A General Argument Against the Universal Validity of the Superposition Principle

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Abstract

We reconsider a well known problem of quantum theory, i.e., the so called measurement (or macro-objectification) problem, and we rederive the fact that it gives rise to serious problems of interpretation. The novelty of our approach derives from the fact that the relevant conclusion is obtained in a completely general way, in particular, without resorting to any of the assumptions of ideality which are usually done for the measurement process. The generality and unescapability of our assumptions (we take into account possible malfunctionings of the apparatus, its unavoidable entanglement with the environmment, its high but not absolute reliability, its fundamentally uncontrollable features) allow to draw the conclusion that the very possibility of performing measurements on a microsystem combined with the assumed general validity of the linear nature of quantum evolution leads to a fundamental contradiction.

1 Introduction

In most of the treatises on the foundations of Quantum Mechanics, when discussing the measurement problem and its possible solutions, the authors make use of the von Neumann scheme for an *ideal measurement process* [1] since, due to its simplicity, it allows to grasp immediately why the standard solution to such a problem is not satisfactory. The von Neumann argument goes as follows.

Consider a microscopic system S and one of its observables O. Let o_n be its eigenvalues (we suppose the spectrum of O to be purely discrete and non?degenerate), and $|o_n\rangle$ the corresponding eigenvectors.

Let us call M the apparatus devised to measure the observable O of the microsystem S. One supposes that M has a ready-state $|M_0\rangle$, i.e., a state in which the apparatus is ready to measure the considered property, plus a set of *mutually orthogonal states* $|M_n\rangle$ (and orthogonal to the ready?state), which correspond to *different macroscopic configurations* of the instrument, like different positions of a pointer along a scale.

Finally, one assumes that the interaction between the microsystem S and the apparatus M is *linear* (since the Schrödinger equation is supposed to govern all natural processes) and that it yields a perfect correlation between the initial state of S and the final state of the apparatus, i.e.,

$$\text{Initial state:} \quad |o_n\rangle \otimes |M_0\rangle \longrightarrow \text{ Final state:} \quad |o_n\rangle \otimes |M_n\rangle \tag{1.1}$$

in this way one is sure that, if the final state of the apparatus is $|M_n\rangle$ (i.e.,the pointer, for example, is in the n-th position along the scale), he can say that the state of the particle is $|o_n\rangle$, and that the observable O has the value o_n.

Within such a context, the measurement problem arises when the initial state of the particle, previous to the measurement, is not just one of the vectors $|o_n\rangle$ like in eq. (1.1), but a superposition of them, for example:

$$|m+l\rangle = \frac{1}{\sqrt{2}}[|o_m\rangle + |o_l\rangle]$$

In this case, due to the *linearity* of the quantum evolution, the final state of the microsystem+apparatus is given by:

$$|m+l\rangle \otimes |M_0\rangle = \frac{1}{\sqrt{2}}[|o_m\rangle + |o_l\rangle]\,|M_0\rangle \longrightarrow \frac{1}{\sqrt{2}}[|o_m\rangle\,|M_m\rangle + |o_l\rangle\,|M_l\rangle] \tag{1.2}$$

This final state is an entangled state of the microscopic system and of the apparatus, and it is well known that (if one assumes that the theory is complete, i.e., that the wave-function contains all the information about the system) in the considered case it is not *even in principle* legitimate to state that the properties associated to the states $|M_m\rangle$ or $|M_l\rangle$ are possessed by the apparatus (the same holds true for the microsystem): as a consequence, the apparatus is not in any macroscopic definite configuration. This is the essence of the quantum measurement problem.

The *standard* way out from this difficulty is given by the so-called *wavepacket reduction postulate*, which states that "at the end of the measurement process" the final vector in (1.2) reduces to one of its terms

$$|o_m\rangle\,|M_m\rangle \quad \text{or} \quad |o_l\rangle\,|M_l\rangle$$

with a probability given by the square modulus of the coefficient associated to that term (1/2 for both outcomes, in our example).

