The phenomenon of state reduction

P. Hájíček

November 14, 2013 arXiv

Abstract

The standard quantum theory leaves open where, when and under which objective conditions state reductions occur. Three of the recently published ideas to complete the theory are developed and improved. First, the disturbance of measurement due to identical particles in the environment can be avoided if the meters do not react to states of the environment particles but do react to the prepared states of the registered particle. Such meters are mathematically described by the so-called truncated positive operator valued measures. The difference between the states of the registered particles and the states from the environment is mathematically described by the so-called separation status. Second, the separation status is refined from a domain of space to a domain of space, momentum and energy. Third, starting from the previously introduced distinction between ancillas, screens and detectors, further study of experiments suggests the conjecture that a loss of the separation status is the objective condition for the occurrence of the a state reduction. The conjecture is falsifiable and a test is suggested using superconductor currents.

1 Introduction

It is well known that the quantum theory of measurement is in an unsatisfactory state [1, 2]. For example, the ideas of quantum decoherence theory [3] have brought some progress but they do not solve the problem of quantum measurement without any additional assumptions, e.g., the Everett interpretation [4, 5, 6].

A measurement on microscopic systems can be split into preparation and registration. Registration devices are called *meters*. The available empirical facts and the spirit of quantum mechanics suggest that the process of registration has the following fascinating properties:

1. Registered value r is in general only created by the interaction of the object system with the meter during the registration. Unlike the measurement in a classical theory, registrations do not reveal already existing values. An important aspect of the creation is a state reduction[1].

[1] The creation is a hypothesis that is neither disproved nor generally accepted.

2. As a rule, repeated experiments give different outcomes r from a well-defined set of possible alternatives $\mathbf{R}$. Each outcome is then created with probability P_r such that $\sum_{r \in \mathbf{R}} \mathsf{P}_r = 1$. The resulting randomness, or the so-called QM *indeterminism* occurs only during registrations. This is different from other quantum processes, which are governed by Schrödinger equation in a deterministic way.

3. There are correlations between outcomes given by distant meters. As the outcomes are only created by registrations, a spooky action at a distance between the meters turns out to be necessary. Indeed, the quantum formalism suggests that the correlations between the registered values are encoded in the states but the values themselves are not! This is called QM *non-locality* or *non-separability* and it again seems to appear only via registrations (e.g., Einstein-Podolski-Rosen experiment [7] with two particles as well as non-locality in registrations of a single particle [8]). There has been a lot of work on this feature since the beginnings of quantum mechanics and it is strongly suggested by a number of theoretical and experimental results: Contextuality [9, 10], Bell inequalities [11], Hardy impossibilities [12], Greenberger-Horne-Zeilinger equality [13], etc.

These properties are not logically self-contradictory. Moreover, they are testable and have been confirmed by numerous experiments as well as theoretical analysis. For many physicists, however, they are unacceptable for taste and traditional reasons. We just take them as "facts of life". Nevertheless, they give us a strong motivation to focus research on the phenomenon of registration.

In several recent papers [14]-[21], a reformulation of quantum mechanics was proposed that was not radical but that made the theory easier to understand. The present paper is a continuation of our work on quantum measurement [16, 19, 20]. Our strategy is to observe carefully what happens in real experiments and to formulate some general hypotheses, which have then an empirical rather than a speculative character.

In [16] it has been shown that quantum mechanical theory implies a strong disturbance of any registration that could be mathematically truly described by a self-adjoint operator (such as position, momentum, energy, spin and orbital angular momentum). The origin of the disturbance is the existence of systems of the same type as the registered system $\mathcal{S}$ in the world (not just in a neighborhood of $\mathcal{S}$). For example, according to the standard quantum mechanics, such measurements on an electron are disturbed because of the existence of other electrons. This theoretical observation clearly contradicts the long and successful praxis of experimenting. The fact that such disturbances do not occur in real measurements must then be understood as a proof that the current theory of observables and registrations need some corrections.

In [16, 19, 20], an attempt at a correction of measurement theory is described that focuses on disturbances due to *remote* particles. For example, the registration of a spin operator of an electron prepared in our laboratory had to be (theoretically) disturbed by an electron prepared in a distant laboratory. The idea was that it is not the spin operator that is really measured, but a different quantity that can be constructed from the spin operator by some process of localization. Such constructs have been called D-local observables, D being some region of space. However, in order that D-local observables be measurable on, say, an electron, the electron must also be prepared in such a way that the influence of all other electrons on the measurement apparatus inside a region D of space is negligible (e.g., the wave functions of all these electrons practically vanish in D). We said then that the electron has a *separation status*[2] D. On such a basis, a whole general[3] mathematical theory has been constructed [20].

Another important fact is that a particle prepared with a certain local separation status loses the status if it arrives in a region of space where its wave function has a non-zero overlap with wave functions of other particles of the same type. Then, the particle itself does not make sense as an individual system in a prepared state because there are only (anti-)symmetrized states of the whole system. We recognize and highlight such changes of separation status as important changes in the physics of the studied systems. A change

[2]To prevent confusion, let us recall that the word "separation" has two very different uses in the literature on quantum mechanics. First, it is the "quantum non-separability", as in point 3 above. Second, it is associated with the notion of the "cluster separability" [22]. The "separation status" was developed in connection with the principle of cluster separability [16].

[3]For example, we speak here about wave functions for the sake of simplicity, but wave functions are not sufficiently general in two respects: they represent pure states and refer to a particular frame, the Q-representation.

of separation status can be understood as an objective fact: it can be observed but is itself independent of any observer. Notice that the separation status is a new property of a quantum system that is very different from all other properties used by standard quantum mechanics such as states or values of observables in that it cannot be determined by the state of the system alone.

A careful study of many real experiments in [19, 20] has shown that meters contain macroscopic detectors and screens. Further observations show that the registration processes include separation status losses in the detectors and screens on the one hand and state reductions on the other. We have therefore proposed a general rule that associates a definite state reduction to a given process containing separation status losses. The rule is sufficiently specific in the sense that the final state is uniquely determined. The hypothesis is empirical in its nature: it is not derived by some theoretical procedure but is postulated and justified by observations. It is sufficiently specific so that it is testable and has some predictive power.

