The informationally-complete quantum theory

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Abstract

Quantum mechanics is a cornerstone of our current understanding of nature and extremely successful in describing physics covering a huge range of scales. However, its interpretation remains controversial for a long time, from the early days of quantum mechanics to nowadays. What does a quantum state really mean? Is there any way out of the so-called quantum measurement problem? Here we present an informationally-complete quantum theory (ICQT) and the trinary property of nature to beat the above problems. We assume that a quantum system?s state provides an informationally-complete description of the system in the trinary picture. We give a consistent formalism of quantum theory that makes the informational completeness explicitly and argue that the conventional quantum mechanics is an approximation of the ICQT. We then show how our ICQT provides a coherent picture and fresh angle of some existing problems in physics. The computational content of our theory is uncovered by defining an informationally-complete quantum computer.

1. Introduction

The unease of understanding quantum theory (QT) began at the very beginning of its establishment. The famous Bohr-Einstein debate [1, 2] inspired a lively controversy on quantum foundations. QT is surely an empirically successful theory, with huge applications ranging from subatomic world to cosmology. However, why does it attract such a heated debate over its whole history? The controversial issues on quantum foundations mainly focus on two aspects: (Q1) What does a wave function (or a quantum state) really mean? (Q2) Is the so-called quantum measurement problem [3-8] really a problem? The first axiom of the standard QT states that a system's wave function provides a complete description of the system. But accepting the wave function as QT's central entity, what is the physical meaning of the wave function itself? In this regard, there are two alternatives that the quantum state might be either a state about an experimenter's knowledge or information about some aspect of reality (an 'epistemic' viewpoint), or a state of physical reality (an 'ontic' viewpoint). A recent result [9] on this issue seems to support the reality of quantum states, yet with ongoing controversy [10, 11].

On the other hand, the quantum measurement problem is perhaps the most controversial one on quantum foundations. According to the orthodox interpretation (namely, the Copenhagen interpretation [4]) of QT, the quantum state evolves deterministically according to the Schrödinger equation in a superposition of different states, but actual measurements always collapse, in a truly random way, the physical system into a definite state, with a probability given by the probability amplitude. When, where, and how the quantum state really collapses are out of the reach of QT as it is either 'uninteresting or unscientific to discuss reality before measurement' [11]. In such an orthodox interpretation, classical concepts are necessary for the description of measurements in QT, although the measurement apparatus can indeed be described quantum mechanically, as done by von Neumann [12, 13]. At a cosmological scale, the orthodox interpretation rules out the possibility of assigning a wave function to the whole universe, as no external observer could exist to measure the universe.

Facing with these interpretational difficulties, various interpretations on QT were proposed by many brilliant thoughts, such as the hidden-variable theory [14-16] (initiated by the famous Einstein-Podolsky-Rosen paper [1] questioning the completeness of QT), many-worlds interpretation [17, 18], the relational interpretation [19, 20], and the decoherence theory [5], to mention a few. Thus, '*questions concerning the foundations of quantum mechanics have been picked over so thoroughly that little meat is left.*"[11] The discovery of Bell's inequalities [15] (recently questioned from the many-worlds interpretation [18]) and the emerging field of quantum information [21] might be among a few exceptions. The recent development of quantum information science sparks the information-theoretical understanding of the quantum formalism [22-25].

According to our classical world view, there exists a world that is objective and independent of any observa- tions. By sharp contrast, what is observed on a quantum system is dependent upon the choice of experimental arrangements; mutually exclusive (or complementary) properties cannot be measured accurately at the same time, a fact known as the complementarity principle. In particular, which type of measurements one would like to choose is totally a *free will* [26] or a *freedom of choice* [16, 27, 28]. Such a freedom of choice underlies the Pusey- Barrett-Rudolph theorem [9] and the derivation of Bell's inequalities [16, 27, 28]. However, one could ask: What does a free will or a freedom of choice really mean and whose free will or freedom of choice?

Inspired by this question and the above-mentioned progresses, here we present an informationally-complete quantum theory (ICQT) by removing the concept of free will or freedom of choice. The ICQT is based on the *informational completeness principle*: A quantum system's state provides an informationally- complete description of the system. In other words, quantum states represent an informationally-complete code of any possible information that one might access. Current QT is *not* informationally-complete and thus suffers from so many interpretational difficulties. After working out the informational completeness explicitly in our formalism, we show that informationally-complete physical systems are characterized by dual entanglement pattern, emergent dual Born rule and dual dynamics. The computational content of our theory is uncovered by defining an informationally-complete quantum computer with potential of outperforming conventional quantum computers. Moreover, we consider the possible conceptual applications of our theory, hoping to shed new light on some existing problems in physics. In particular, the ICQT offers an exciting possibility of a coherent theory unifying matter and gravity in an informationally complete quantum picture.