It is obvious that the postulate of wave?packet reduction contradicts the general validity of the Schrödinger equation and consequently it does not represent a satisfactory solution to the measurement problem. Accordingly, many other (more or less satisfactory) solutions have been proposed [2, 3, 4, 5, 6]. In spite of this, various people [8, 7, 9] have suggested that the problem does not derive from the structure of quantum mechanics (in particular from the linear character of the quantum evolution), or from the wavepacket reduction postulate, but from adopting the over-simplified model of the measurement process put forward by von Neumann. If one takes into account a more realistic model, they argue, the measurement problem would turn into a false one, and there would be no need to modify the interpretation of Quantum Mechanics, or to put forward a new theory.

In particular, the following assumptions have been criticized:

- That the measuring apparatus can be prepared in a precise state $|M_0\rangle$: since the instrument is a macroscopic object with many degrees of freedom, it is impossible to know its precise state at any given time.

- That one can safely neglect the interactions between the apparatus and the surrounding environment. In fact, the interaction with the environment (which is referred to as decoherence) produces essentially a randomization of the phases associated to the different components of the wave?function, a process which can be seen as an *apparent* collapse of the wave?function into one of these components.

- That the final states of the apparatus, corresponding to perceptively different macroscopic configurations of the apparatus itself, are orthogonal: actually, different states usually correspond to different *positions* of some component of the instrument, and since no wave-function can have compact support in configuration space (because of the quantum evolution), wave-functions corresponding to different states cannot, in general, be orthogonal.

- That the final state of the apparatus gets perfectly correlated to the initial state of the microscopic system: this is an highly idealized characteristic which is not shared by any realistic physical instrument.

Just to give some examples of the above situation, we consider it appropriate to mention explicitly some statements by distinguished physicists, supporting the idea that the measurement problem is a false one:

1. H. Primas, during a conference in Finland [7], has stated:

   > ... the measurement problem of quantum mechanics as it is discussed by theoreticians and philosophers is an extremely ill posed problem ... Measurements of the first kind are unrealistic and completely irrelevant for experimental science ... Replace in all textbooks and philosophical treatises on quantum mechanics the word 'measurement' by the proper expression 'measurement of the first kind' and add a footnote: measurements of the first kind are idealizations which never play any role in experimental science.

   In this article we will prove rigorously that also when consideration is given to very reasonable and realistic schemes of measurement processes, the problem cannot be avoided. As a consequence, Primas' statements are not pertinent.

2. In an article appeared in Physics Today [8], W. Zurek claims that, because of the interaction with the environment, the

   > ... coherent superposition of ... states [like those of eq. (1.2)] ... is continuously reduced to a mixture. A preferred basis of the detector, sometimes called a "pointer basis", has been singled out ... *Decoherence prevents superpositions of the preferred basis states from persisting.*

   Despite what Zurek claims, it is well known that decoherence, *by itself*, does not "prevent superpositions of the preferred basis states from persisting", and the theorem we will prove in this paper will show that Zurek's conclusions are incorrect.

3. In a quite recent book [9], some of the authors (whose conceptual position with respect decoherence is slightly different from the one of refs. [7,8]) seem to consider decoherence a satisfactory explanation of why ordinary macroscopic objects appear to behave classically. For example, in Section 8.3 where decoherence and spontaneous localization models [2] are compared, we find it written:

   > ... stochastic collapse models may be an interesting concept for the purpose of illustrating and discussing the measurement prob-

> lem in quantum theory, but there seems to be no phenomenological necessity for its introduction, neither in connection with the appearance of classical properties (*for which environmental decoherence already provides a satisfactory explanation*), nor prompted by new phenomena.

We stress again that decoherence does not represent by itself a solution to the macro-objectification problem in general, and to the measurement problem in particular: only if one adopts a precise interpretation (like bohmian mechanics, decoherent histories, ...), decoherence can be of help in solving these problems. Unfortunately, all the above authors forget to inform us of the interpretation they adopt (which, of course, cannot be the standard one, since it includes the postulate of wavepacket reduction which, as we have said, is contradictory).

As we have already mentioned, in this paper we will consider a completely general and realistic model for the measurement process, and we will show that superpositions of different macroscopic configurations of macro-objects cannot be avoided within a strict quantum mechanical scheme. Correspondingly, the appearance of macroscopic situations which are incompatible with our definite perceptions about the objects of our experience is inescapable. This "impasse" can only be eliminated either by adopting a precise and unambiguous interpretation which differs from the orthodox one, or by modifying the theory itself[1].