The resulting theory, which is described with many detail in [20], suggests the way in which the quantum measurement theory could and ought to be corrected. However, it represents an idealized model whose practical applications are limited. On the one hand, it only focuses on the space aspects of quantum systems working exclusively with regions in the eigenspace of the position operator and so violates the transformation symmetry of quantum mechanics. On the other, the spatial separation status is rather difficult to be prepared. We can, e.g., never achieve perfect vacuum in the cavities where the object systems are moving.

One can wonder whether the separation of particles in the momentum space could play a similar role as that in the position one studied in [20]. In fact, it is straightforward to built up a mathematical formalism in the momentum space that is completely analogous to the formalism in the position space. To see the physical meaning of such a construction, imagine that the detector used in an experiment has an energy threshold E_0. Then the particles with the kinetic energy lower than the threshold, including particles within the detector itself, cannot influence the detector. It is easy to achieve momentum separation status and, in fact, most measurements work exactly in this way.

The main aim of the present paper is to introduce quantities that are better suited to describe real experiments than the D-local observables and give a new definition of separation status that seems to be more satisfactory than the old one.

The plan of the paper is as follows. Section 2 gives a brief account of the standard theory of quantum measurement that is being used today for study of real experiments. This is a good starting point for our analysis. Section 3 contains a new discussion of the disturbance of registration due to identical particles. Section 4 introduces the *truncated positive operator valued measures* (TPOVM) as the quantities describing real experiments. Section 5 defines the *extent* of a quantum system in a given state as some domain in the space spanned by three space coordinates, three components of momentum and total energy. The notion of extent is then used to define a new kind of separation status. Section 6 recapitulates and reformulates some older ideas using the new notion of separation status. First, ancillas, screens and detectors are structure elements of a given meter and must be distinguished from each other. Second, the reading of a meter is postulated to be a signal from a detector. Third, detected systems lose their separation status within screens and detectors. Fourth, the standard unitary evolution is corrected by a specific new rule in the case that separation statuses are lost. Section 7 describes a simple model of the Stern-Gerlach measurement within our theory illustrating how the new rule works. Section 8 shows that the new rule suggests a specific direction of investigation in the field of experiments with superconducting currents and that this investigation might suggest a test of the rule. The last section gives a summary of the paper.

2 The standard theory of measurement

In this section, we give a short review of the standard theory of measurement as it is employed in the analysis of many measurements today and as it is described in, e.g., [5, 23, 24]. The emphasis is on being close to experiments and on physical meaning rather than on mathematical formalism.

The standard theory splits a measurement process into three steps.

1. Initially, the *object system* $\mathcal{S}$ on which the measurement is to be done is prepared in state $\mathsf{T}_{\mathcal{S}}^{in}$ and the *meter* $\mathcal{M}$, that is the aparatus performing

the measurement, in state $\mathsf{T}^{in}_{\mathcal{M}}$. These two preparations are independent so that the composite $\mathcal{S}+\mathcal{M}$ is then in state $\mathsf{T}^{in}_{\mathcal{S}} \otimes \mathsf{T}^{in}_{\mathcal{M}}$. $\mathsf{T}^{in}_{\mathcal{S}}$ and $\mathsf{T}^{in}_{\mathcal{M}}$ are *state operators* (sometimes also called density matrices).

2. An interaction between $\mathcal{S}$ and $\mathcal{M}$ suitably entangles them. This can be theoretically represented by unitary map U, called *measurement coupling*, that defines the evolution of system $\mathcal{S}+\mathcal{M}$ during a finite time interval. Accordingly, at the end of the time interval, the composed system is supposed to be in state

$$\mathsf{U}(\mathsf{T}^{in}_{\mathcal{S}} \otimes \mathsf{T}^{in}_{\mathcal{M}})\mathsf{U}^{\dagger}$$

3. Finally, *reading* the meter gives some *definite* value r of the measured quantity. If the same measurements are repeated many times independently from each other, then all readings form a set, $r \in \mathbf{R}$. $\mathbf{R}$ is not necessarily the spectrum of an observable (s.a. operator), in particular, it need not contain only real numbers ($\mathbf{R}$ need not be a subset of $\mathbb{R}$). The experience with such repeated measurements is that each reading $r \in \mathbf{R}$ occurs with a definite probability, P_r.

One of the most important assumptions of the standard theory is that, after the reading of the value r, the object system $\mathcal{S}$ is in a well-defined state,

$$\mathsf{T}^{out}_{\mathcal{S}r}$$

called *conditional* or *selective* state. This is a generalization of Dirac's postulate:

> A measurement always causes a system to jump in an eigenstate of the observed quantity.

Such a measurement is called *projective* and it is the particular case when $\mathsf{T}^{out}_{\mathcal{S}r} = |r\rangle\langle r|$ where $|r\rangle$ is the eigenvector of a s.a. operator for a non-generated eigenvalue r.

The average of all conditional states after registrations, a *proper mixture* (see, e.g., [25], p. 101 and the discussion in [20]),

$$\sum_r \mathsf{P}_r \mathsf{T}^{out}_{\mathcal{S}r}$$

is called *unconditional* or *non-selective* state. It is described as follows: "make measurements but ignore the results". One also assumes that

$$\sum_r \mathsf{P}_r \mathsf{T}^{out}_{\mathcal{S}r} = \mathsf{Tr}_{\mathcal{M}}\left(\mathsf{U}(\mathsf{T}^{in}_{\mathcal{S}} \otimes \mathsf{T}^{in}_{\mathcal{M}})\mathsf{U}^\dagger\right)$$

where $\mathsf{Tr}_{\mathcal{M}}$ denotes a partial trace defined by any orthonormal frame in the Hilbert space of the meter.

In the standard theory, the reading is a mysterious procedure. If the meter is considered as a quantum system then to observe it, another meter is needed, to observe this, still another is and the resulting series of measurements is called *von-Neumann chain*. At some (unknown) stage including the processes in the mind (brain?) of observer, there is the so-called *Heisenberg cut* that gives the definite value r. Moreover, the conditional state cannot, in general, result by a unitary evolution. The transition

$$\mathsf{Tr}_{\mathcal{M}}\left(\mathsf{U}(\mathsf{T}^{in}_{\mathcal{S}} \otimes \mathsf{T}^{in}_{\mathcal{M}})\mathsf{U}^\dagger\right) \mapsto \mathsf{T}^{out}_{\mathcal{S}r}$$

in each individual registration is called "the first kind of dynamics" [26] or "state reduction" or "collapse of the wave function". We will use the name "state reduction".

