Objective Reality of Pointer States

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Abstract

It is shown that the nature of quantum states that emerge from decoherence is such that one can *measure* the expectation value of any observable of the system in a single measurement. This can be done even when such pointer states are a priori unknown. The possibility of measuring the expectation value of any observable, without any prior knowledge of the state, points to the objective existence of such states.

The emergence of classical world from quantum mechanics has been a subject of endless debate for about a century now [1, 2]. If one believes that quantum mechanics is a fundamental theory governing the dynamics of all particles, it is only a natural inference that classical physics should emerge from it. However, the unitary nature of the Schrödinger equation appears to allow states which are never observed in our familiar classical world. Quantum mechanics allows existence of superposition of macroscopically separated states, whereas classical mechanics doesn't allow a particle to be delocalized. There have been attempts of a very diverse nature to resolve the issue, but there is no consensus on it to date.

One way the emergence of classicality can be understood is by recognizing that macroscopic objects are almost impossible to shield from their environment. The interaction with the environment is unavoidable and leads to decoherence [3-5]. The decoherence approach has become very popular because it attempts to explain the emergence of classicality while staying within conventional quantum mechanics, and has demonstrated predictive power. Controlled influence of environment has been successfully studies in numerous experiments.

The process of decoherence can be represented in the following way. Let the state of a system and its environment be written as

$$|\psi\rangle = \left(\sum_p c_p |p\rangle\right) |\mathcal{E}^0\rangle \tag{1}$$

where the state of the system is expanded in terms of certain basis states $|p\rangle$ which we shall call *pointer states*, and $|\mathcal{E}^0\rangle$ is the state of the environment. The meaning of pointer states will be clear in the following discussion.

Assume a Hamiltonian and time evolution of the form [6]

$$\mathscr{H} = \sum_p |p\rangle \langle p| \otimes \mathscr{H}^{(p)} \tag{2}$$

$$\mathcal{U}_t = \sum_p |p\rangle \langle p| \otimes \mathcal{U}_t^{(p)} \tag{3}$$

where $\mathcal{U}_t^{(p)} = e^{-i\mathscr{H}^{(p)}t/\hbar}$ and the $\mathscr{H}^{(p)}$ are certain unspecified Hermitian operators involving the environment. The state (1), with the above specified time-evolution, evolves into

$$|\psi(t)\rangle = \sum_p c_p |p\rangle \mathcal{U}_t^{(p)} |\mathcal{E}^0\rangle \tag{4}$$

If one wants to look only at the system and forget about the environment, it is useful to write the reduced density matrix, which is obtained by writing the density matrix for (4), and tracing over the states of the environment, and has the form

$$\rho_r(t) = \sum_{p,p'} c_{p'}^* c_p |p\rangle \langle p'| \langle \mathcal{E}^0 | \mathcal{U}_t^{(p')\dagger} \mathcal{U}_t^{(p')} |\mathcal{E}^0\rangle \tag{5}$$

Under the kind of time evolution specified above, the diagonal components of the density matrix, in the basis $|p\rangle$, remain unchanged, while off-diagonal elements are reduced by a factor $\langle \mathcal{E}^0 | \mathcal{U}_t^{(p')\dagger} \mathcal{U}_t^{(p')} |\mathcal{E}^0\rangle \leq 1$. The time dependence of the suppressing terms will, in general, depend on the specific model of the environment and its interaction with the system, but for a wide variety of models the suppressing terms have been found to rapidly decay over short time scales[4, 7-10].

Over a time scale, called decoherence time-scale, the off-diagonal terms in (5) disappear for all practical purposes, and one is left with a diagonal density matrix of the system

$$\rho_r(t) = \sum_p |c_p|^2 |p\rangle \langle p| \tag{6}$$

The off-diagonal elements which are associated with quantum superpositions are no longer present. Thus the system *appears* to behave classically. This special set of states is selected by the interaction with the environment to emerge as classical states. In the literature they have come to be known as pointer states, and this pro- cess of environment-induced selection is called *einselection* [11, 12]. The Hamiltonian in (2) is a generic form which leads to the system ending up in diagonal density matrix in the pointer states basis. In general, a microscopic Hamiltonian is needed to find out what states form the pointer basis. What should be the pointer states for a particular system, is a question that is not easy to answer. This question has been answered only for a few cases. For example, the pointer states of a simple harmonic oscillator are believed to be coherent states [13], and those for a free particle have been exactly shown to be minimum uncertainty Gaussian states [14]. There are some indications that Gaussian states emerge as pointer states for a particle in Stern-Gerlach experiment [15]. In the limit of weakest interaction with the environment, energy eigenstates have been shown to emerge as pointer states [16].

A question one might ask is whether such pointer states exist out there on their own or do they need an observer to bring them into existence. If decoherence is indeed the mechanism for the emergence of classicality from quantum mechanics, the pointer states should have an objective existence. Ollivier, Poulin and Zurek have addressed this issue from the point of view that pointer states leave their imprint on the environment, which can be read out by an observer without disturbing the state [17]. Here we take a different approach and ask if the the pointer states themselves have some properties which make them robust enough to allow getting information from them without disturbing them. According to the standard quantum mechanics lore, one cannot get any information about an unknown quantum state. The process of measurement necessarily destroys the original quantum state, unless of course, in the trivial case, it is already in an eigenstate of the observable being measured. In the following we will show that pointer states are special in the sense that they allow a *measurement* of the expectation value of any observable without destroying the state.