2 The Microscopic System

Let us start our general discussion of the measurement process by analyzing the microscopic system whose properties we want to measure; we consider for simplicity the simplest system upon which non-trivial measurements can be performed, i.e.,a system (S) whose associated Hilbert space ($\mathcal{H}_S$) is two?dimensional - like the one describing the spin of an electron, or the polarization states of a photon - and we consider an observable O having two different eigenvalues; let us call $|u\rangle$ and $|d\rangle$ the eigenstates associated to these

[1] An explicit proof that releasing the request of an ideal measurement does not allow to circumvent the measurement problem can be found in the well known book by d'Espagnat [10]; however, his proof is much more complex and much less general than the one we are going to present here.

eigenvalues. For definiteness, we will consider a single such system and we will call "spin" its degree of freedom; we will say that the particle has "spin Up" when it is in state $|u\rangle$, and that it has "spin Down" when it is in state $|d\rangle$. Besides these two states, also their superpositions can be taken into account, like for example:

$$|u + d\rangle = \frac{1}{\sqrt{2}}[|u\rangle + |d\rangle]$$

a vector describing a new state, "spin Up + spin Down", of the particle. Without any loss of generality, we will assume that, by resorting to appropriate procedures, one can "prepare" the system S in any one of the above considered states.

We remark that we could have considered more general physical systems, like compound ones, and observables having a more complicated spectrum (e.g. the continuos spectrum of position). Anyway, in accordance with the generally accepted position that microsystems can be prepared in a precise quantum state and with the (nowadays) common experimental practice to handle single particles and to measure their (discrete) spin (or polarization, in the case of photons) states, we have chosen to work with very simple microsystems as the one we are considering here. Moreover, we also assume that, after the preparation, the system is in a precise and known state, and that it can be treated as isolated from the rest of the world, at least until the measurement process begins[2]. We stress that if one denies these assumptions it is not clear what he takes quantum theory to be about.

3 The Measuring Apparatus

A measuring apparatus is a *macroscopic* system which, interacting with the microsystem whose properties one is interested in ascertaining, ends up into a state more or less correlated with the eigenstates of the observable it is devised to measure. The different possible outcomes of the measurement are supposed to be correlated to *perceptively different macroscopic configurations* of a part of the apparatus, e.g. different positions of the pointer (for analog

[2] In mathematical terms, we assume that, prior to the measurement process, the wave-function of the universe factorizes into the wave-function of the particle times the wave-function of the rest of the world.

instruments), different numbers on a display (for digital ones), different spots on a photographic plate, different plots on a screen, and so on. For simplicity, in what follows we will assume that the apparatus has a pointer movable along a scale, whose position registers the result of the measurement.

Contrary to microsystems, the measuring apparatus, being a macroscopic object, has many degrees of freedom, most of which - in particular the microscopic ones - we cannot control at all; and of the macroscopic ones, like the position of the pointer, we can have only a very limited control. Moreover, the apparatus, due to its dimensions, is always interacting with the environment (whose degrees of freedom are also essentially out of control). Following this line of reasoning, one can remark that the apparatus - or at least its constituents - existed quite a long time before the measurement is performed, so it had all the time to interact (even if only weakly) with a large part of the universe, or perhaps with all of it. All these interactions make, to a large extent, unknown and uncontrollable the state of the macroscopic systems which enters into play. In spite of this difficulty, in order to keep our analysis as general as possible, we will take them all into account.

According to the above discussion, we should, in general, speak of different situations of the "whole universe", even though our "reading" refers only to the degrees of freedom of the pointer; accordingly, we shall indicate the state vectors we will deal with in the following way:

$$|A\alpha\rangle$$

These vectors belong to the Hilbert space associated to the apparatus, the environment, and in the most general case to the whole universe. A is a label which indicates that the pointer of the apparatus is in a specific macroscopic configuration, i.e.,one which we perceive and we identify with a specific position along the scale. In first approximation, we could say that A is essentially the value x characterizing the "projection operator" $|x\rangle\langle x|$ ($|x\rangle$ being an "improper" state vector of the Hilbert space of the pointer) giving the exact position of (e.g., the centre of mass of) the pointer along the scale. But it is evident that no system can be prepared in such a state (since it is impossible to measure a continuous variable with a perfect accuracy); and even if it were possible to do so, the hamiltonian evolution would immediately change that state: thus the pointer cannot ever be in an eigenstate of $|x\rangle\langle x|$.