The state reduction is an empirical fact but its theory is incomplete. First, the time and location of the Heisenberg cut is not known. Second, if there are two different kinds of dynamics, there ought to be also objective conditions under which each of them is applicable. At the present time, no such objective conditions are known. For example, for the state reduction, the condition of the presence of an observer is not objective and the condition that a quantum system interacts with a macroscopic system is not necessary.

The standard theory describes a general measurement mathematically by two quantities. The first is a *state transformer* $\mathcal{O}_r$. $\mathcal{O}_r$ enables us to calculate $\mathsf{T}^{out}_{\mathcal{S}r}$ from $\mathsf{T}^{in}_{\mathcal{S}}$ by

$$\mathsf{T}^{out}_{\mathcal{S}r} = \frac{\mathcal{O}_r(\mathsf{T}^{in}_{\mathcal{S}})}{\mathsf{Tr}(\mathsf{T}^{in}_{\mathcal{S}})}$$

$\mathcal{O}_r$ is a so-called *completely positive map* that has the form [27]

$$\mathcal{O}_r(\mathsf{T}) = \sum_k \mathsf{O}_{rk}\mathsf{T}\mathsf{O}^\dagger_{rk} \tag{1}$$

for any state operator T, where O_{rk} are some operators satisfying

$$\sum_{rk} \mathsf{O}_{rk}^{\dagger}\mathsf{O}_{rk} = 1$$

Equation (1) is called *Kraus representation*. A given state transformer $\mathcal{O}_r$ does not determine, via Eq. (1), the operators O_{rk} uniquely.

The second quantity is an operator E_r called *effect* giving the probability to read value r by

$$\mathsf{P}_r = \mathsf{Tr}\left(\mathcal{O}_r(\mathsf{T}_{\mathcal{S}}^{in})\right) = \mathsf{Tr}\left(\mathsf{E}_r\mathsf{T}_{\mathcal{S}}^{in}\right)$$

The set $\{\mathsf{E}_r\}$ of effects E_r for all $r \in \mathbf{R}$ is called *positive operator valued measure* (POVM). Every POVM satisfies two conditions: positivity,

$$\mathsf{E}_r \geq 0$$

for all $r \in \mathbf{R}$, and normalization,

$$\sum_{r\in\mathbf{R}} \mathsf{E}_r = 1$$

One can show that $\mathcal{O}_r$ determines the effect E_r by

$$\mathsf{E}_r = \sum_k \mathsf{O}_{rk}^{\dagger}\mathsf{O}_{rk}$$

The definition of POVMs that is usually given is more general: $\mathsf{E}(X)$ is a function on the Borel subsets $X \subset \mathbf{R}$. The simplified formalism that we use in the present paper can be easily generalized in this way.

In the standard theory, the state transformer of a given registration contains all information that is necessary for further analysis and for classification of measurements. Such a classification is given in [5], p. 35. Thus, the formalism of the state transformers and POVMs can be considered as the core of the standard theory. Notice that these quantities are independent of any further assumptions about state reduction. In this way, the standard theory can work in practice and ignore the incompleteness of the state reduction theory.

3 Disturbance by identical particles

Let us first briefly recall the argument of Ref. [16] about the disturbance of registration due to identical particles. Consider two distant laboratories, A and B, and suppose that each of them prepares an electron in states $\psi(\vec{x}_A)$ and $\phi(\vec{x}_B)$, respectively (we are leaving out the spin indices and we work in Q-representation for the sake of simplicity). Then everyday experience shows that A can do all manipulations and measurements on its electron without finding any contradictions to the assumption that the state is $\psi(\vec{x}_A)$. Analogous statements hold about B.

However, according to the standard quantum theory, the state of the two particles must be

$$2^{-1/2}(\psi(\vec{x}_A)\phi(\vec{x}_B) - \phi(\vec{x}_A)\psi(\vec{x}_B)) \tag{2}$$

Suppose next that A makes a measurement of the position of the electron. Standard quantum mechanics associates position observable with the multiplication operator $\vec{x}_A$ for the A electron and with a symmetrized multiplication operator

$$\vec{x}_A + \vec{x}_B \tag{3}$$

for the two electrons because the meter cannot distinguish the contributions of two identical particles from each other. Hence, the average of the measurement results must be

$$\int_{\mathbb{R}} d^3x_A\, \vec{x}_A|\psi(\vec{x}_A)|^2 + \int_{\mathbb{R}} d^3x_B\, \vec{x}_B|\phi(\vec{x}_B)|^2 \tag{4}$$

which differs from what one would expect if the state of the electron were just $\psi(\vec{x}_A)$, and the difference even increases with the distance of the laboratories.

In this connection, the following question is often asked. Suppose we have a system $\mathcal{S}$ composed from two subsystems $\mathcal{S}_1$ and $\mathcal{S}_2$ containing indistinguishable particles. Two methods of dealing with $\mathcal{S}$ can be imagined. First, we use only states that are symmetric over all indistinguishable bosons and antisymmetric over all indistinguishable fermions of $\mathcal{S}$ (*exchange symmetry*), and use only operators of $\mathcal{S}$ that preserve the exchange symmetry of the states. Second, for both states and operators of $\mathcal{S}$, we use (tensor) products $\psi_1\psi_2$ and a_1a_2 without symmetrizing and antisymmetrizing, where ψ_j and

a_j is a state and an operator of $\mathcal{S}_j$. The question is, which method is correct for the above problem?

However, if there really were two different methods that could deal with the same system and that could give different results, then the theory would be self-contradictory. To prevent that, a clear criterion ought to be stated determining which of the two methods is to be applied to any given system.

We shall argue that there is no such dilemma: under certain conditions, call them C, the two methods are equivalent (give the same averages). Consider system $\mathcal{S}_1$ on which a measurement by a meter is done, its environment $\mathcal{S}_2$ (containing the meter) and the composite $\mathcal{S} = \mathcal{S}_1 + \mathcal{S}_2$. Let condition C read: The meter is such that it reacts to the prepared states of $\mathcal{S}_1$ but does not react to the states of any particle of the same type as $\mathcal{S}_1$ in the environment $\mathcal{S}_2$. If C is satisfied, both methods give the same results for the measured average. At the same time, the registration is not disturbed by the particles in the environment.