Let us now explore the process of measurement on a system which is undergoing decoherence. We assume that the system of interest is interacting with the environment. Let us represent the pointer states of the system by $\{|p\rangle\}$. In addition, the system is assumed to be interacting with an apparatus. So, the system is expected to undergo decoherence because of its interaction with the environment, and also cause a shift in the state of the apparatus, which essentially constitutes the process of measurement. The Hamiltonian of the system, apparatus and the environment is given by

$$\mathscr{H} = H_A + gQ_SQ_A + \sum_p |p\rangle\langle p| \otimes \mathscr{H}^{(p)} \tag{7}$$

where H_A is the Hamiltonian of the apparatus and the second term on the right hand side represents the interaction of the apparatus with the system, with a strength parametrized by g. Operators Q_S and Q_A are the operators of the system and the apparatus, respectively, through which the interaction takes place. The "free" Hamiltonian of the system is ignored because the measurement interaction is supposed to act for a short time, and dominate the time evolution during that period. The last term represents the interaction between the system and the environment, and is taken to be of a form which leads to the states $\{|p\rangle\}$ emerging as pointer states, as demonstrated in the preceding discussion. We assume that the apparatus is so constructed that Q_A commutes with the free Hamiltonian of the apparatus, i.e., $[Q_A, H_A] = 0$, so that we can have eigenstates $|a_i\rangle$ such that $Q_A|a_i\rangle = a_i|a_i\rangle$ and $H_A|a_i\rangle = E_i^a|a_i\rangle$.

The initial state of the system, apparatus and environment is assumed to be

$$|\Psi_0\rangle = \left(\sum_p c_p|p\rangle\right)|\mathcal{E}^0\rangle|\phi_a\rangle \tag{8}$$

where the first term represents the unknown state of the system, written in terms of the pointer state basis, and $|\phi_a\rangle$ is the initial state of the apparatus. It should be emphasized that at this point, neither the initial state of the system, nor the pointer basis is known. Here $|\mathcal{E}^0\rangle$ is the initial state of the environment.

For the measurement process, we let the apparatus interact with the system for a time T. In addition to this, the system is continually interacting with the environment. The combined state of the system, apparatus and environment, after a time T, is given by

$$|\Psi_T\rangle = e^{-i\frac{T}{\hbar}\left[H_A + gQ_SQ_A + \sum_{p'}|p'\rangle\langle p'|\mathscr{H}^{(p')}\right]}\left(\sum_p c_p|p\rangle\right)|\mathcal{E}^0\rangle|\phi_a\rangle \tag{9}$$

Introducing a complete set of states $\sum_{p''}|p''\rangle\langle p''|$ before the initial state, the above can be rewritten as

$$|\Psi_T\rangle = \sum_{p''} e^{-i\frac{T}{\hbar}\left[H_A + g\langle p''|Q_S|p''\rangle Q_A + \mathscr{H}^{(p'')}\right]} |p''\rangle \langle p''| \left(\sum_p c_p |p\rangle\right) |\mathcal{E}^0\rangle |\phi_a\rangle$$

$$= \sum_p e^{-i\frac{T}{\hbar}\left[H_A + g\langle p|Q_S|p\rangle Q_A + \mathscr{H}^{(p)}\right]} c_p |p\rangle |\mathcal{E}^0\rangle |\phi_a\rangle \tag{10}$$

Since $\mathscr{H}^{(p)}$ commutes with the other terms in the exponent, the above can be simplified to

$$|\Psi_T\rangle = \sum_p e^{-i\frac{T}{\hbar}\left[H_A + g\langle p|Q_S|p\rangle Q_A\right]} c_p |p\rangle |\phi_a\rangle e^{-i\frac{T}{\hbar}\mathscr{H}^{(p)}} |\mathcal{E}^0\rangle \tag{11}$$

The fact that H_A commutes with Q_A further allows us to separate the first exponent into two parts

$$|\Psi_T\rangle = \sum_p e^{-i\frac{T}{\hbar}H_A} c_p |p\rangle e^{-i\frac{gT}{\hbar}\langle p|Q_S|p\rangle Q_A} |\phi_a\rangle \mathcal{U}_T^{(p)} |\mathcal{E}^0\rangle \tag{12}$$

Let us now assume that the apparatus is prepared in an initial state which is a wave-packet of eigenstates $|r\rangle$ of an operator R_A such that $[R_A, Q_A] = i\hbar$. The wave-packet may be centered at (say) r_0. It is straightforward to see what the effect of $e^{-i\frac{gT}{\hbar}\langle p|Q_S|p\rangle Q_A}$ on $|\phi_a(r_0)\rangle$ will be. It will simply translate the packet by an amount $gT\langle p| Q_S |p\rangle$, thus yielding an apparatus state which is shifted by an amount proportional to the expectation value of the system observable Q_S

$$|\Psi_T\rangle = \sum_p e^{-i\frac{T}{\hbar}H_A} c_p |p\rangle |\phi_a(r_0 - gT\langle Q_S\rangle_p)\rangle \mathcal{U}_T^{(p)} |\mathcal{E}^0\rangle \tag{13}$$

where $\langle Q_S\rangle_p = \langle p| Q_S |p\rangle$.