We could try to improve our model by taking into account not precise positions along the scale, but small intervals $\Delta(x) = [x - \delta, x + \delta]$, and claiming that "the pointer is at position x" when the wave-function is an eigenstate of the projection operator which projects onto the interval[3] $\Delta(x)$ of the position of the centre of mass. If one takes such a position, the label A characterizing our general state $|A\alpha\rangle$ refers to any wave-function having such a property, of course with the interval $\Delta(x)$ replaced by the interval $\Delta(A)$: as a consequence, for the considered state we can claim that "the pointer is at position A". However, also this approach is not viable since the hamiltonian evolution transforms any wave-function with compact support into a wave-function with a non?compact one; this fact gives rise to what has sometimes been called the "tail problem", a problem which cannot be avoided, and which renders rather delicate the task of making precise the idea of "an object being somewhere" within a quantum mechanical framework.

In the light of the above discussion, we consider a very general physical situation: we will call V_A the set of all (normalized) vectors $|A\alpha\rangle$ for which we are allowed to say that "the pointer of the apparatus is at position A" or, stated differently, that "the universe is in a configuration which we perceive as one corresponding to the statement: the pointer is at A". We do not put any restriction to the vectors belonging to V_A: they can represent wave-functions with or without tails, more or less localized in space, and so on; we do not even resort to projection operators to characterize these states. The only physical requirement we put forward is that, if the pointer admits two macroscopically different positions along the scale (let us call them A and B), then any two vectors corresponding to such different configurations must be "almost orthogonal". This requirement can be translated into the following mathematical relation: denoting as V_B the set of all normalized vectors corresponding to the statement "the pointer is at B" while V_A, as before, contains all the vectors corresponding to the statement "the pointer is at A", we must have:

$$\inf_{\substack{|A\alpha\rangle\in V_A \\ |B\beta\rangle\in V_B}} \| \, |A\alpha\rangle - |B\beta\rangle \, \| \geq \sqrt{2} - \eta \qquad \eta << 1 \tag{3.1}$$

i.e.,the minimum distance between the vectors of the two above sets cannot differ too much from $\sqrt{2}$, which is the distance between two orthogonal

[3] Of course, here we are considering for simplicity a one?dimensional situation; the argument can be easily generalized to the three?dimensional case.

normalized states. We recall that the orthogonality request of the standard measurement theory is done to be sure to be dealing with strictly mutually exclusive situations. Obviously such a request can be partially released (as we are doing here) but not given up completely if one wants to be able to "read" the outcome in a fundamentally non-ambiguous way. It is evident that (3.1) is a necessary requirement if one pretends that different macroscopic positions of the pointer (and of any other system) represent mutually exclusive configurations of the object[4].

Let us now comment on the second parameter α characterizing our states: this is an index which takes into account all other degrees of freedom that are out of control[5]; thus, two vectors labeled by A, but with different values for α, refer to the "same" macroscopic configuration for the pointer (or, in general, of the "part of the universe we perceive"), while they describe two different states for the rest of the universe (e.g., given a certain atom of the pointer, it might be in the ground state when the state is $|A\alpha\rangle$, while it might be in an excited state when it is $|A\beta\rangle$.

Since the microscopic particle has two spin?states, if we want to use the apparatus to distinguish them we have to assume that the pointer admits at least two macroscopically different positions (U and D) along the scale[6]; the previous argument requires to assume that there exist two sets V_U and V_D: the first one contains all the vectors corresponding to the situation in which the pointer can be said to point at "U", while the second set contains all those vectors associated to the statement "the pointer is at D". Moreover,

[4]Obviously, here we are making reference to a genuinely quantum description (with the completeness assumption). In alternative interpretations or formulations of the theory, orthogonality is not necessary to guarantee macroscopic differences. Typically, in hidden variables theories one could have non orthogonal wave-functions and different values for the hidden variables such that the associated physical situations are macroscopically different.