The meter can then be (at least approximately) described by a mathematical quantity $\{\mathsf{E}_r\}$ of the following kind: $\{\mathsf{E}_r\}$ is in every respect similar to a POVM with the exception that all effects E_r annihilate all states of the particles in the environment $\mathcal{S}_2$ but some of the effects do not annihilate the prepared states of $\mathcal{S}_1$. This is what we shall call "truncated POVM". Clearly, $\mathcal{S}_1$ must also have the following property: the states of $\mathcal{S}_1$ on the one hand and the states of all particles in $\mathcal{S}_2$ that are indistinguishable from $\mathcal{S}_1$ on the other must be sufficiently different. We shall try to formulate this mathematically with the help of "separation status".

Let us now return to the two experiments. Consider the same situation as in Experiment 1 with meter $\mathcal{M}$ being a detector that registers the position $\vec{x}_A$ of electron $\mathcal{S}_A$. That is, the outcome of any registration by $\mathcal{M}$ is a triple $\vec{x}_A$ of coordinates with respect to some Cartesian frame chosen by laboratory A.

First, it is clear that the outcome of a possible registration by any real meter cannot be any element of the coordinate space $\mathbb{R}^3$. For example, let $\mathcal{M}$ be a $2a\times 2a$ quadratic photographic plate positioned orthogonally to the x^3 axis of the frame at $x^3 = 0$ so that the coordinates x^1 and x^2 inside the plate satisfy $-a < x^1 < a$ and $-a < x^2 < a$. Then the outcome of any registration must lie in the set

$$\mathbf{D} = (-a, a) \times (-a, a) \times (-\delta, \delta)$$

for some $\delta > 0$ that characterizes the width of the plate. This is only a small subset of the spectrum $\mathbb{R}^3$ of the position operator $\vec{\mathsf{x}}$.

Let us denote the spectral measure of $\vec{\mathsf{x}}$ by $\Pi[\mathbf{X}]$ (Π is an orthogonal projection acting on the Hilbert space of $\mathcal{S}_1$ and $\mathbf{X}$ is a Borel subset of $\mathbb{R}^3$, see, e.g., [28]). Meter $\mathcal{M}$ can measure only a small part of the spectrum, namely $\mathbf{D}$ in such a way that it does not react to any state of $\Pi[\mathbb{R}^3 \backslash \mathbf{D}](\mathbf{H})$[4]. Such a state cannot excite the photo-plate so that a black point appears on it. We could try to describe the meter mathematically by TPOVM with effects $\{\Pi[\mathbf{X}] | \mathbf{X} \subset \mathbf{D}\}$.

Let us assume next that states prepared in laboratory B are very well approximated by elements of $\Pi[\mathbb{R}^3 \backslash \mathbf{D}](\mathbf{H})$ and that we use the method that respects the full exchange symmetry. Then there will be no disturbance of the registrations by $\mathcal{M}$ due to activity in laboratory B because the second term in Equation (4) would vanish. The result,

$$\int_{\mathbf{D}} d^3 x_A \, \vec{x}_A |\psi(\vec{x}_A)|^2$$

will then be the same as if we do not respect the exchange symmetry and describe the state of the two electrons by tensor product $\psi(\vec{x}_A)\phi(\vec{x}_B)$ and the registered operator by $\{\Pi[\mathbf{X}] | \mathbf{X} \subset \mathbf{D}\}$. The two methods yield the same average and there is no dilemma.

What is the bearing of the above considerations on the theory of quantum observables? The standard quantum mechanics defines observables of a system $\mathcal{S}$ as the self-adjoint operators on the Hilbert space of $\mathcal{S}$. Some mathematical physicists (e.g., Ludwig, Bush, Lahti and Mittelstaed) define observables as POVMs. The spectral measures of s.a. operators are POVMs and in this sense, the latter definition is a generalization of the former one.

Wiseman and Milburn [5] propose a different idea: On the one hand, self-adjoint operators are general theoretical quantities used by the theory for many different purposes, e.g., to describe the dynamics (Hamiltonian), as elements of C*-algebras to form a basic space of quantum mechanics, to

[4] The set difference $A \backslash B$ is defined by $A \backslash B = \{x : x \text{ in } A \text{ and } x \text{ not in } B\}$.

construct some POVMs, etc. They can be called observables, because their eigenvalues can be measured. On the other hand, the structure of s.a. operators is too restricted so that they are not suitable to describe truly what is measured in many genuine measurements. For that purpose, POVMs seem to be more suitable and one can try to describe mathematically any given registration apparatus by some POVM. This distinction between observables and POVMs is helpful in practice. For example, one can then define a POVM that measures an observable by the requirement that all its effects are functions of the observable and one can give many examples of measurements that are not of this kind [5], p. 38.

However, the example of the position operator above goes even further: Although elements of its spectrum can, in principle, be measured, any real registration apparatus can measure only a restricted part of the spectrum. Hence, it may have a mathematical description that is even more precise than a POVM.

4 Truncated POVMs

In the previous section, we have seen that real meters must be such that they do not react to some states in the sense that the probability to register any value on such a state is zero. Strictly speaking, a meter of this kind cannot be described by any POVM because of the condition of normalization. In the present section, we introduce new mathematical quantities that are more suitable to describe meters.

