FRACTAL APPLICATIONS IN LANDSCAPE ECOLOGY

1. Fractal Geometry: Pure Fractals

A. What is Not a Fractal?

Consider any geometric object with increasing detail (i.e. finer grain). For non-fractal geometric shapes, enlargements (dilations) become smoother. A curve becomes a straight line (defining the tangent at point P). We can say that the curve is "attracted by dilation" to a straight line. Every standard (non-fractal) shape converges under dilation (i.e. increasing detail) to a "universal attractor."

Another way of thinking about this is that for standard geometric objects, "increasingly accurate measurements based upon successive scale reductions give series converging to a limit: the true extent of the object." (Li 2000 p. 34). In other words, the object has some fundamental grain, and we "lose the shape" as we consider finer scales. If we are looking at a square with length X, we see only line segments or points as we view it with window sizes less than X^2.

B. What is a Fractal?

For objects characterized by fractal geometry, shapes remain unchanged as the scale of observation is progressively refined. "The structure of every piece holds the key to the whole structure" (Mandelbrot 1989, p. 4).

Increasingly accurate measurements no longer converge to the true extent of the object, since the object (e.g. a curve) does not converge to a single rectifiable length or area. Instead, the process of repeatedly zooming in to observe increasing detail will generate infinite series with common dimension D, the fractal dimension. Unlike standard geometric objects, fractal objects have a non-linear dimension.

A classic example of this is a coastline. When smaller units (e.g. pieces of string on a planar map) are used to measure a coastline, the apparent length of the coastline becomes greater. For example, a Spanish encyclopedia gives the length of the common border between Spain and Portugal as 987 km, whereas the Portuguese, using finer measuring units, arrived at a length of 1214 km for the same border (Zeide 1991).

Fractal patterns are generated by iterative processes. Simple rules for pattern generation can generate rich, complex structures. Since the same pattern generation rule is applied at all scales, pattern at different scales is self-similar. Self-similarity may also be referred to as scale invariance.

Self-similarity means that spatial objects or time series phenomena reveal an underlying simple form that repeats itself at different scales of observation. The level of variation (i.e. shape) at all scales can be described by a single parameter (fractal dimension).

- Fractal dimension of pure fractals:

$$D = \frac{\ln N}{\ln r}$$

where D = fractal dimension, N = the number of steps used to measure a unit length, and r is the scale ratio.

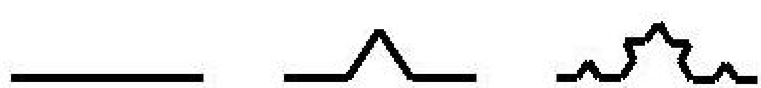
Another, similar way of describing the fractal dimension by looking across scales (i.e. scale 1 and scale 2) is:

$$D = \frac{\ln(L2/L1)}{\ln(S1/S2)}$$

where L1, L2 measure lengths of curves in units and S1, S2 measure the sizes of the units (i.e., scales) used in the measurements.

Fractal dimension of a linear fractal curve varies between D=1 and D=2, since when D=2, it is in fact 2-dimensional and has become an area. A straight line has D =1 since N = r. As D gets greater than 1, it is beginning to "fill space" in the second dimension (i.e. has developed curvature). When considering volumes, the value of D ranges between 2 (completely smooth 2-D surface) and 3 ("infinitely crumpled three-dimensional object", Turner et al. 2001 p. 117).

Simple example of a fractal, the Deterministic Koch Curve (3 iterations)



D = log(4)/log(3) = 1.2619.

Note that coastlines of the world appear to be fractal with dimension of approximately 1.25, very close to that of the Koch Curve. Another example of a fractal pattern is the Sierpinski gasket (Fig. 1). There are endless examples of fractal geometric patterns that have been derived, and it is relatively simple to develop computer programs allowing generation of highly sophisticated fractal patterns. The question then becomes: do we observe these fractal patterns in nature, or are they simply human constructs?