[5]From the mathematical point of view, α stands for the eigenvalues of a complete set of commuting observables for the whole universe, exception made for the position of the pointer.

[6]The idea is that, if we perform the measurement and we find the pointer in the position labeled by U, then we can claim the "the result of the measurement is that the spin of the particle is Up"; similarly, if we find the pointer in D, then we can say that "the spin of the particle is Down".

these two sets must be almost orthogonal in the sense of (3.1):

$$\inf_{\substack{|U\alpha\rangle\in V_U \\ |D\beta\rangle\in V_D}} \| \, |U\alpha\rangle - |D\beta\rangle \, \| \geq \sqrt{2} - \eta \qquad \eta << 1 \tag{3.2}$$

One interesting property of V_U and V_D (which is shared by any couple of sets satisfying (3.2)) is that they have no vectors in common: in fact, it is easy to see that if V_U and V_D had such a common vector, then the minimum distance between them would be zero, a fact which would contradict (3.2). From the physical point of view, this property is obvious since a vector belonging both to V_U and to V_D would be a vector for which we could claim both that "the pointer points at U" and that "the pointer points at D", a contradictory situation since "U" and "D" correspond to macroscopically different situations.

4 The Preparation of the Apparatus

A measuring instrument must be prepared before one performs a measurement, i.e.,one has to arrange the apparatus in such a way that it is ready to interact with the microscopic system and give a result; following the discussion of the previous section, it is evident that the initial state vector must carry an index α which takes into account the state of the rest of the universe: accordingly, we will denote the initial state vector as $|A_0\alpha\rangle$, where A_0 indicates that the pointer "is" in the ready (A_0) state.

Anyway, we remember that, besides the measuring instrument, we have also to prepare the microsystem in a precise state, and moreover we have assumed that after the preparation and immediately before the measurement process, the microsystem itself is isolated from the rest of the universe; the initial state vector for the whole universe can then be written as:

$$|A_0\alpha\rangle = |\text{spin}\rangle \otimes |A_0\bar{\alpha}\rangle$$

where $\bar{\alpha}$ specifies the state of the whole universe, with the exception of the initial state of the micro-particle and the initial "position" of the pointer; $|\text{spin}\rangle$ is the initial state vector of the particle.

Obviously, also in the process of preparing the apparatus we cannot control the state of the universe so that we do not know the precise initial state

$|A_0\bar{\alpha}\rangle$: in fact, in any specific situation any value for the index $\bar{\alpha}$ will occur with a given probability $p(\bar{\alpha})$, which in general is unknown to us - but of course, it has to satisfy appropriate requirements we will discuss in what follows. Accordingly, the initial setup, for the apparatus and the microscopic particle, will be described as follows:

$$\text{Initial Setup } = \{|\text{spin}\rangle \otimes |A_0\bar{\alpha}\rangle\}$$

where $p(\bar{\alpha})$ gives the probability distribution of the remaining, uncontrollable, degrees of freedom.

5 The Measurement Process

If one assumes that Quantum Mechanics governs all physical systems, the measurement process, being an interaction between two quantum systems, is governed by a unitary operator $U(t_I, t_F)$. Suppose the initial state of the microsystem is $|u\rangle$ and the one of the apparatus (plus the rest of the universe) is $|A_0\bar{\alpha}\rangle$, then, during the measurement, the whole universe evolves in the following way:

$$|u\rangle \otimes |A_0\bar{\alpha}\rangle \longrightarrow U(t_I, t_F)[|u\rangle \otimes |A_0\bar{\alpha}\rangle] = |F\,u\,\bar{\alpha}\rangle \tag{5.1}$$

while, if the initial state of the microsystem is $|d\rangle$, one has

$$|d\rangle \otimes |A_0\bar{\alpha}\rangle \longrightarrow U(t_I, t_F)[|d\rangle \otimes |A_0\bar{\alpha}\rangle] = |F\,d\,\bar{\alpha}\rangle \tag{5.2}$$

Some comments are needed.

