distribution in nuclear physics. (E)
function for an arbitrary system is shown to become a sum over classical periodic orbits in the semiclassical limit, thus yielding what is now called the general trace formula, in particular when the orbits are unstable as in the anisotropic Kepler problem. (I)


198. “The Classical Quantization of a Hamiltonian with Ergodic Behavior,” M. C. Gutzwiller, Phys. Rev. Lett. 45, 150–153 (1980). The trace formula is shown to agree with Selberg’s for a particle on a surface of constant negative curvature, and is used to calculate the energy levels of the anisotropic Kepler problem by adding over the binary code for the classical periodic orbits. (A)

199. “A Rule for Quantizing Chaos?” M. V. Berry and J. P. Keating, J. Phys. A 23, 4839–4849 (1990). The trace formula is rewritten to look like the Riemann zeta function, so that the Riemann–Siegel method for computing can be used. That leads to a drastic reduction of the infinite sum (or product) and suggests a duality between short and long periodic orbits. (A)


202. “Semiclassical Quantization of Multidimensional Systems,” E. B. Bogomolny, Nonlinearity 5, 805–866 (1992). The quantization of an arbitrary system with $k$ degrees of freedom is reduced to a map of a $(k−1)$-dimensional surface on itself, which can be directly expressed in classical terms and leads to the periodic orbits. (A)

203. “Prebifurcation Periodic Ghost Orbits in Semi-Classical Quantization,” M. Kus, F. Haake, and D. Delande, Phys. Rev. Lett. 71, 2167–2171 (1993). Corrections to the trace formula are necessary in the neighborhood of a classical bifurcation in order to smooth out the singularity; these are explained in terms of complex solutions in phase space, and confirmed in the case of the “kicked top.” (A)


G. Maps and billiards

I. Articles in scientific journals

209. “Quantizing a Classically Ergodic System: Sinai’s Billiard and the KKR method,” M. V. Berry, Ann. Phys. (N.Y.) 131, 163–216 (1981). A pioneering article in the quantum-mechanical treatment of ergodic billiards, including not only the first numerical computations but also a new derivation of the trace formula for billiards of the Sinai type. (A)

210. “Chaos, Quantum Recurrences, and Anderson Localization,” S. Fishman, D. R. Grempel, and R. E. Prange, Phys. Rev. Lett. 49, 509–512 (1982). The periodically kicked rotor is mapped into the motion of a particle through a one-dimensional lattice with a random local potential so that its eigenstates become localized à la Anderson rather than the usual Bloch waves in a regular lattice. This unexpected idea demonstrates that the energy in the quantal kicked rotor does not increase linearly with time as it would classically. (A)


215. “Semiclassical Dynamics of Chaotic Motion: Unexpected Long-Time Accuracy,” S. Tomsovic and E. J. Heller, Phys. Rev. Lett. 67, 664–667 (1991). The quantum mechanical wave function starting in a fixed point at time 0 is constructed with the help of the classical trajectories in the stadium billiard, and the result is found to follow the exact wave for a long time in spite of the classical chaos. (E)


219. “Exact and Quasiclassical Fredholm Solutions of Quantum Billiards,” B. Georgeot and R. E. Prange, Phys. Rev. Lett. 74, 2851–2854 (1995). Although the frequency distribution in vibrating membranes was first established by H. Weyl in 1912 with the help of integral equations, this seems to be the first mathematically correct treatment with the help of the Fredholm method, and yields convergent expressions for the trace formula. (A)


H. General atomic physics

I. Textbook and monograph


2. Conference proceedings and collections

227. "Quantum Coherence—30 Years of the Aharonov–Bohm Effect,' Conference at the University of South Carolina, edited by J. S. Anandan (World Scientific, Singapore, 1990). A fine survey of the state-of-the-art concerned with geometric phases, experiments on the AB effect in resistive devices, and philosophical as well as practical issues related to Bell’s inequality. (A)

228. Irregular Atomic Systems and Quantum Chaos, edited by J.-C. Gay, Comments At. Mol. Phys., volume 25; and as book by Gordon and Breach, Philadelphia, 1992. A fairly complete and up-to-date collection of articles, written for this volume, and centered around the quantum-mechanical few-body problem in atomic physics. The connection with recent high-precision experiments is brought out more than usual. (A)