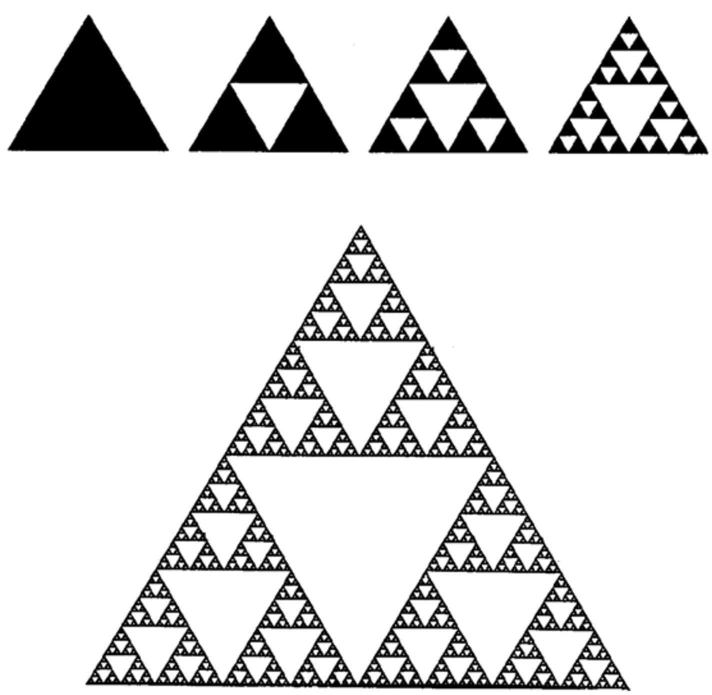


Fig. 1. *from Mandelbrot 1989, Fig. 1.* The Sierpinski gasket. The four small diagrams at top show the iterative process used in constructing the fractal. The basic step of the construction is to divide a given black triangle into four sub-triangles, and then to "white out" the middle fourth. The diagram at bottom shows the process at an "advanced" stage. Clearly each of the three reduced "gaskets" is simply one-third of the overall shape, which is repeated over different scales. Therefore the overall pattern is self-similar and the fractal dimension of the shape is scale-independent.

2. Fractals in Nature

There is ever-growing evidence of fractal patterns in nature. This is true for coastlines, as already mentioned; the boundaries of clouds; coral reefs; breeding bird home ranges; tree crowns; and of course certain landscape mosaic patterns. Figure 2 gives an example, from the Turner et al. 2001 text, of nested soil patterns that look similar across scales and so might well be described by fractal measures.

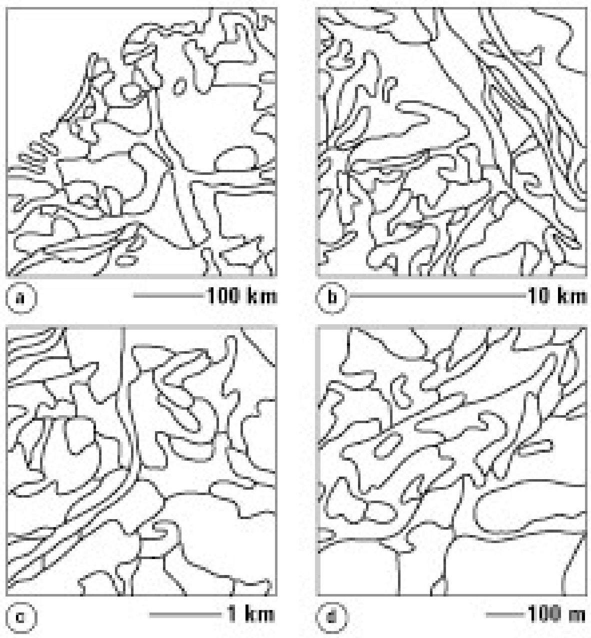


Fig. 2, *from Turner et al. 2001* Figure 5.10. Nested maps of soil patterns in northwest Europe at scales ranging from 100 km to 100 m. Note how natural variation may look similar across scales. Redrawn from Burroughs, 1986.

Fractal patterns in nature are seldom perfect geometric fractals such as the Koch Curve. Rather, they are statistical fractals, with a great deal of "noise" and stochastic variation (Fig. 3).

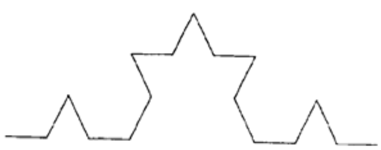


Figure 1. Deterministic Koch curve, after three iterations.

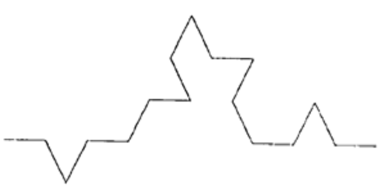


Figure 2. Random Koch curve, after three iterations. The direction of each new triangle was chosen by flipping a coin.