- Note that in the above equations (5.1) and (5.2) the index $\bar{\alpha}$ distinguishes various possible and uncontrollable situations of the measuring apparatus in its "ready" state. Once the initial state is fully specified also the final one, being the evolution is unitary, is perfectly and unambiguously determined. Accordingly, such a state is appropriately characterized by the same index $\bar{\alpha}$. Note also that, while the state $|A_0\bar{\alpha}\rangle$ belongs to the Hilbert space of the whole universe exception made for the micro-particle, the state $|F\,d\,\bar{\alpha}\rangle$ now includes also the particle.

- Contrary to what one does in the ideal-measurement scheme of von Neumann, we do not assume that the final state is factorized; thus, in general
$$|F\,u\,\bar{\alpha}\rangle \neq |u\rangle \otimes |A_U\bar{\alpha}\rangle$$

- In particular, we do not suppose that the final state of the microsystem be the same as the initial one: we allow the measurement process to modify in a significant way the state of the particle; it could even destroy the particle.

The only thing we require is that *the measuring apparatus is reliable to a high degree*, i.e.,that it can safely be used to measure the state of the microsystem since, in most cases, it gives the correct answer. This means that if the initial state of the microsystem (prior to the measurement) is $|u\rangle$, then the final state $|F\,u\,\bar{\alpha}\rangle$ must belong *in most of the cases* to V_U, while if the initial state of the particle is $|d\rangle$, then the final state $|F\,d\,\bar{\alpha}\rangle$ must *almost always* belong to V_D. Note that by not requiring full reliability, we take into account also the possibility that the measuring instrument gives the wrong results, though pretending that such mistakes occur quite seldom.

It is possible to formalize the above reliability requests in the following way. Let us consider the set K of all subsets J of the possible values that the index $\bar{\alpha}$ can assume and let us equip it with the following (natural) measure:

$$\mu(J) = \sum_{\bar{\alpha}\in J} p(\bar{\alpha})$$

Let us also define the two following sets:

$$\begin{array}{l} J_U^- = \{\bar{\alpha} \text{ such that: } |F\,u\,\bar{\alpha}\rangle \notin V_U\} \\ J_D^- = \{\bar{\alpha} \text{ such that: } |F\,d\,\bar{\alpha}\rangle \notin V_D\} \end{array}$$

J_U^- is the sets of all the indices $\bar{\alpha}$ such that the states $|F\,u\,\bar{\alpha}\rangle$ do not correspond to the outcome *"the pointer is at position U"*, despite the fact that prior to the measurement the state of the particle was $|u\rangle$. Similarly, J_D^- corresponds to the states $|F\,d\,\bar{\alpha}\rangle$ for which we cannot claim that *"the pointer is in D"*, even if the initial state of the system was $|d\rangle$. Let also $J_U^+ = CJ_U^-$ be the complement set of J_U^-, and $J_D^+ = CJ_D^-$ the complement of J_D^-.

Given this, the requirement that the instrument is reliable can be mathematically expressed in the following way:

$$\mu(J_U^-) \leq \epsilon \qquad \mu(J_D^-) \leq \epsilon \quad , \quad \epsilon << 1 \tag{5.3}$$

i.e.,the probability that the final position of the pointer does not match the initial spin-value of the particle is very small, this smallness being controlled by an appropriate parameter ϵ expressing the efficiency of the measuring device and which, as such, can change (always remaining very small) with the different actual measurement procedures one can devise.

Of course, it is easy to derive also limits on the measure for the complements of the above sets:

$$\mu(J_U^+) \geq 1 - \epsilon \qquad \mu(J_D^+) \geq 1 - \epsilon$$

We need to take into account also the two sets: $J^- = J_U^- \cup J_D^-$ and $J^= C J^- = J_U^+ \cap J_D^+$; they satisfy the following relations:

$$\mu(J^-) \leq 2\epsilon \qquad \mu(J^+) \geq 1 - 2\epsilon$$

Again, all these limits simply state that, being the apparatus reliable, the probability that - at the end of the measurement process - the pointer is not in the correct position is very small, *if* the initial state of the particle was either $|u\rangle$ or $|d\rangle$.