2. Informationally complete states for d-dimensional systems

The orthodox quantum measurement theory [3-8] was proposed by von Neumann and can be summarized as follows. For an unknown d-dimensional quantum state $|\psi\rangle_{\mathcal{S}}$ of a quantum system $\mathcal{S}$ to be measured, a measurement apparatus ('a pointer') $\mathcal{A}$ is coupled to the system via a unitary operator $\hat{U}_{\mathcal{S}\mathcal{A}}(\hat{s},\hat{p})$. Here $\hat{s}$ is system's observable whose eigenstate with respect to the eigenvalue s_j reads $|j,\mathcal{S}\rangle$, namely,

$$\hat{s}\,|j,\mathcal{S}\rangle = s_j\,|j,\mathcal{S}\rangle \quad (j = 1,2,....,d)$$

$\hat{p}$ is the momentum operator which shifts pointer's $\hat{q}$-reading ($[\hat{q},\hat{p}] = i$). Assuming that the pointer is initialized in a 'ready' state $|0,\mathcal{A}\rangle$ and expanding

$|\psi, \mathcal{S}\rangle$ in terms of $|j, \mathcal{S}\rangle$ as

$$|\psi, \mathcal{S}\rangle = \sum_j c_j |j, \mathcal{S}\rangle$$

then the system and the apparatus are mapped into

$$\hat{U}_{\mathcal{SA}}(\hat{s}, \hat{p}) |\psi, \mathcal{S}\rangle |0, \mathcal{A}\rangle = \sum_j c_j |j, \mathcal{S}\rangle |q_j, \mathcal{A}\rangle$$

To ideally measure $\hat{s}$, one has to assume that $\mathcal{A}$ must have at least d macroscopically distinguishable pointer positions (plus the ready position corresponding to $|0, \mathcal{A}\rangle$, and the pointer state $|q_j, \mathcal{A}\rangle$ and the measured states $|j, \mathcal{S}\rangle$ have a one-to-one correspondence (namely, they are perfectly correlated). The above is the usual pre-measurement progress. The orthodox interpretation of the measurement can only predict the collapse of a definite state $|j, \mathcal{S}\rangle$ with a probability $|c_j|^2$ given by the probability amplitude c_j; the collapse occurs in a truly random way. For latter convenience, we call $\hat{s}, \hat{p})$ as an observable pair. It is interesting to note that a factorizable structure of the 'measurement operation' $\hat{U}_{\mathcal{SA}}(\hat{s}, \hat{p})$ was discovered in the context of the dynamical approach to quantum measurement problem [6, 7].

To avoid the quantum measurement problem, here we take a key step by assuming explicitly informational completeness, whose meaning will be clear below, in our formalism of describing nature. To this end, starting from a separate state $|\psi, \mathcal{S}\rangle |\phi, \mathcal{A}\rangle$, we introduce the third system, called the 'programming system' $(\mathcal{P})$ hereafter. We assume that $\mathcal{P}$ has $D_{\mathcal{P}}$ dimensions spanned by $D_{\mathcal{P}}$ orthogonal states, called programming states

$$|r, \mathcal{P}\rangle \quad (r = 1, 2, ..., D_{\mathcal{P}} - 1)$$

where $D_{\mathcal{P}}$ is to be determined by informational completeness. Let us define a unitary programming operation

$$\hat{U}_{\mathcal{P}(\mathcal{SA})}(\hat{s}, \hat{p}) = \sum_{r=0}^{D_{\mathcal{P}}-1} |r, \mathcal{P}\rangle \langle r, \mathcal{P}| \hat{U}_{\mathcal{SA}}(\hat{s}, \hat{p})$$

which means that if $\mathcal{P}$ is in $|r, \mathcal{P}\rangle$, then do a unitary measurement operation $\hat{U}_{\mathcal{SA}}(\hat{s}_r, \hat{p}_r)$ on $\mathcal{SA}$. Now suppose that $\mathcal{P}$ is prepared in an initial state

$$|\chi, \mathcal{P}\rangle = \sum_r g_r |r, \mathcal{P}\rangle$$

Then the state of the whole system $\mathcal{PSA}$ reads

$$|\mathcal{P}(\mathcal{SA})\rangle = \sum_{r=0}^{D_{\mathcal{P}}-1} g_r \, |r, \mathcal{P}\rangle \, |r, \mathcal{SA}\rangle \tag{1}$$

where

$$|r, \mathcal{SA}\rangle = \hat{U}_{\mathcal{SA}}(\hat{s}_r, \hat{p}_r) \, |\psi, \mathcal{S}\rangle \, |\phi, \mathcal{A}\rangle$$

For a given $|r, \mathcal{P}\rangle$, the observable pair, denoted by $(\hat{s}_r, \hat{p}_r)$, to be measured is determined by the Schmidt form of $|r, \mathcal{P}\rangle$. Note that $|\mathcal{P}(\mathcal{SA})\rangle$ can also be written in a Schmidt form with positive real coefficients [29] Hereafter we suppose that the Schmidt decomposition of $|\mathcal{P}(\mathcal{SA})\rangle$ have been done.

Now the key point of our formalism is to require that the programming system $\mathcal{P}$ encodes all possible, namely, informationally complete, measurement operations that are allowed to act upon the $\mathcal{SA}$-system. To be 'informationally complete', all programmed measurement operations $\hat{U}_{\mathcal{SA}}(\hat{s}_r, \hat{p}_r)$ can at least achieve the measurements of a complete set of operators for $\mathcal{S}$; for the d-dimensional system, the complete set has d^2 operators[30], i.e., the minimal $D_{\mathcal{P}} = d^2$. However, as these $D_{\mathcal{P}}$ states $|r, \mathcal{SA}\rangle$ of $\mathcal{SA}$ in the Schmidt form can at most have d independent ones, in general they cannot be locally distinguished with certainty. Note that informationally complete set of operators or measurement are important for quantum state tomography [31, 32].