Fig. 3. from *Johnson et al. 1995, Figs 1-2.*

True fractals are infinite mathematical sets, while fractal patterns in nature are of course finite (discussed in Johnson et al. 1995). At the fine scale, they are limited by some fundamental building block, even if it is down to the atom or subatomic particle. At the coarse scale, they are limited to the range of scales at which similar pattern-creating processes act to form their characteristic patterns.

3. Fractals as Landscape Metrics

Fractal relationships have been used generally to describe shape complexity of patches in landscapes. The fractal power law equation relating patch area to perimeter is:

$$A = (kP)^d$$

where A = patch area, P = some measure of patch perimeter, k is a constant that is dependent upon the measure used for P, and d = fractal dimension.

If the length of one side of an object is used to estimate P, k = 1.0, and

$$d = \frac{\ln A}{\ln P}$$

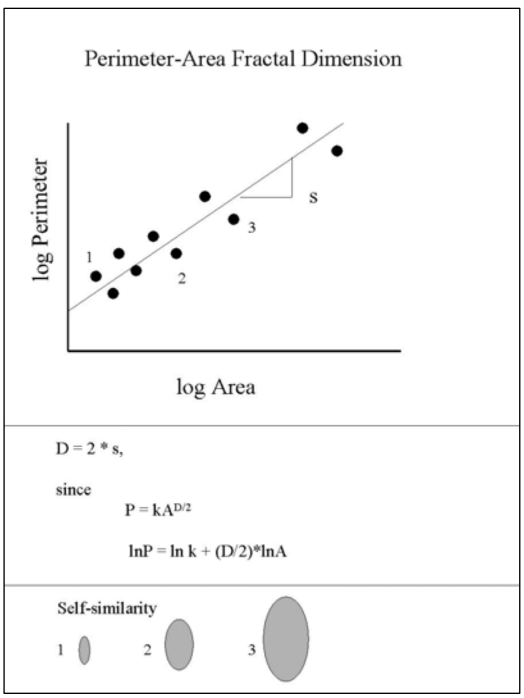
Otherwise,

$$d = \frac{\ln A}{\ln P + \ln k}$$

There are various different formulas for calculating fractal dimension of actual landscapes. One commonly used formulation is the perimeter-area fractal dimension, sometimes called the double log fractal dimension.

<u>Perimeter-Area Fractal Dimension</u>

For analyzing empirical landscapes, we can regress the natural logarithm of patch perimeter vs. the natural logarithm of patch area for all patches. The fractal dimension will be estimated as double the slope of the regression line, since:



$$P = kA^{D/2} \qquad \text{(fractal power law)}$$

$$\ln P = \ln k + (D/2) * \ln A$$

where,
P = patch perimeter,
A = patch area,
D = fractal dimension
K = constant (intercept in regression)

- This is a measure of patch complexity: greater values indicate more complex shapes
- D = 1 for simple Euclidean shapes (circles if vector, rectangles if raster)
- D = 2 for the most complex, convoluted shapes with plane-filling perimeters
- The double log fractal dimension is very sensitive to sample size, since it is based on regression analysis
- The regression approach assumes that self-similar patterns exist across various sizes of landscape patches

Fractal dimensions have been used as a metric of the complexity of landscape patterns, in the sense of comparing different landscapes, analyzing changes of a particular landscape over time, and for comparing shape complexity of patches of different size. Human-influenced landscapes often exhibit simpler patterns, as evidenced by lower values of the fractal dimension (D). In a classic study, Krummel et al. (1987) measured the fractal dimension of deciduous forest patches in Mississippi. They found that smaller patches associated with forest management (i.e. woodlots) had simpler boundaries (smaller D) than larger patches whose boundaries were influenced more by natural processes (Fig. 4).