This is the central result of this work. It can be interpreted in more ways than one. In the conventional treatment of decoherence one believes that one state from the pointer states basis emerges as the reality, which happens with a probability. One can write a density matrix for the state represented by (13), and take a trace over the degrees of freedom of the environment. Doing that we find

$$\rho_r(p, p') = \sum_{p,p'} c_p c_{p'}^* |p\rangle \langle p'| e^{-i\frac{T}{\hbar}H_A} |\phi_a(r_0 - gT\langle Q_S\rangle_p)\rangle$$

$$\langle \phi_a(r_0 - gT\langle Q_S\rangle_p)| e^{i\frac{T}{\hbar}H_A} \langle \mathcal{E}^0| \mathcal{U}_T^{(p)\dagger} \mathcal{U}_T^{(p)} |\mathcal{E}^0\rangle \tag{14}$$

For realistic decoherence, the factor $\langle\mathcal{E}^0|\mathcal{U}_T^{(p)\dagger}\mathcal{U}_T^{(p)}|\mathcal{E}^0\rangle$ is expected to decay to close to zero over a very short time scale. One is then left with a density matrix which is approximately diagonal in the pointer basis

$$\rho_r(p,p') \approx \sum_p |c_p|^2 |p\rangle\langle p| e^{-i\frac{T}{\hbar}H_A} |\phi_a(r_0 - gT\langle Q_S\rangle_p)\rangle \\ \langle\phi_a(r_0 - gT\langle Q_S\rangle_p)| e^{i\frac{T}{\hbar}H_A} \quad (15)$$

The above implies that a particular pointer state $|p\rangle$ emerges as a reality with a probability $|c_p|^2$, while the apparatus state gets shifted by an amount proportional to the expectation value of the observable of the system being measured (Q_S) in that particular state. Thus we end up measuring the expectation value of Q_S in the state which emerged out of decoherence, which we had no knowledge of. Note that the measurement process doesn't disturb the state - the only natural disturbance is due to decoherence.

Another way to interpret the result would be the following. Let us denote the states of the environment appearing in the final expression as $|\mathcal{E}_p\rangle = \mathcal{U}_T^{(p)}|\mathcal{E}^0\rangle$. Beyond the decoherence time-scale, the states $|\mathcal{E}_p\rangle$ corresponding to different p are expected to be nearly orthogonal to each other. The combined state of the system, apparatus and environment has the form

$$|\Psi_T\rangle = \sum_p e^{-i\frac{T}{\hbar}H_A} c_p |p\rangle |\phi_a(r_0 - gT\langle Q_S\rangle_p)\rangle |\mathcal{E}_p\rangle \quad (16)$$

Looking at the sum over the states $|p\rangle$ one might wonder that the state appears entangled. However, from the point of view of decoherence, when the results gets correlated with certain orthogonal states of the environment, the measurement is considered complete. The terms corresponding to different $|p\rangle$ may be considered as independent *branches* which are robust (no possibility of recoherence), each representing an independent classical reality [4]. For example, in k'th branch the measured expectation values of the observable will correspond to only one state $|k\rangle$. A kind of modified many-worlds interpretation might be needed here, if one doesn't believe in a real collapse of the quantum state [18].

All this is possible because of the very nature of pointer states, namely because of their being entangled with certain orthogonal states of the environment. In all this analysis, at no stage did we need any information about the states $|p\rangle$ - their being pointer states is enough to provide the possibility of measuring the expectation value of any observable. This analysis has demonstrated that it is possible to *measure* the expectation value of any observable of a system if it has undergone decoherence and pointer states have emerged as its quantum states. This definitely points to the objective existence of pointer states.

The result (16) has another very interesting meaning, which can have some practical use. In a conventional measurement, the state of the apparatus gets shifted by an amount proportional to an *eigenvalue* of the observable being measured. Our result says that if the system being measured is undergoing decoherence, then the shift in the apparatus state will be proportional, not to the *eigenvalue* but, to the *expectation value* of the observable being measured.

In conclusion, we have shown that pointer states allow the *measurement* of the expectation value of any observable of choice, with a single quantum measurement. The measurement process does not disturb an emerging pointer state. Just the expectation value of the arbitrarily chosen observable is picked out by the measuring apparatus. Since we are able to measure the expectation value of any observable of our choice in a pointer state which is emerging because of decoherence, it is sufficient reason to assign them an objective existence. As a corollary, we find that if a conventional quantum measurement is made on a system undergoing decoherence, what will be measured is not the eigenvalue of the observable of interest, rather it?s expectation value. It might be pertinent to mention that the only other case where the possibility of measuring expectation value in a single measurement exists is that of protective measurements [19-23]. However, there are several constraints which severely re- strict a realization of the same.

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