Another trick in the above discussion is that, to enable the informationally-complete programmed measurements, it seems that one needs $D_{\mathcal{P}}$ different measurement apparatuses. Hereafter we take a step further by dropping this specific measurement model by regarding the $\mathcal{A}$-system as a single system with $D_{\mathcal{A}}(\geq d)$ dimensions. In this case we have $D_{\mathcal{P}} \geq dD_{\mathcal{A}}$. The step is necessary for seeking a model-independent and intrinsic description of the whole system $\mathcal{PSA}$.

To have an easy understanding of our informationally-complete description of physical systems, some remarks are necessary. First, we note that the third system are also included in other interpretations of QT, such as the many-worlds interpretation [17, 18] and the relational interpretation [19, 20]. However, the third system in our formalism plays a role that is dramatically different from those interpretations. Actually, imposing informational completeness into our quantum description of nature distinguishes our theory from all previous interpretations of QT. Second, the fact that $|r, \mathcal{SA}\rangle$, as entangled, can always be written in a Schmidt form implies a symmetric role played by $\mathcal{S}$ and $\mathcal{A}$; such a distinction could be a convenience. Meanwhile, the role of $\mathcal{P}$ is dramatically different from that of either $\mathcal{S}$ or $\mathcal{A}$. But the combined system $\mathcal{SA}$ and $\mathcal{P}$ play a symmetric role. We anticipate that such a feature could have profound consequences, particularly for the internal consistency of the theory. We will find that this is indeed the case when we consider the dynamics of the ICQT.

3. The emergent dual Born rule

How to acquire information and which kind of information to acquire are two questions of paramount importance. According to the ICQT, on one hand, the only way to acquire information is to interact (i.e., entangle) the system $\mathcal{S}$ and the apparatus $\mathcal{A}$ with each other; no interaction leads to no entanglement and thus no information. This is in a similar spirit as the relational interpretation [19, 20], which treats the quantum state as being observer-dependent, namely, the state is the relation between the observer and the system. On the other hand, the programming system $\mathcal{P}$, by interacting with $\mathcal{SA}$, dictates the way (actually, the informational-complete way) on which kind of information to acquire about the system $\mathcal{S}$. For instance, if the whole system is programmed to measure $\hat{s}_r$, then $\mathcal{S}$ and $\mathcal{A}$ interact with each other to induce the programmed measurement operations $\hat{U}_{\mathcal{SA}}(\hat{s}_r, \hat{p}_r)$. This process generates the entangled state $|r, \mathcal{SA}\rangle$ with which $\mathcal{A}$ 'knows, in a completely coherent way, all information about $\mathcal{S}$ in the basis of $\hat{s}_r$; the amount of entanglement contained in $|r, \mathcal{SA}\rangle$ quantifies the amount of information acquired during this measurement. Also, $\mathcal{P}$ 'knows', also in a completely coherent way, the information about which kind of information (here $|r, \mathcal{SA}\rangle$, $\mathcal{A}$ has about $\mathcal{S}$; the amount of the $\mathcal{P}-(\mathcal{SA})$ entanglement quantifies the amount of information on which kind of measurements to do. All information is coherently and completely encoded there.

Note that information here is characterized by dual entanglement—the $\mathcal{P}-(\mathcal{SA})$ entanglement (maximally, $\ln D_{\mathcal{P}}$) and the $\mathcal{S}-\mathcal{A}$ entanglement (maximally, $\ln D_{\mathcal{S}}$) contained in $|r, \mathcal{SA}\rangle$. As both $|r, \mathcal{SA}\rangle$ and $|\mathcal{P}(\mathcal{SA})\rangle$ are pure states, their entanglement is uniquely quantified by the usual entanglement entropy [29, 33]. This immediately identifies each of the squared coefficients of their Schmidt decompositions as a probability to reconcile with Shannon's definition of entropy. Put differently, in our informationally-complete description of physical systems, entanglement is all *the* information; classical terms like probability arise in our description only when we are so used to classical concept of information.

The status of quantum states in the informational completeness formalism thus represent a complete reality of the whole system ($\mathcal{P}$, $\mathcal{S}$, and $\mathcal{A}$). Such a reality picture is only possible by taking in account the informational completeness explicitly into our formalism. Quantum states do exist in a world that is informational and objective. Whatever an observation might be, they always encode information pertaining to that observation, without invoking observers or having to appeal to any mysterious mechanisms to account for wave function collapse; there is simply no wave function collapse. Here quantum states are all relative, but their information encoded in dual entanglement is invariant under the changes of local bases.