Let us define *truncated POVMs* (TPOVMs) as follows. In general, any given experiment Exp on system $\mathcal{S}$ using meter $\mathcal{M}$ works with a limited set $\mathbf{T}_{\text{Exp}} = \{\mathsf{T}_1, \mathsf{T}_2, ..., \mathsf{T}_K\}$ of states in which $\mathcal{S}$ is prepared before registrations. We assume that there is subspace $\mathbf{H}_{\text{Exp}}$ of Hilbert space $\mathbf{H}$ of $\mathcal{S}$ satisfying two conditions. First,

$$\Pi[\mathbf{H}_{\text{Exp}}]\mathsf{T}\Pi[\mathbf{H}_{\text{Exp}}] = \mathsf{T} \tag{5}$$

for all $\mathsf{T} \in \mathbf{T}_{\text{Exp}}$, where $\Pi[\mathbf{H}_{\text{Exp}}] : \mathbf{H} \mapsto \mathbf{H}_{\text{Exp}}$ is an orthogonal projection. Second, $\mathbf{H}_{\text{Exp}}$ is minimal, that is any subspace of $\mathbf{H}$ that satisfies (5) must contain $\mathbf{H}_{\text{Exp}}$. In fact, for most experiments, $\mathbf{H}_{\text{Exp}}$ is a finite-dimensional subspace of $\mathbf{H}$ (many examples of finite-dimensional $\mathbf{H}_{\text{Exp}}$'s can be found in [5]).

Definition 1 *Any TPOVM associated with experiment Exp is a set* $\{\mathsf{E}'_r\}$ *of s.a. operators satisfying*

$$\mathsf{E}'_r \geq 0$$

for all $r \in \mathbf{R}$ *and*

$$\sum_r \mathsf{E}'_r = \Pi[\mathbf{H}_{\text{Exp}}]$$

Example Let E_r, $r \in \mathbf{R}$, be POVM. Then,

$$\mathsf{E}'_r = \Pi[\mathbf{H}_{\text{Exp}}]\mathsf{E}_r\Pi[\mathbf{H}_{\text{Exp}}] \quad , \quad r \in \mathbf{R}$$

is a TPOVM.

We have the desired property: states T annihilated by $\Pi[\mathbf{H}_{\text{Exp}}]$ satisfy $\text{Tr}(\mathsf{T}\mathsf{E}'_r) = 0$ for any r Another example of a TPOVM is described in Sec. 7.

5 Separation status

The foregoing section introduced quantities that are not necessarily disturbed by environmental particles during registrations. However, further conditions must be satisfied in order that a registration is not disturbed.

First, we need some estimate of the regions in the coordinate, momentum and energy spaces within which a system in a fixed state, on which no registration is done, can relatively strongly influence the registration on other quantum systems.

Definition 2 *Let* $\mathcal{S}_\tau$ *be a system of* N *particles of type* τ *in state* T. *Let* a_k *be an observable of the* k*-th particle. Let*

$$\bar{a} = \text{Tr}\left(\mathsf{T}\frac{\sum_k \mathsf{a}_k}{N}\right) \tag{6}$$

and

$$\Delta a = \sqrt{\text{Tr}\left(\mathsf{T}\frac{\sum_k (\mathsf{a}_k - \bar{a})^2}{N}\right)} \tag{7}$$

where $\sum_k \mathsf{a}_k$ and $\sum_k (\mathsf{a}_k - \bar{a})^2$ *are symmetrized one-particle operators that preserve the exchange symmetry of states.*

The extent $Ext(\mathsf{T})$ of T is the domain of $\mathbb{R}^7$ defined by the Cartesian product of intervals,

$$Ext(\mathsf{T}) = \prod_i^3 (\bar{x}^i - \Delta x^i, \bar{x}^i + \Delta x^i) \prod_i^3 (\bar{p}^j - \Delta p^j, \bar{p}^j + \Delta p^j)(\bar{H} - \Delta H, \bar{H} + \Delta H) \quad (8)$$

where $\bar{x}^i$ and Δx^i, are determined by Equations (6) and (7) for $\mathsf{a}_k = \mathsf{x}^i_k$, x^i_k being the icomponent of the position operator, $\bar{p}^j$ and Δp^j are similarly determined by components of the momentum operator and $\bar{H}$ and ΔH by the Hamiltonian, of the k's particle in $\mathcal{S}_\tau$.

For example, consider two bosons in state $\mathsf{T} = |\Psi\rangle\langle\Psi|$, where

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi 1\rangle \otimes |\phi 2\rangle + |\phi 1\rangle \otimes |\psi 2\rangle)$$

$|\psi\rangle$ and $|\phi\rangle$ are two orthogonal vector states in the common Hilbert space of both bosons satisfying $\langle\psi\,|\,\phi\rangle = 0$ and the symbol $|\psi k\rangle$ means that the state $|\psi\rangle$ is occupied by the k-th particle. For any observable a, a short calculation gives

$$\bar{a} = \frac{\langle\psi|\,\mathsf{a}\,|\psi\rangle + \langle\phi|\,\mathsf{a}\,|\phi\rangle}{2}$$

and

$$\Delta a = \sqrt{\frac{1}{2}\left(\Delta^2_\psi a + \Delta^2_\phi a + \frac{1}{2}(\langle\psi|\,\mathsf{a}\,|\psi\rangle - \langle\phi|\,\mathsf{a})^2\,|\phi\rangle\right)} \quad (9)$$

where

$$\Delta^2_\chi a = \langle\chi|\,\mathsf{a}^2\,|\chi\rangle - \langle\chi|\,\mathsf{a}\,|\chi\rangle^2 \quad (10)$$

for any state χ. We can see that the extent includes not only the "a-sizes" $\Delta_\psi a$ of individual particles but also the "a-distances" $|\langle\psi|\,\mathsf{a}\,|\psi\rangle - \langle\phi|\,\mathsf{a}|$ between different particles in $\mathcal{S}_\tau$.

Now, we can give a definition of the separation status:

Definition 3 *Given a system $\mathcal{S}$, let $\mathcal{S}_\tau$ be the subsystem of $\mathcal{S}$ containing all particles in $\mathcal{S}$ of type τ. Similarly, let $\mathcal{E}_\tau$ be the subsystem of the environment of $\mathcal{S}$ that contains all particles of type τ and let $\mathsf{T}_\mathcal{E}$ be the state of the environment. We say that $\mathcal{S}$ prepared in state T has a separation status if the extents of T_τ and $\mathsf{T}_{\mathcal{E}\tau}$ have empty intersection for all τ. Here,T_τ and $\mathsf{T}_{\mathcal{E}\tau}$ are partial traces of T and $\mathsf{T}_\mathcal{E}$ over all particles of the type different from τ.*

To give some physical interpretation to this formalism, consider meter $\mathcal{M}$ that is able to register systems of the same type as $\mathcal{S}$. Suppose that, in order to be registered by $\mathcal{M}$, $\mathcal{S}$ has to be at some time inside $\mathcal{M}$ and its kinetic energy must lie in the interval (E_0, ∞) defined by threshold E_0 of the meter. Similarly, the momentum must have the direction in which $\mathcal{S}$ must arrive at the meter in order to be registered. These are condition on the extent of $\mathcal{S}_\tau$ for all types τ.