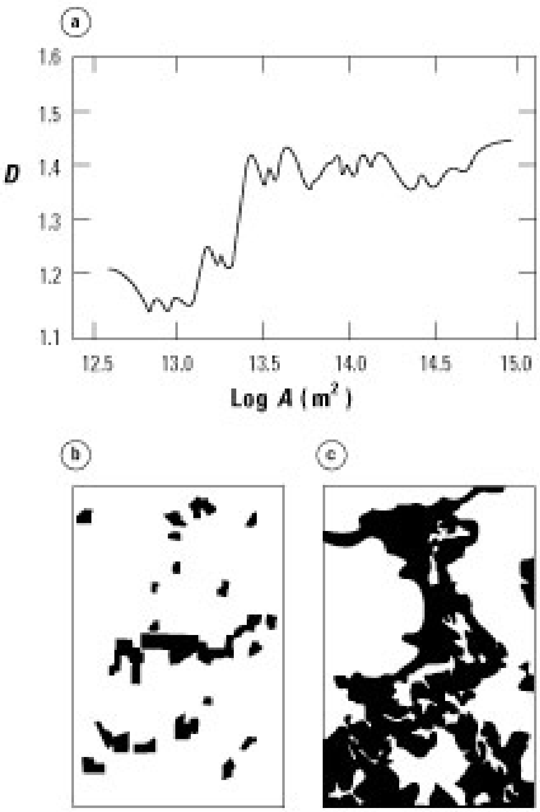


Figure 4. *from Turner et al., Figure 5.11, redrawn from Krummel et al., 1987.* (a) Fractal dimension (D) of forest patches in the vicinity of Natchez, Mississippi, as a function of patch size. (b) Section of the original map illustrating how small patches tend to be simple in shape. (c) Section of the original map illustrating the more complex shapes associated with the larger patches.

Figure 4 was estimated by 306 successive regressions of log(P) on log(A).! The first regression included the 200 smallest forest patches, with successive regressions formed by removing the smallest and adding the next largest patch.

Similarly, Mladenoff et al. (1993) used the method of Krummel et al. (1987) to calculate the perimeter-area fractal dimension over a range of patch sizes, in a comparison of old-growth and anthropogenically disturbed forest landscapes in Wisconsin (Figure 5). They found that the old-growth landscape (Sylvania) had a characteristic pattern of simpler patch shapes for smaller patches, and complex patch shapes for larger patches. The anthropogenically disturbed landscape (Border Lakes) had lost this characteristic pattern, with patches of all sizes having statistically similar fractal dimension. Essentially, the Border Lakes forest had lost the "large, landscape-integrating" patches of old-growth hemlock that formed the matrix of the natural landscape, providing connectivity (Mladenoff et al. 1993).

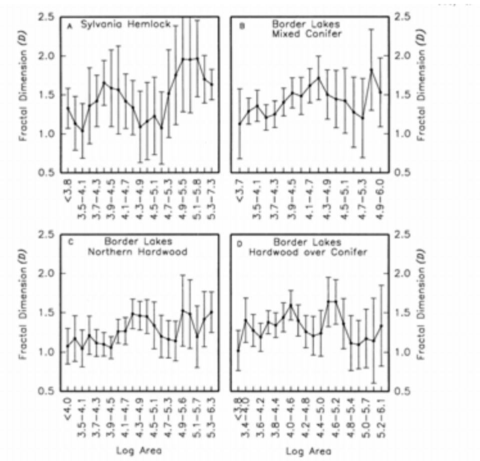


FIG. 3. Plots of fractal dimension (D) vs. area (measured in square metres) for dominant Border Lakes and Sylvania landscape forest types. Vertical bars are 95% confidence limits. Overall fractal dimensions of Border Lakes ecosystem types (B, C, D) are significantly different from those of Sylvania (A; $P < .005$). Within a single type, only Sylvania hemlock (A) contained different patch size areas of significantly different fractal dimension ($P < .05$; log(Area) < 4.6 vs. log(Area) 5.2–7.3).

Figure 5, *from Mladenoff et al. 1993, Fig. 3.* See figure legend.

4. Fractals and Spatial Scaling

Since fractals are by definition self-similar, identification of "natural breaks" in the distribution of fractal dimension values over scales (e.g. Figs. 4, 5) should help to identify natural domains of scale (Wiens 1989) over which pattern-process relationships remain relatively constant. A change in the fractal dimension of a pattern may indicate that different pattern-producing processes have emerged as dominant. Rapid change in fractal dimension with small changes in measurement scale may indicate chaotic transitions between scale domains.

Similarly, where fractal dimension is invariant over a range of measurement scales and patterns are therefore self-similar, we should be able to extrapolate our observations across scales if we only know the pattern at one scale and the fractal dimension. Fractals should help us to find the "scaling parameter" or scaling function for translating across scales. This is the Holy Grail of many landscape ecologists. We don't seem to know how to do this yet. But herein lies the promise of fractal applications in ecology. Otherwise, the fractal dimension may be just another fancy way to quantify the relative complexity (i.e. space-filling characteristics) of a given ecological pattern.