In certain sense, $\mathcal{P}$ and $\mathcal{A}$ act like a 'quantum being' ('qubeing') who holds coherently all the informationally- complete programmes on how to entangle $\mathcal{S}$ and $\mathcal{A}$. In this way, the qubeing has all the information about $\mathcal{S}$. However, our human beings, unlike the qubeing, only have limited ability to acquire information, with limited precisions, limited degrees of freedom, limited information detection and storage, and so on; or simply we are so used to and familiar with classical concepts on information and physical systems. For example, an experimenter, Alice, would like to acquire information about $|\psi, \mathcal{S}\rangle$. First of all, she has to *decide* which kind of information she would like to know. After making a decision, she needs then to *observe* (that is, to interact with) the apparatus readily entangled with $\mathcal{S}$. In principle, Alice's decision and observation are all physical processes which should be described quantum mechanically. Nevertheless, Alice is macroscopic, and has so many quantum degrees of freedom and limited ability (lack of full knowledge of the entire system). In this case, she has to ?trace out? her quantum degrees of freedom involved in her decision (interaction with $\mathcal{P}$), leading to a mixed state

$$\sum_r |g_r|^2 \, |r, \mathcal{P}\rangle \, |r, \mathcal{SA}\rangle \, \langle r, \mathcal{SA}| \, \langle r, \mathcal{P}|$$

This state allows a probability interpretation about Alice's freedom of choice: Each of her decisions $|r, \mathcal{P}\rangle$ occurs with a probability of $|g_r|^2$. As far as a particular choice $|r, \mathcal{P}\rangle$ has been made, again she has to trace out her quantum degrees of freedom involved in her observation (interaction with $\mathcal{A}$). This then leads to the usual Born rule about $|\psi, \mathcal{S}\rangle$ for the given measurement.

To summarize the above picture, the world view of the ICQT is fascinating. If we regard the system $\mathcal{S}$ as an indivisible part of the qubeing $\mathcal{PA}$, the whole system $\mathcal{PSA}$ then represents an informationally complete and objective entity; it seems that the qubeing has its own 'consciousness' to encode and access all its information. The trinary picture (the division of $\mathcal{S}$, $\mathcal{A}$, and $\mathcal{P}$) of physical systems arises here as a new feature of the ICQT, as shown in Fig. 1.

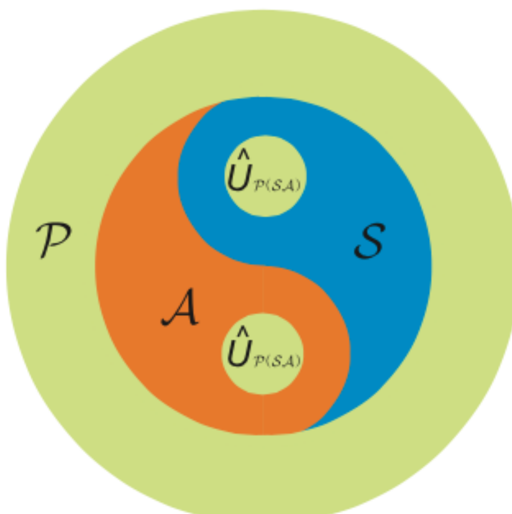

Figure 1: **The trinary picture of the world**. The division of system $\mathcal{S}$, measurement apparatus $\mathcal{A}$, and programming system $\mathcal{P}$ naturally arises in the informationally complete description of physical systems. 'The pattern of Taiji' shows in the most intuitive manner the $\mathcal{S} - \mathcal{A}$ interaction (entanglement), while the green discs inside and outside the Taiji pattern represent the programmed measurement operation $\hat{U}_{\mathcal{P}(\mathcal{SA})}$ between $\mathcal{P}$ and $\mathcal{SA}$. In ancient China, Taoists regarded the Taiji pattern as a ?diagram of the universe?. The trinary picture of the world shown here is ubiquitous in the sense that our world, on the most fundamental ground, is made up of a trinity: gravity ($\mathcal{P}$), elementary particles ($\mathcal{S}$) and gauge fields ($\mathcal{A}$); the trinity should be describable by an informationally-complete quantum theory.

To retain the informationally complete description of nature, such a trinary picture seems to be unavoidable. The limitation of informational contents in dual entanglement could be tentatively called 'the trinity principle', instead of the conventional complementarity principle, to put the trinary property of physical systems on the most fundamental ground.

The loss of the trinary picture of describing physical systems leads to the *emergent dual Born rule*, i.e., the probability description on which kind of observables to measure and on which eigenvalue of the observable to measure, due to, e.g., lack of full knowledge of the entire system in our ICQT. The conventional von Neumann entropy quantifies this dual loss of information. In other words, the conventional Born rule arises as a consequence of the sacrifice of informationally-complete description in the trinary picture; the sacrifice leads to a *partial* reality of physical system as described by conventional QT.

4. The informationally-complete dynamics

According to the above picture of nature, single free systems are simply meaningless for acquiring information and thus in the 'entanglement-vacuum' state. The '$\mathcal{S}+\mathcal{A}$' description in the usual QT is also inadequate because of its informational incompleteness. Therefore, the dynamics of the ICQT has to obey the informational completeness principle within the trinary picture and thus will be dramatically different from the usual picture.

Before considering the informationally-complete dynamics, let us introduce an important conception of *dual measurability* : the $\mathcal{P}-(\mathcal{S}\mathcal{A})$ measurability and the programmed $\mathcal{S}\mathcal{A}|_{\mathcal{P}}$ measurability. The former means the ability of measuring $\mathcal{P}$ with $\mathcal{S}\mathcal{A}$ and vice verse; the latter means the ability of measuring $\mathcal{A}$ with $\mathcal{S}$ and vice versa, under a given programmed measurement operation of $\mathcal{P}$. The $\mathcal{P}-(\mathcal{S}\mathcal{A})$ measurability (the programmed measurability $\mathcal{S}\mathcal{A}|_{\mathcal{P}}$ leads to $D_{\mathcal{P}}=D_{\mathcal{A}}D_{\mathcal{S}}\,(D_{\mathcal{A}}=D_{\mathcal{S}}=D)$.