230. Chaos in Atomic Physics, R. Blümel and W. P. Reinhardt (Cambridge U.P., New York, 1997). This textbook starts with some typical examples of classical chaos, and goes on to discuss in detail the main manifestations in atomic physics, especially, in hydrogen and helium as well as scattering in simple molecules including the latest research. (I)

3. Articles in scientific journals

231. "Phase-Integral Approximation in Momentum Space and the Bound States of an Atom—I and II," M. C. Gutzwiller, J. Math. Phys. 8, 1979–2000 (1967) and 10, 1004–1020 (1969). A fairly systematic effort to get the semiclassical approximation of the Feynman path integral in different coordinate systems, and apply the results to the energy levels and eigenstates in a spherically symmetric potential; the relation between caustics in classical mechanics and special phases in quantum mechanics is established. (A)

232. "The Quantization of a Classically Ergodic System," M. C. Gutzwiller, Physica D 5, 183–207 (1982). The energy levels for the anisotropic Kepler problem in two dimensions are calculated by adding over the periodic orbits, including the effect of reflection symmetries. (A)


237. "Resonances and Recurrences in the Absorption Spectrum of an Atom in an Electric Field," J. Gao and J. B. Delos, Phys. Rev. A 49, 869–880 (1994). Since the electric field does not destroy integrability, the old quantization rules apply, and their connection with periodic orbits is used as first pointed out by Gutzwiller, and then by Berry and Tabor. (I)

238. "The Classical Limit of an Atom," M. Nauenberg, C. Stroud, and J. Yeazell, Sci. Am. 270, 44–49 (June 1994). Pulses of laser light make giant atoms whose properties come from both worlds, classical and quantal (cf. further references to articles by the authors at the end). (E)


I. Hydrogen and Rydberg atoms

1. Textbooks and monograph


2. Conference proceedings and collections


3. Articles in scientific journals

248. ‘‘Hydrogen Atom in a strong Magnetic Field.’’ Y. Yafet, R. W. Keyes, and E. N. Adams, J. Phys. Chem. Solids I, 137–142 (1956). Although this research was carried out during the ‘‘modern’’ period, its results are important for understanding the recent experiments. (E)


256. ‘‘Long-Period Orbits in the Stark Spectrum of Lithium,’’ M. Courtney, H. Jiao, N. Spellmeyer, and D. Kleppner, Phys. Rev. Lett. 73, 1340–1343 (1994). Recurrence spectra show some very long periodic orbits along the direction of the electric field. (I)


258. ‘‘The Importance of Resonances in Microwave ‘Ionization’ of Excited Hydrogen Atoms,’’ P. M. Koch and K. A. H. van Leuwen, Phys. Rep. 255, 289–403 (1995). The experiments on the ionization of highly excited H atoms by microwaves are reviewed in detail; they are the first where the effects of chaos in the classical system could be demonstrated. (I)

259. ‘‘Nonstationary, Nondispersive Wave Packets in a Rydberg Atom,’’ A. F. Brunello, T. Uzer, and D. Farrelly, Phys. Rev. Lett. 76, 2874–2877 (1996). Wave functions in a circularly polarized microwave field and a magnetic field yield eigenstates that are scarred by periodic orbits from the classically chaotic system. (I)


J. Helium and planetary atoms

1. Textbook

261. Analytic Perturbation Theory for the Two-Electron Problem—From Perturbation Theory for Linear Operators, T. Kato (Springer-Verlag, New York, 1966), pp. 410–413. The higher-order terms in the perturbation expansion for the eigenvalues of the atomic two-electron problem are shown to converge provided the nuclear charge is larger than 4. (A)

2. Articles in scientific journals


K. Molecular physics

The items in this and some of the following sections came to the attention of the author without any systematic effort on his part to understand all the problems in this area that can be treated with the help of classical mechanics. They are presented here in order to encourage the reader to go to the library of the physics and chemistry departments, and find out the many fields where the quantal nature of physics and chemistry can be illustrated in terms of classical concepts.