We assume first: every measurement on a system $\mathcal{S}$ with no separation status will be disturbed by particles in its neighborhood. Second, experiments on $\mathcal{S}$ can be arranged so that they will be only negligibly disturbed by environment particles if $\mathcal{S}$ has a separation status. The TPOVM of the registration will then practically not react to states with disjoint extent.

Separation status has been defined differently in [16]: first, as operators a_k, just the components of the position operators were considered and second, supports of the wave function or generalization of support to any state operator in the position representation were used to define the extents. Such definition is useful only in special cases because first, the volumes where the particles are prepared have only an imperfect vacuum and there can still be many particles of the same type there and second, most wave functions have unbounded support. Definition 3 is free from these flaws.

We can interpret what has been said as yet as follows. Standard quantum mechanics as it is usually presented seems incomplete:

1. It admits only two separation statuses for any system $\mathcal{S}$:
    (a) $\mathcal{S}$ is isolated. Then all states of $\mathcal{S}$ would have separation status and all registrations could be described by POVMs. But this is self-contradictory because $\mathcal{S}$ cannot be isolated if it interacts with a meter.
    (b) $\mathcal{S}$ is a member of a larger system containing particles identical to $\mathcal{S}$. Then there are no individual physical states and observables for $\mathcal{S}$.
2. It ignores the existence of separation-status changes.

Indeed, the separation status of a system $\mathcal{S}$ depends on the state of $\mathcal{S}$, and the state changes with time (in the Schrödinger picture, but all definitions can be transformed to the Heisenberg picture). Separation-status changes have two important features:

1. They are objective phenomena that happen independently of any observer, and can be distinguished from other quantum mechanical processes.

2. Losses of separation status seem to be associated with state reductions. This gives us some hope that state reductions would indeed occur only if some objective conditions were satisfied.

6 Theory of meter reading

Let us now explain in more detail how registered systems can lose their separation status in meters and how this can be associated with state reduction. To this aim, we must refine a bit the language used for the description of meters. The words that name parts or structural elements of meters will be *field, screen, ancilla* and *detector*. It is clear what field and screen is but ancillas and detectors will have slightly different meaning from that of common use. In many modern experiments, in particular in non-demolition and weak measurements, but not only in these, the following idea is employed. The object system $\mathcal{S}$ interacts first with a quantum system $\mathcal{A}$ that is prepared in a suitable state. After $\mathcal{S}$ and A become entangled, A is subject to further registration and, in this way, some information on $\mathcal{S}$ is obtained. Subsequent measurements on $\mathcal{S}$ can but need not be made. The state of $\mathcal{S}$ is influenced by the registration just because of its entanglement with $\mathcal{A}$. The auxiliary system $\mathcal{A}$ is usually called *ancilla*.

It seems, however, that any registration on microscopic systems has to use *detectors* in order to make features of microscopic systems visible to humans. Detector is a macroscopic system containing *active volume* $\mathcal{D}$ and *signal collector* $\mathcal{C}$ in thermodynamic state of metastable equilibrium. Notice that the active volume is a physical system, not just a volume of space. Interaction of the detected systems with $\mathcal{D}$ triggers a relaxation process leading to macroscopic changes in the detector that are called *detector signals*. For the theory of detectors, see, e.g., [29, 30]. Thus, our notion of detector is less general while that of ancilla is more general than what is often assumed.

For example, consider a ionisation gas chamber that detects a particle $\mathcal{S}$ so that $\mathcal{S}$ first enters the active volume $\mathcal{D}$ of the chamber and then $\mathcal{S}$ can leave $\mathcal{D}$ again and be subject to further registrations. When in $\mathcal{D}$, $\mathcal{S}$ interacts with several gas atoms that become ionized. The microscopic subsystem of freed electrons can also be viewed as an ancilla $\mathcal{A}$ with an initial separation status defined by energy, the ancilla electrons having positive energy while the other electrons of $\mathcal{D}$ are bound and have negative energy. $\mathcal{A}$ "is detected" subsequently by the rest of the detector, that is, $\mathcal{A}$ interacts with $\mathcal{D}$ and $\mathcal{C}$ and is involved in a dissipative process in which the initial separation status is lost.

According to the above assumption, measurements on ancillas need detectors. Thus we are lead to the following hypothesis [16]:

Pointer Hypothesis *Any meter for microsystems must contain at least one detector and every reading of the meter can be identified with a signal from a detector.*

This assumption makes the reading of meters less mysterious.

In the above example of ionization chamber, the state of the ancilla that is prepared by the interaction with the object system has, initially, a separation status: it can be distinguished from other systems of atoms within $\mathcal{D}$ and, therefore, registered without disturbance by other particles. However, in the process of interaction with $\mathcal{D}$ and $\mathcal{C}$ and the relaxation process, its energy is dissipated and its position is smeared so that it loses its separation status. We assume next:

Active-Volume Hypothesis *Active volume $\mathcal{D}$ of the detector detecting system $\mathcal{S}'$ contains many particles in common with $\mathcal{S}'$. The state of $\mathcal{S}' + \mathcal{D}$ then dissipates so that $\mathcal{S}'$ loses its separation status.*

Thermodynamic relaxation is necessary to accomplish the loss. $\mathcal{S}'$ might be the objects system or an ancilla of the original experiment.

Finally, we propose the following assumption:

Separation Status Hypothesis *Let the Schrödinger equation for the composite $\mathcal{S} + \mathcal{M}$ leads to a linear superposition of alternative evolutions such* *that some of the alternatives contain losses of separation status of the object system or ancilla(s). Then, there is a state reduction of the linear superposition to the proper mixture of the alternatives.*

Separation Status Hypothesis can be considered as a dynamical law in the sense that it determines the correction to unitary evolution *uniquely*. This has been shown in [19, 20] for a large class of scattering and registration processes. That is why the law can be viewed as an empirical rule. However, there is as yet no proof that the class contains all conceivable dynamical processes: there could be processes to which the rule either fails to be applicable or to which it is applicable but gives wrong results.