After the above preparation, now we give a definition of informationally-complete physical systems: A physical system is said to be informationally-complete if and only if (the use of 'if and only if' will be explained below) it is consisted of $\mathcal{S}$, $\mathcal{A}$ and $\mathcal{P}$ described as a trinity such that the $\mathcal{P}-(\mathcal{S}\mathcal{A})$ measurability and the programmed measurability $\mathcal{S}\mathcal{A}|_{\mathcal{P}}$ are satisfied. As a result of this definition, the $\mathcal{P}-(\mathcal{S}\mathcal{A})$ measurability measurability implies the existence of D^2 independent *informationally complete entanglement operations* in the Hilbert spaces of both $\mathcal{P}$ and $\mathcal{SA}$. Meanwhile, under the given programming state of $\mathcal{P}$, the programmed measurability $\mathcal{SA}|_{\mathcal{P}}$ implies the existence of D independent informationally incomplete entanglement operations in the Hilbert spaces of both $\mathcal{S}$ and $\mathcal{A}$. The two sets of D independent informationally incomplete operations for $\mathcal{S}$ and $\mathcal{A}$ jointly define D^2 independent informationally entanglement operations for $\mathcal{SA}$.

Let us suppose that the informationally-complete system $\mathcal{PSA}$ has a general Hamiltonian $\hat{H}_{\mathcal{PSA}}$. We assume that the whole system evolves according to a Schrödinger-like equation (we take $\hbar = 1$), namely,

$$i\frac{d}{dt}|\mathcal{PSA},t\rangle = \hat{H}_{\mathcal{PSA}}|\mathcal{PSA},t\rangle$$

In general.

$$\hat{H}_{\mathcal{PSA}} = \hat{H}_{\mathcal{P}} + \hat{H}_{\mathcal{S}} + \hat{H}_{\mathcal{A}} + \hat{H}_{\mathcal{PS}} + \hat{H}_{\mathcal{PA}} + \hat{H}_{\mathcal{SA}} + \hat{H}_{\mathcal{P(SA)}}$$

where the subscripts label the corresponding systems. Now our problem is to determine how the informational completeness principle constrains the form of $\hat{H}_{\mathcal{PSA}}$ and thus the dynamics of the $\mathcal{PSA}$-system.

We note that for any time t, there always exists an orthonormal basis

$$\{|e_n(t),\mathcal{P}\rangle\,; n = 0,1,...,D_{\mathcal{P}}-1\}$$

called the 'Schmidt basis' for $\mathcal{PSA}$, such that $|\mathcal{PSA},t\rangle$ is in a Schmidt form. We can associate each $|e_n(t),\mathcal{P}\rangle$ as an eigenstate of $\mathcal{P}$-system's observable $\hat{e}_{\mathcal{P}}(t)$ with eigenvalue $e_n(t)$, possibly time-dependent. Without loss of generality, one can choose $\hat{e}_{\mathcal{P}}(t)$ to be $\hat{H}_{\mathcal{P}}(t)$; other choices are equivalent up to local unitary transformations on $\mathcal{P}$ and $\mathcal{SA}$. It is easy to verify that the following Hamiltonian obeys the informational completeness principle:

$$\hat{\mathcal{I}}_{\mathcal{PSA}} = \hat{H}_{\mathcal{P}} + \hat{H}_{\mathcal{P(SA)}} = \hat{H}_{\mathcal{P}}(t) + \sum_{n=0}^{D_{\mathcal{P}}-1}|e_n,\mathcal{P}\rangle\langle e_n,\mathcal{P}|\,\hat{H}_{\mathcal{SA}|_{\mathcal{P}}}(e_n,t) \qquad (2)$$

where we have omitted the time-dependence of $e_n(t)$ for brevity. Here

$$\hat{H}_{\mathcal{SA}|_{\mathcal{P}}}(e_n,t) = \hat{U}^{-1}_{\mathcal{SA}|_{\mathcal{P}}}[e_n,t]\,i\frac{d}{dt}\hat{U}_{\mathcal{SA}|_{\mathcal{P}}}[e_n,t]$$

is an informationally-complete set of $\mathcal{SA}$ and as such, $\hat{H}_{\mathcal{S}} + \hat{H}_{\mathcal{A}} + \hat{H}_{\mathcal{PS}} + \hat{H}_{\mathcal{PA}} + \hat{H}_{\mathcal{SA}}$ has been included therein. $\hat{\mathcal{I}}_{\mathcal{PSA}}$ induces a dynamical evolution

$$|\mathcal{PSA}, t\rangle = \hat{U}_{\mathcal{PSA}} |\mathcal{PSA}, t = 0\rangle$$

The evolution operator $\hat{U}_{\mathcal{PSA}}$ always has a factorizable structure

$$\hat{U}_{\mathcal{PSA}} = \hat{U}_{\mathcal{P}}(t)\hat{U}_{\mathcal{SA}|_{\mathcal{P}}}$$

such that $(\forall e_n)$

$$i\frac{d}{dt}\hat{U}_{\mathcal{P}}(t) = \hat{H}_{\mathcal{P}}(t)\hat{U}_{\mathcal{P}}(t)$$
$$i\frac{d}{dt}\hat{U}_{\mathcal{SA}|_{\mathcal{P}}}(e_n, t) = \hat{H}_{\mathcal{SA}|_{\mathcal{P}}}(e_n, t)\hat{U}_{\mathcal{SA}|_{\mathcal{P}}}(e_n, t) \tag{3}$$

In this way, the dynamical evolutions of $\mathcal{P}$ and $\mathcal{SA}$ are mutually defined, in accordance with the informational completeness principle.