It is useful to remark that, having taken into account the possibility that the measuring instrument can make mistakes, we can easily include also the possibility that it fails to interact at all with the microsystem, thus giving no result: in such a case, the pointer remains in the "ready-state", and the corresponding vector belongs to the set J^-. In fact, let us consider the set V_0 associated to the "ready-state", as we did for the two sets V_U and V_D referring to the "U" and "D" positions of the pointer. By the same argument as before, V_0 is disjoint from the two sets V_U and V_D, since the "ready-state" is macroscopically different from the "U" and "D" states; consequently, if the vector, at the end of the measuring process, belongs to V_0, it cannot belong either to V_U or to V_D.

We have mentioned the possibility that the apparatus misses to detect the particle because such an occurrence affects (and in some case in an appreciable way) many experimental situations (e.g. the efficiency of photodetectors is usually quite low). This does not pose any problem to our treatment: in fact, we can easily circumvent this difficulty by simply disregarding (just as it is common practice in actual experiments) all cases in which a detector should register something but it doesn?t. The previous analysis and the sets we have identified by precise mathematical criteria must then be read as referring exclusively to the cases in which the apparatuses register an outcome.

6 Proof of the Theorem

In the previous section we have reconsidered the measurement scheme, first analyzed by von Neumann, and we have reformulated it on very general grounds; in fact, the only requirements at the basis of our discussion are the following two:

1. That the quantum evolution of any physical system is *linear*, since it is governed by the Schrödinger equation;

2. That any two sets, like V_U and V_D, containing vectors corresponding to *different macroscopic configurations* of a macro-object are almost oathogonal:

$$\inf_{\substack{|U\alpha\rangle \in V_U \\ |D\beta\rangle \in V_D}} \| \, |U\alpha\rangle - |D\beta\rangle \, \| \geq \sqrt{2} - \eta \qquad \eta << 1 \tag{6.1}$$

We think that everybody would agree that *any real* measurement situation (if it has to be described in quantum mechanical terms), shares these two properties[7].

Starting with these very simple premises we can now easily show that quantum mechanics must face the problem of the occurrence of superpositions of macroscopically different states of the apparatus, and in general of a macro-system.

In our terms, the "measurement problem" arises (as usual) when the initial spin?state of the particle is not $|u\rangle$ or $|d\rangle$, as we have considered in the previous sections, but a superposition of them, like the state $|u + d\rangle$ of section 2, which can be easily prepared in a laboratory. In such a case, due to the linearity of the evolution, the final state of the particle + apparatus system will be:

$$|u + d\rangle \otimes |A_0 \bar{\alpha}\rangle \longrightarrow U(t_I, t_F)[|u + d\rangle \otimes |A_0 \bar{\alpha}\rangle]$$
$$|F\, u + d\, \bar{\alpha}\rangle = \frac{1}{\sqrt{2}}[|F\, u\, \bar{\alpha}\rangle + |F\, d\, \bar{\alpha}\rangle]$$

[7]As already remarked, request (2) can be violated in hidden variables theories. On the other hand, request (1) is purposely violated in dynamical reduction theories. Since both theories account for the objectification of macroscopic properties, they must necessarily violate one of the two requests.

It is now very simple to prove that for each $\bar{\alpha}$ belonging to J^+, $|F\,u + d\,\bar{\alpha}\rangle$ *cannot* belong either to V_U or to V_D. In fact, let us suppose that it belongs to V_U (the proof in the case in which it is assumed to belong to V_D is analogous); since the distance between $|F\,u + d\,\bar{\alpha}\rangle$ and $|F\,d\,\bar{\alpha}\rangle$ is:

$$\begin{aligned} \| \, |F\,u + d\,\bar{\alpha}\rangle - |F\,d\,\bar{\alpha}\rangle \, \| &= \| 1/\sqrt{2}\,|F\,u\,\bar{\alpha}\rangle + (1/\sqrt{2} - 1)\,|F\,d\,\bar{\alpha}\rangle \, \| \\ &\leq \frac{1}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}} = 1 \end{aligned} \tag{6.2}$$