Some more research is necessary.

The three Hypotheses form a basis of our theory of measurement. They generalize some empirical experience. Moreover, they are rather specific and, therefore, testable. In fact, they cannot be disproved by purely logical argument but rather by an experimental counterexample. For the same reason, they also show a specific direction in which experiments are to be proposed and analyzed: if there is a state reduction, does then a loss of separation status take part in the process? What system loses its status? How the loss of the status can lead to state reduction?

In fact, our theory remains rather phenomenological with respect to the last question in that it suggests no detailed model of the way from a separation status change to a state reduction. Such a model would require some new physics and we believe that hints of what the new physics can be will come from attempts to answer the above questions for suitable experiments.

7 Stern-Gerlach story retold

In this section, we shall modify the textbook description (e.g., [22], p. 14) of the Stern-Gerlach experiment. In this way, the above ideas can be explained and illustrated.

A silver atom consists of 47 protons and 61 neutrons in the nucleus and of 47 electrons around it, but we consider only its mass-center and spin degrees of freedom and denote the system with these degrees of freedom by $\mathcal{S}$. Let $\vec{\mathrm{x}}$

be its position and $\vec{\mathsf{p}}$ its momentum and S_z the z-component of its spin with eigenvectors $|j\rangle$ and eigenvalues $h\hbar/2$, where $j = \pm 1$.

Let $\mathcal{M}$ be a Stern-Gerlach apparatus with an inhomogeneous magnetic field in a region D that splits different z-components of spin of a silver atom arriving in D with a momentum in a suitable direction. Let a photo-emulsion film with energy threshold E_0 be placed orthogonally to the split beam. The emulsion is the active volume $\mathcal{D}'$ of $\mathcal{M}$ and it may be also the signal collector if the hit emulsion grains can be made directly visible.

First, let $\mathcal{S}$ be prepared at time t_1 in a definite spin-component state

$$|\vec{p}, \Delta\vec{p}\rangle \otimes |j\rangle \tag{11}$$

where $|\vec{p}, \Delta\vec{p}\rangle$ is a Gaussian wave packet so that $\mathcal{S}$ can be registered by $\mathcal{M}$ within some time interval (t_1, t_2). Let state (9) have a separation status at t_1 and $\mathcal{D}'$ is in initial metastable state $\mathsf{T}_{\mathcal{M}}(t_1)$ at t_1.

Interaction of $\mathcal{S}$ with $\mathcal{M}$ is described by measurement coupling U (see Section 2). The time evolution within (t_1, t_2) is:

$$\mathsf{U} N \Pi(|\vec{p}, \Delta\vec{p}\rangle \langle\vec{p}, \Delta\vec{p}| \otimes |j\rangle \langle j| \otimes \mathsf{T}_{\mathcal{M}}(t_1) \Pi \mathsf{U}^\dagger = \mathsf{T}_j(t_2)$$

where Π is antisymmetrization on the Hilbert space of silver atom part of $\mathcal{S} + \mathcal{D}'$ and N is a normalization factor because Π does not preserve normalization. States $\mathsf{T}_j(t_2)$ are determined by these conditions.

This evolution includes a thermodynamic relaxation of $\mathcal{D} + \mathcal{S}'$. States $\mathsf{T}_j(t_2)$ describe subsystem $\mathcal{S}$ that has lost its separation status. Then, individual states of $\mathcal{S}$ do not make sense: *neither the conditional state nor the state transformer exist for* $\mathcal{S}$. (These notions are, in fact, applicable only for some parts of some measurements with ancillas.)

State $\mathsf{T}_j(t_2)$ also describes a detector signal. The signals will be concentrated within one of two strips on the film, each strip corresponding to one value of j. We make the following assumption. The strips lie in two space regions D'_1 and D'_2 that are sufficiently separated so that the extents of the silver atom subsystems within them do not overlap. Then, the two possible separation status losses can be distinguished. Although there is only one separation status, losses of separation status may differ from each other.

Suppose next that the initial state of $\mathcal{S}$ at t_1 is

$$|\vec{p}, \Delta\vec{p}\rangle \left(\sum_j c_j |j\rangle\right)$$

with

$$\sum_j |c_j|^2 = 1$$

As it is linear, unitary evolution U gives

$$N\mathsf{U}\Pi\left[|\vec{p}, \Delta\vec{p}\rangle \langle\vec{p}, \Delta\vec{p}| \otimes \left(\sum_j c_j |j\rangle\right)\left(\sum_{j'} c_{j'} \langle j'|\right) \otimes \mathsf{T}_{\mathcal{M}}(t_1)\right]\Pi\mathsf{U}^\dagger$$
$$= \sum_{jj'} c_j c^*_{j'} \mathsf{T}_{jj'}(t_2)$$

a quadratic form in $\{c_j\} \in \mathbb{C}^2$. Coefficients $\mathsf{T}_{jj'}(t_2)$ of the form are operators on the Hilbert space of $\mathcal{S} + \mathcal{D}$.

The operator coefficients are state operators only for $j' = j$. From the linearity of U, it further follows that

$$\mathsf{T}_{jj}(t_2) = \mathsf{T}_j(t_2)$$

Now we postulate the following correction to the Schrödinger equation:

1. The loss of separation status of $\mathcal{S}$ disturbs the standard quantum evolution so that, instead of

   $$\sum_{jj'} c_j c^*_{j'} \mathsf{T}_{jj'}(t_2)$$

   state

   $$\sum_j |c_j|^2 T_j(t_2)$$

   results.

2. States $T_j(t_2)$ are uniquely determined by the experimental arrangement: the measurement coupling and the losses of separation status in the meter.

3. The sum is not only a convex combination but also a proper mixture of the signal states $T_j(t_2)$. That is, the system $\mathcal{S} + \mathcal{M}$ is always in one particular state $T_j(t_2)$ after each individual registration and the probability for that is $|c_j|^2$.

The described model is simple because the silver atoms are both the object systems and components of the detector. If the detector contained no silver, we would have to insert an intermediate step suggested by the example of a ionization gas chamber in Section 6.