The Hamiltonian $\hat{\mathcal{I}}_{\mathcal{PSA}}$ as given above respects the $\mathcal{P} - (\mathcal{SA})$ measurability. If we also require the programmed measurability $\mathcal{SA}|_{\mathcal{P}}\,(\forall e_n)$, the evolution governed by $\hat{H}_{\mathcal{SA}|_{\mathcal{P}}}(e_n, t)$ will also acquire the factorizable structure as

$$\hat{H}_{\mathcal{SA}|_{\mathcal{P}}}(e_n) = \hat{H}_{\mathcal{S}|_{\mathcal{P}}}(e_n, t) + \sum_{i=0}^{D} |\varepsilon_i(e_n), e)n, \mathcal{S}\rangle \langle\varepsilon_i(e_n), e)n, \mathcal{S}| \,\hat{H}_{\mathcal{A}}[e_n, \varepsilon_i(e_n), t]$$

with

$$\hat{H}_{\mathcal{A}} = \hat{U}_{\mathcal{A}}^{-1} i\frac{d}{dt}\hat{U}_{\mathcal{A}}[e_n, \varepsilon_i(e_n), t]$$

Here the Schmidt basis for $\mathcal{S}|_{\mathcal{P}}$ is $\{|\varepsilon_i(e_n), e)n, \mathcal{S}\rangle\}$, where $\{|\varepsilon_i(e_n), e)n, \mathcal{S}\rangle\}$ is an eigenstate of $\hat{H}_{\mathcal{S}|_{\mathcal{P}}}(e_n, t)$ with eigenvalue $\varepsilon_i(e_n)$ for given e_n. The $\mathcal{SA}|_{\mathcal{P}}$ dynamics is then similar to the $\mathcal{P} - \mathcal{SA}$ dynamics considered above. Such a dual dynamics of the whole system $\mathcal{PSA}$ is an attribute of the trinary description and quite distinct to the usual Schrödinger evolution.

5. Relation with conventional QT

What is the relation between (3) and the usual Schrödinger equation? Here the dynamical evolution is such that systems $\mathcal{P}$ and $\mathcal{SA}$ are mutually defined, and also systems $\mathcal{S}$ and $\mathcal{A}$. In an informationally-complete quantum field theory, if we regard $\mathcal{S}$ as particle fields and $\mathcal{A}$ as gauge fields, and $\mathcal{SA}$ together as matter field, then we immediately recognize that system $\mathcal{P}$ must be the gravitational field, nothing else. If we think this way, an amazing picture (Fig. 1) of our world arises: The gravitational field and matter field are mutually defined and entangled?no matter, no gravity (spacetime) and vice verse, and for each of their entangled patterns, particle fields and their gauge fields are likewise mutually defined and entangled. If this is indeed what our nature works to obey the informational completeness principle, the conventional QT will be an approximation of our ICQT when nature?s programming system, i.e., gravity, 'hides' its quantum effects. Such an approximation leads to the approximate Schrödinger equation and the probability description of current QT, which, with its current form, is informationally incomplete. This is in the exact sense that classical Newtonian mechanics is an approximate theory of special relativity when a physical system has a speed much less than the speed of light. On the other hand, no matter how weak the gravity is, it is forced to be there by the informational completeness principle, to play a role for the internal consistency of the theory.

If we take the above argument seriously, then the ICQT captures the most remarkable trinity of nature, namely, the division of nature by particles, their gauge fields and gravity (spacetime), though the role of the Higgs field needs a separate consideration. The previous two sections argued the necessity of the informational completeness in the trinary description. Here we see that it is also sufficient: We do not have to invoke more programming systems to programe $\mathcal{PSA}$ simply because we do not have spacetime (gravity) out of spacetime (gravity).

As we showed elsewhere [34], following the above argument indeed leads to a consistent framework of unifying spacetime and matter, without the fundamental inconsistencies [35] between gravity and conventional quantum field theory. With the theoretical input from loop quantum gravity predicting the quantized area [20, 35-38], the informationally complete quantum field theory naturally gives the holographic principle [39-41]. Such a strong limit on the allowed states of the trinary system in any finite spacetime regime, as imposed by the ICQT, paves the way to escape the infrared and ultraviolet singularities (divergences) that occur in conventional quantum field theory.

The natural position of gravity in the ICQT cannot be accidental and may be a strong evidence supporting our informationally-complete description of nature. Thus, at the most fundamental level, our reasoning suggests once again the trinary picture of our world consisting of particles, gauge fields and gravity as a trinity. Loss of this trinity in our description leads to dual probability description as we argued above. It is surprise to see that nature singles out gravity as a programming field, which plays a role that is definitely different from matter field. This also indicates the distinct roles of matter-matter (particles and their gauge fields) entanglement and spacetime-matter entanglement.

6. Informationally-complete quantum computation and simulation

A new theory should make new predictions or/and motivate new applications. Of course, previous interpretations of QT are very important for a better understanding of the theory. However, no interpretations make new predictions or/and motivate new applications. Now we argue that our ICQT indeed motivates new applications if we consider its computational power. Even though gravity would play certain role in our future understanding of nature, artificial informationally- complete quantum systems are realizable as a quite good approximation.