we get a contradiction, because $|F\,u + d\,\bar{\alpha}\rangle$ belongs to V_U and $|F\,d\,\bar{\alpha}\rangle$ belongs to V_D, and relation (6.1) must hold between any two vectors of these two sets: this proves that $|F\,u + d\,\bar{\alpha}\rangle$ cannot belong either to V_U or to V_D. Of course, by the same argument, we can also prove that, for all $\bar{\alpha} \in J^+$, the index of the apparatus cannot be in any other macroscopic position different from "U" and "D". We have shown that, for all $\bar{\alpha} \in J^+$ and for all measurements processes in which the apparatus registers an outcome, the vector $|F\,u + d\,\bar{\alpha}\rangle$ *does not allow us to assign any macroscopic definite position to the index of the apparatus, not even one different from "U" or "D"*. Stated differently, the large majority of the initial apparatus states, when they are triggered by the superposition $|u + d\rangle$, end up in a state which does not correspond to any definite position (in our general language of section 3, to any definite situation of the part of the universe we perceive), i.e.,one paralleling our definite perceptions.

We believe that our formulation represents the most general proof of the unavoidability of the macro-objectification problem: to have a consistent picture one must accept that in one way or another the linear nature of the dynamics must be broken. We are fully aware that such a conclusion is an old one which is largely shared by those interested in the foundational aspects of quantum mechanics. But we also think that our derivation of this result is useful and interesting for the absolutely minimal (and physically unavoidable) requests on which it is based, i.e., that one can prepare microscopic systems in precise states which are eigenstates of a quantum observable and that when this is done and the considered observable is measured, one can get reliable information about the eigenvalue of the observable itself, by appropriate amplification procedures leading to perceivably different macroscopic situations of the universe.

References

[1] J. von Neumann: Mathematisce Grundlagen der Quantummechanik, Springer-Verlag (1932). English translsation: Mathematical Foundations of Quantum Mechanics, Princeton University Press (1955).

[2] G.C. Ghirardi, A. Rimini, T. Weber: Phys. Rev. D 34, 470-491 (1986). G.C. Ghirardi, P. Pearle, A. Rimini: Phys. Rev. A 42, 78-89 (1990). G.C. Ghirardi, R. Grassi, P. Pearle: Found. Phys. 20, 1271-1316 (1990). G.C. Ghirardi, R. Grassi, F. Benatti: Found. Phys. 25, 5-38 (1995).

[3] D. Bohm: Phys. Rev. 85, 166-179, 180-193 (1952). D. Dürr, S. Goldstein, N. Zanghí, in: Bohmian mechanics and quantum theory: an appraisal, Boston Studies in the Philosophy of Science 184, 21-44, Kluwer Academic Publishers, Dordrecht (1996). M. Daumer, D. Dürr, S. Gold- stein and N. Zanghí: Erkenntnis 45, 379-397 (1997).

[4] E.P. Wigner, in: The scientist speculates, W. Heinemann, London (1961).

[5] H. Everett III: Rev. Mod. Phys. 29, 454 (1857). J.A. Wheeler Rev. Mod. Phys. 29, 463 (1957).

[6] R.B. Griffiths: Journ. Stat. Phys. 36, 219-272 (1984). R. Omnès: Journ. Stat. Phys. 53, 893-932, 933-955, 956-975. (1988) M. Gell-Mann and J.B. Hartle, in: Proceedings of the 3rd international symposium on the Foundations of quantum mechanics in the light of new technology, S. Kobayashi et al. eds., Physical society of Japan, Tokio (1990).

[7] H. Primas, in: Symposium on the Foundations of Modern Physics 1990, Joensuu, Finland. Discussion Sections edited by K.V. Laurikainen and J. Viiri.

[8] W. Zurek: Physics Today, October 1991, pp. 36-44.

[9] D. Giulini, E. Joos, C. Kiefer, J. Kumpsch, I.-O. Stamatescu, H.D. Zeh: Decoherence and the Appearance of a Classical World in Quantum Theory, Springer-Verlag (1996).

[10] B. d'Espagnat: Conceptual foundations of quantum mechanics, 2nd ed., Addison Wesley Publishing Company, Reading, MA (1989).