1. Textbook


2. Conference proceedings and collections


3. Surveys


4. Articles in scientific journals


L. Nuclear physics and spins

1. Monograph and survey


296. Quantum Chaos—A New Paradigm of Nonlinear Dynamics, K. Nakamura (Cambridge U.P., Cambridge, 1993). In spite of its general title this monograph concentrates on the chaotic motion of small spin systems as well as spin waves in solids. The treatment is quite systematic and complete with many references, but it requires a fair degree of mathematical sophistication. (I)

2. Conference proceedings


3. Articles in scientific journals

306. “Semiclassical Periodic Orbit Theory for Identical Particles,” Hans A. Weidenmüller, Phys. Rev. A 48, 1819–1823 (1993). The trace formula is generalized to include a system of several particles interacting while they obey the Pauli exclusion principle. (E)

M. Field theory

Instantons are a special manifestation of the much older solitons that go back to some simple hydrodynamics at the beginning of the nineteenth century. Mathematicians discussed them as solitary waves due to gravity on the interface between water and air. Physicists picked up the idea in the 1950s and 1960s, and it finally entered particle physics with a 1974 paper listed below. Their implicit hope was that elementary particles would be directly related to soliton-like solutions of their classical nonlinear field theories. But Derrick’s theorem (cf. below) shows that solitons occur only in 1+1 dimensions, and, quite generally, they are tied to the possibility of completely integrating the equations of motion, which is a very unlikely circumstance at best (cf. below).

1. Conference proceedings

309. “The Principles of Instanton Calculus: A Few Applications,” J. Zinn-Justin, in Recent Advances in Field Theories and Statistical Mechanics, edited by J. B. Zuber and R. Stora. Les Houches Lectures XXXIX (Elsevier, Amsterdam, 1984), pp. 39–172. Rather down-to-earth collection of significant examples with explicit calculations that lead to instantons, such as Phi-Four, i.e., the scalar field phi to the fourth power, in various dimensions, the “false vacuum,” large-order perturbation theory. (I)

2. Articles in scientific journals


312. “Quantum Meaning of Classical Field Theories,” R. Jackiw, Rev. Mod. Phys. 49, 681–706 (1977). Review of the then recent work on quantizing the vibrations around the simplest solution of a classical field theory. (I)

N. Clusters

1. Articles in scientific journals

317. “Model Nuclei in the Form of Metal Clusters,” S. Bornholm and J. Pedersen, Nucl. Phys. News 1, 18–22 (1991). The asymptotic distribution of energy levels in a finite volume is used to explain the occurrence of small clusters in certain sizes and shapes. (E)

O. Statistical mechanics

1. Textbooks and monographs


2. Conference proceedings


3. Articles in scientific journals

322. “Supersymmetry and the theory of disordered metals,” K. B. Efetov, Adv. Phys. 32, 53–127 (1983). The random element now is the distribution of impurities (not the motion due to heat); the modes of the electron diffusion, the distribution of energy levels, and the localization of electron states is investigated with the help of a new approach that is called the supersymmetry method, or also the nonlinear sigma model, and has an obvious connection with classical mechanics. (A)

353. Quantum Complexity in Mesoscopic Systems. Papers from the conference at the Center for Nonlinear Studies at Los Alamos, New Mexico 1994, edited by Alan R. Bishop, Robert E. Ecke, and Ronnie Mainieri, Physica D 83 (1995). The quantum manifestations of classical chaos are presented along with the recent results from the experiments on mesoscopic systems, going from the question of decoherence to the quantum Hall effect, and including the quantum corrolas on the surface of a copper crystal. (I)


3. Articles in scientific journals


R. Quantum optics

1. Textbooks and monographs


373. Elements of Quantum Optics (2nd ed.), P. Meystre and M. Sargent III (Springer-Verlag, Berlin, 1990). Readable textbook for graduate students willing to face some algebra, covering wide variety of topics from bistability to squeezed states. (I)

374. Optical Coherence and Quantum Optics, L. Mandel and E. Wolf (Cambridge U.P., Cambridge, 1995). The most up-to-date and complete textbook in this area, explaining all the mathematical details very explicitly, e.g., Chap. 9 on semiclassical Theory of Photoelectric Detection of Light. (A)

2. Collections


3. Articles in scientific journals


S. The interplay between classical and quantum mechanics

List of titles for a Reprint Book from the Resource Letters in AJP