What is an informationally-complete quantum computer (ICQC)? We define the ICQC as an artificial informationally-complete quantum systems, or a quantum intelligent system (qubeing), which has an informationally-complete trinary structure consisting of $\mathcal{S}$, $\mathcal{A}$, and $\mathcal{P}$. The ICQC starts from an initial state

$$|\text{ICQT}\rangle_0 = |\psi, \mathcal{S}\rangle\, |\phi, \mathcal{A}\rangle\, |\chi, \mathcal{P}\rangle$$

As usual, the $\mathcal{S}$ system has n qubits, and thus dimensions of 2^n. To be well defined, we also use qubits to make up the $\mathcal{A}$ system and the $\mathcal{P}$ system; $\mathcal{A}(\mathcal{P})$ has $n_{\mathcal{A}}(n_{\mathcal{P}})$ qubits and dimensions of $2^{n_{\mathcal{A}}}(2^{n_{\mathcal{P}}})$. To fulfil the informational completeness principle, we have $n_{\mathcal{A}} = n$ and $n_{\mathcal{P}} = 2n$. Our ICQC then works by applying certain patterns of universal quantum logic gates (single-qubit and two-qubits ones), determined by quantum algorithm pertaining to the question under study. Generally speaking, as an artificially controllable quantum system the patterns of gates are allowed to exhaust all unitary operations on the whole $\mathcal{PAS}$ system, which we denote collectively by

$$\hat{V}_{\mathcal{PAS}}\hat{U}(\mathcal{P},\mathcal{A},\mathcal{S},\mathcal{AS},\mathcal{PA},\mathcal{PS},\mathcal{PAS})$$

At the end of running the ICQT, we perform the programmed measurement operation $\hat{U}_{\mathcal{P}(\mathcal{SA})}$ on $\mathcal{PAS}$. The resulting final state of the ICQC reads

$$|\text{ICQT}\rangle = \hat{U}_{\mathcal{P}(\mathcal{SA})}\hat{V}_{\mathcal{PAS}}\,|\text{ICQT}\rangle_0$$

Here

$$\hat{U}_{\mathcal{P}(\mathcal{SA})} = \sum_{p=0}^{4^n-1} |p,\mathcal{P}\rangle\langle p,\mathcal{P}|\,\hat{U}(p,\mathcal{A},\mathcal{S})$$

and all $\hat{U}(p,\mathcal{A},\mathcal{S})$ span a complete operator set for $\mathcal{S}$.

Is the ICQC defined above the usual quantum computer merely with more $(n + n_{\mathcal{A}} + n_{\mathcal{P}} = 4n)$ qubits? The answer is definitely 'not' because of their conceptual difference. To see this, we prepare each qubit of $\mathcal{S}$ in the initial state

$$|+,\mathcal{S}\rangle = \frac{1}{\sqrt{2}}(|0,\mathcal{S}\rangle + |1,\mathcal{S}\rangle)$$

such that $|\psi,\mathcal{S}\rangle$ is in a superposition of all 2^n bit-values with equal probability amplitudes:

$$|\psi,\mathcal{S}\rangle = \frac{1}{\sqrt{2^n}}\sum_{x=0}^{2^n-1}|x,\mathcal{S}\rangle$$

The initial states of $\mathcal{A}$ and $\mathcal{P}$ are likewise prepared:

$$|\phi,\mathcal{A}\rangle = \sum_{x=0}^{2^n-1}|y,\mathcal{A}\rangle$$

and

$$|\chi,\mathcal{P}\rangle = \sum_{x=0}^{4^n-1}|z,\mathcal{P}\rangle$$

Such a coherent superposition of conventional quantum com????puter's initial states is believed to be the very reason of quantum algorithm?s speedup [21, 42]. We make a further simplification by doing nothing anymore on $\mathcal{A}$ and $\mathcal{S}$, namely, the ICQC only acts $\hat{U}_{\mathcal{P}(\mathcal{SA})}$ on the initial state:

$$\hat{U}_{\mathcal{P}(\mathcal{SA})}\,|\psi,\mathcal{S}\rangle\,|\phi,\mathcal{A}\rangle\,|\chi,\mathcal{P}\rangle$$

For such a simplified ICQC, $\hat{U}_{\mathcal{P}(\mathcal{SA})}$ can code all possible programmed measurement operations upon algorithm?s speedup [21, 42]. We make a further simplification by doing nothing anymore on . These operations are actually all allowed quantum algorithms and their outputs on n-qubit state $|\psi, \mathcal{S}\rangle$, in the terminology of conventional quantum computing. Then we immediately see that in the ICQC, one has *dual parallelism*: Parallelism in initial states as usual *and* parallelism of programmed operations (algorithms and outputs). In other words, *an ICQC with* $4n$ *qubits could compute all algorithms of usual quantum computers with* n *qubits*. Due to this particular dual parallelism enabled by the ICQC, it is reasonable to expect much higher computational power.

Actually, the ICQC is, by definition, the most powerful computational machine on qubit systems in the sense of informational completeness; otherwise it is informationally incomplete. Finding algorithms on the ICQC to explicitly demonstrate the computational power of the ICQC is surely a future interesting problem. Also, computational complexity and error-tolerance in the ICQC framework are two important issues. If nature does use the informational completeness as a guiding principle, it computes the world we currently know; such a world could be simulated and thus comprehensible by the ICQC in principle.