5. Fractals and Generation of Artificial Landscapes

A practical application of fractals is the generation of artificial landscapes that, while of precisely known fundamental characteristics, exhibit more realistic patterns than simple random landscapes. We will be applying fractals in this manner when we use the RULE software in our "Neutral Models" laboratory exercise. Such artificial landscapes (i.e. neutral landscape models, topic of a future lecture) may be represented using multifractal maps, which seem realistic because they are generated by a fractal algorithm that generates spatially correlated patterns of patchiness (Gardner 1999). Therefore, such maps are useful when investigators wish to simulate movement and dispersal of organisms (With et al. 1997). One widely used algorithm for multifractal maps uses a midpoint displacement algorithm similar to that shown, above, for the Koch curve. The actual midpoint is perturbed by a random variate, and the degree of randomness is controlled by an input parameter, H (range: 0 – 1.0). When H is low, maps will be highly fragmented; when H is high, maps will be highly aggregated (Fig. 6; Gardner and Walters 2002).

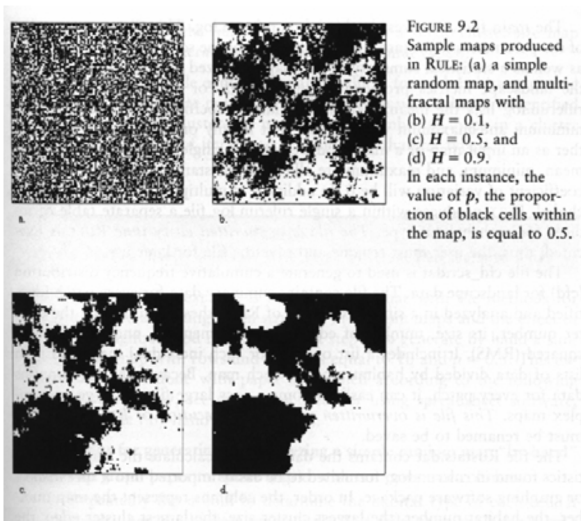


Fig. 6. *from Gardner and Walters 2002.* See figure legend.

6. References

Gardner, R.H. 1999. RULE: A program for the generation of random maps and the analysis of spatial patterns. In J.M. Klopatek and R.H. Gardner, eds. Landscape Ecological Analysis: Issues and Applications. Springer-Verlag. pp. 280-303.

Gardner, R.H., and S. Walters. 2002. Neutral landscape models. In S.E. Gergel and M.G. Turner, eds. Learning Landscape Ecology: A Practical Guide to Concepts and Techniques. Springer-Verlag. pp. 112-128.

Haskell, J.P., Ritchie, M.E, and H. Olff. 2002. Fractal geometry predicts varying body size scaling relationships for mammal and bird home ranges. Nature 418: 527-530.

Johnson, G. D., A. Tempelman, and G. P. Patil. 1995. Fractal based methods in ecology: a review for analysis at multiple spatial scales. Coenoses **10**:123-131.

Krummel, J.R., Gardner, R.H., Sugihara, G., O'Neill, R.V., and P.R. Coleman. 1987. Landscape patterns in a disturbed environment. Oikos 48: 321-324.

Li, Bai-Lian. 2000. Fractal geometry applications in description and analysis of patch patterns and patch dynamics. Ecological Modelling 132: 33-50.

Mandelbrot, B.B. 1989. Fractal geometry: What is it, and what does it do? Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences 423: 3-16.

Milne, B.T. 1991. The utility of fractal geometry in landscape design. Landscape and Urban Planning 21: 81-90.

Mladenoff, D.J., White, M.A., Pastor, J., and T.R. Crow. 1993. Comparing spatial pattern in unaltered old-growth and disturbed forest landscapes. Ecological Applications 3: 294-306.

Turner, M. G., Gardner, R. H., O'Neill, R. V. 2001. *Landscape ecology in theory and practice: Pattern and process*. New York, Springer-Verlag. 404 pp. (Chapter 5)

Wiens, J.A. 1989. Spatial scaling in ecology. Functional Ecology 3: 385-397.

With, K. A. 1997. The application of neutral landscape models in conservation biology. Conservation Biology **11**:1069-1080.

Zeide, B. 1991. Fractal geometry in forestry applications. Forest Ecology and Management 46: 179-188.