Stern-Gerlach experiment measures values of a truncated POVM that consist of two effects,

$$\mathsf{E}_j = |\vec{p}, \Delta\vec{p}\rangle \langle\vec{p}, \Delta\vec{p}| \otimes |j\rangle \langle j|$$

where $j = \pm 1$. Clearly, the set $\{\mathsf{E}_j\}$ lives on a two-dimensional subspace $\mathbf{H}_{\mathrm{Exp}}$ of the Hilbert space of the system $\mathcal{S}$ that is defined by the projection

$$\Pi[\mathbf{H}_{\mathrm{Exp}}] = |\vec{p}, \Delta\vec{p}\rangle \langle\vec{p}, \Delta\vec{p}|$$

8 Experiments with superconductor currents

It is not clear whether our theory is applicable to experiments with quantum systems that are not microscopic such as Bose-Einstein condensates (BEC) (see, e.g., Pointer Hypothesis). However, even an observation "by naked eye" of the fluid helium in a glass vessel needs photons that are scattered off the helium. Thus, we may need microsystems in order to observe properties of quantum systems that are not microscopic. Then, our theory can be applicable to the microsystems.

Let us briefly describe a possible instance of experiments that have not been analyzed in the way proposed at the end of Section 6 and where such an analysis could either disprove our theory or give some new insight on the nature of state reduction.

Consider a single Josephson junction SQUID ring [31], that is a superconducting ring interrupted by transversal layers of oxide which allows the electrons to pass through by tunneling. A quantum model of such a device, based on a number of simplifications and assumptions, which we call "Quantum model of superconductor currents" (QMSC) can be constructed. We shall not go into details of QMSC, referring interested reader to [31] and only list the following properties.

First, QMSC is a quantum system with one degree of freedom which is chosen to be the magnetic flux through the ring created by current of the BEC of the Cooper pairs in the superconductor. Second, QMSC has a Hamiltonian dependent on the external classical magnetic field through the ring. Third, the Hamiltonian has two or more metastable energy levels if the external magnetic flux is chosen properly. Finally, the levels are separated by a potential barrier.

The two possible quantum states create two possible quantum magnetic fluxes through the ring that are added to the external flux. The quantum fluxes can be sufficiently strong to be considered macroscopically distinct so that they could be distinguished by "naked eye", or by a "tiny magnetic needle" [32,6].

The original aim was to prove or disprove that linear superposition of macroscopically distinct states are possible. The hope was that, by a suitable arrangement of the SQUID experiment, one can prepare a state of the QMSC that is a linear superposition of the two metastable states. Some measurements were proposed that would prove the existence of the linear superposition.

The side aspect of such measurements important for us is that the measurements are supposed to be usual quantum measurements inducing a collapse of the wave function. In this case, it will be the collapse to one of the linear superposition components. Now, if there is such a collapse, where is the separation status change that would be necessary if our theory were true? This is definitely a bigger and more specific problem that our theory adds to the difficulty mentioned in [32, 6] of how the tiny needle can influence the macroscopic magnetic field so that it could change appreciably.

However, the notion of a tiny needle occurs only in thought experiments that are based on QMSC. If the real experiments are studied, one finds that they are organized along completely different lines [33, 34]. Such measurements can be done in a more precise way and still confirm the existence of linear superposition but they have less clear results about the wave function collapse. Let us look at some details.

The measurements [33, 34] are made spectroscopically. A microwave radiation is applied to the superconductors and one looks for a resonance at the frequency of the energy difference between the metastable states. The energy that the radiation imparts to the superconducting device is smaller than the height of the potential barrier between the metastable states, hence it requires the tunneling between them and this in turn requires a linear superposition of the states.

Then, however, a different measurement structure emerges: the registered system is a photon rather than the BEC and the measurement apparatus seems to use (a part of) the superconductor device rather than any tiny needle. The most important result for us is that the observed resonance lines show an appreciable width. This is interpreted as a dissipation effect that occurs somewhere within the superconducting device. However, the dissipation is not well understood, in particular, it does not occur within the framework of QMSC [31].

Thus, the new rules of Section 6 suggest a new direction of investigation that concentrates on the dissipation effect. Indeed, some dissipation and some metastable states are postulated by our theory of measurement. The dissipation may take part within a definite subsystem of the apparatus and one can then ask whether a loss of separation status occurs there, what system loses its status and how the status loss is associated with any state reduction. One could also try to suggest another experimental setup that were better adapted to the task of answering these questions.

9 Conclusion

We have shown that the disturbance due to environmental particles makes the experiments that measure any "whole" POVM impossible. In particular, there is no meter that would measure the whole spectrum of s.a. operators such as position, momentum, spin, angular momentum or energy.

The explanation of why real measurements do not seem to be disturbed is, first, that different quantities than POVMs are registered. As such quantities, we have proposed TPOVMs. Second, the preparations of the object systems satisfy an additional condition that is usually not mentioned. To describe the condition, the notion of separation status has been defined in [16]. Here, we have modified the notion so that some problems with the original definition disappear.

The next crucial observation is that the roles of ancilla and detector in registrations of microsystems must be distinguished from each other. We have then conjectured that every meter contains at least one detector and meter readings are always signals of detectors. Moreover, separation statuses must be lost in detectors.

Thus, our theory of measurement give the preparation and registration procedures new importance, even stronger than that of Copenhagen interpretation: they must include changes of separation status. Finally, study of different kinds of real experiments shows that the changes of separation status are associated with state reductions.

What is called "collapse of wave function" could then be roughly explained as the state degradation due to a loss of separation status of the object system or an ancilla by a thermodynamic relaxation process in a detector. Hence, the collapse occurs under specific objective conditions and its origin has a definite place and time. In this way, the theory of state reduction can be made more complete.

The Separation Status Hypothesis of Section 6 is our new dynamical law. The correction to Schrödinger equation defined by it is uniquely determined in a large class of scattering and registration processes [19, 20]. In particular, there is no problem of preferred frame [3]. Work on extensions of the class is in progress.

The hypotheses formulated in Section 6 are testable and have a non-trivial predictive power. As a possible test, experiments with superconductor currents have been proposed.

Acknowledgements

The author is indebted to Jürg Fröhlich, Stefan Jánŏs Petr Jizba, Franck Laloë, Jiří Tolar and Howard M. Wiseman for useful discussions.

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