7. Other conceptual applications

Below we give a few conceptual applications of the informational completeness principle and the ICQT, hoping to shed new light on some long-standing open questions in physics.

An important question is how to understand the occurrence of the classical world surrounding us, including the second law of thermodynamics and the arrow of time, in our new framework characterized by the informational completeness principle and the trinary picture of nature. Though we cannot present quantitative analysis of the problem here, a qualitative and conceptual answer to the problem is quite transparent: For informationally complete quantum systems, always-on interactions lead to the increase of entanglement for $\mathcal{S} - \mathcal{S}$ and $\mathcal{P} - \mathcal{SA}$ systems; the universe as a whole has an increasing entanglement, a kind of entanglement arrow of time. It is easy to verify the entanglement creation by considering the $\mathcal{PSA}$ evolution governed by $\hat{\mathcal{I}}_{\mathcal{PSA}}$ from a separable state. At a thermodynamical and macroscopical scale, tracing out thermodynamically and macroscopically irrelevant degrees of freedom, only as an approximate description of the underlying informationally complete physics, leads to the second law of thermodynamics, the arrow of time, and ultimately, the classical world.

We note related analysis on the role of entanglement in the thermodynamic arrow of time in the framework of conventional [43, 44] or the time-neutral formulation [45] of quantum mechanics. As gravity arguably plays an essential role in our informationally complete description of nature, it is intriguing to see that gravity plays some role in the occurrence of the second law of thermodynamics and the arrow of time, as hinted in the study of black-hole thermodynamics [35, 46-48].

Finally, we briefly discuss potential conceptual applications to cosmology and our human beings, two systems believed to be most complex in the world. Obviously, the conceptual difficulty of applying usual QT to the whole universe disappears in our ICQT. Actually, the ICQT is interpretation-free and does not need an observer as the observer is a part of the universe; the description of the universe by the ICQT would give us all information as it could be. As we will discuss elsewhere, it is possible to describe our human beings as a classical, informationally complete system. In this way, many aspects of human beings could be comprehensible even quantitatively in principle, purely in the informational point of view.

It could well be that the informational completeness should be a basic requirement for any physical systems, classical or quantum. In this regard, classical information theory in its current form is, very likely, also information- ally incomplete and should be reconsidered. It is in this sense that the informational completeness deserves to be named as a principle. It is a missed principle in our current understanding of nature and a rule behind the comprehensibility of the world.

8. Conclusions and outlook

In the present work, we have presented an interpretation-free QT under the assumption that quantum states of physical systems represent an informationally-complete code of any possible information that one might access. To make the informational completeness explicitly in our formalism, the trinary picture of describing physical systems seems to be necessary. Physical systems in trinity evaluate and are entangled both in a dual form; quantum entanglement plays a central role in the ICQT. We give various evidences and conceptual applications of the ICQT, to argue that the ICQT, naturally identifying gravity as nature?s programming system, might be a correct theory capable of unifying matter and gravity in an informationally complete quantum framework. In this sense, the conventional QT will be an approximation of our ICQT when gravity hides its quantum effects. Such an approximation leads to the approximate Schrödinger equation and the probability description of current QT. This is in the exact sense that classical Newtonian mechanics is an approximate description of relativistic systems. The ICQT motivates an interesting application to informationally-complete quantum computing.

As we argued above, current quantum mechanics is *not* informationally-complete and thus suffers from interpretational difficulties. The explicit demand of informational completeness not only removes the concep- tual problem of our current understanding of quantum mechanics, but also leads to a profound constraint on formulating quantum theory. Thus, the ICQT should not be understood simply as another interpretation of current QT. As we noted previously, adding informational completeness requirement into our current quantum formalism leads to serious consequences: Informational completeness not only restricts the way on how to describe physical systems, but also the way how they interact with each other. This will thus give a very strong constraint on what physical processes could have happened or be allowed to happen.

On one hand, the ICQT provides a coherent conceptual picture of, or sheds new light on, understanding some problems or phenomena in current physics, including the intrinsic trinity of particles, gauge fields and gravity, the occurrence of the classical world, the arrow of time, and the holographic principle. On the other hand, some other problems, such as the complementarity principle, quantum nonlocality [18] and quantum communication, should be reconsidered from the viewpoint of the ICQT. All current quantum communication protocols [21, 49, 50] have to make use of classical concepts on information. It is thus very interesting to see how to do communication in the ICQT and, particularly, to see whether or not it is possible to achieve unconditionally secure communication. Possible experimental tests of our theory will be considered elsewhere.

According to the ICQT, the world underlying us is informationally complete, deterministic and thus objective. Such a world view is of course quite different from what we learn from current quantum mechanics, but in some sense, returns to Einstein's world view. Such a viewpoint calls for a reconsideration of our current understanding on physical reality, spacetime (gravity) and matters, as well as their relations. Let us cite the famous Einstein-Podolsky-Rosen paper [1] here: '*While we have thus shown the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible.*' It is too early to judge whether or not our ICQT completes current quantum mechanics in the Einstein- Podolsky-Rosen sense as experiments will be the ultimate judgement. But if nature does work like a description provided by the ICQT, nature will be very funny and more importantly, nature is comprehensible.

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