What Do These Correlations Know about Reality? Nonlocality and the Absurd

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Abstract

In honor of Daniel Greenberger's 65th birthday, I record for posterity two superb examples of his wit, offer a proof of an important theorem on quantum correlations that even those of us over 60 can understand, and suggest, by trying to make it look silly, that invoking "quantum nonlocality" as an explanation for such correlations may be too cheap a way out of the dilemma they pose.

1   INTRODUCTION

Daniel Greenberger has a fine sense of the absurd. He can be breathtakingly funny at the most unexpected moments, leaving you gasping in admiration. I consider it my duty to record for the pleasure of future generations two examples of his wit in which I am privileged to have served as straight man.

My first example tells us something about Danny. I submitted a paper explaining how the three-particle version of the Greenberger-Horne-Zeilinger argument can be reformulated as an extremely simple version of the more complicated Bell-Kochen-Specker theorem. Dimly remembering Danny long ago making incomprehensible remarks about why it might be interesting to think about a particle decaying into a pair of particles each of which then decayed into a pair of its own, I carelessly began a footnote with "Greenberger invented his example to...." In due course the report of an anonymous referee arrived. Here it is in full:

> A very pretty paper. Unfortunately it is rendered unpublishable by virtue of the mischievous footnote 19, which is an affront to what is left of Western Civilization. When three people agree to work together on a project, it is very unfair to single out one for most of the credit. Regrettably, because physicists are hero-worshippers, they often do this anyway, especially if one of the group is very famous. Thus, everyone usually mentions only Einstein in connection with EPR.

> But to accord such treatment to a bonehead like Greenberger is unforgivable. I recommend the paper be sent back to the author for correction ("The GHZ example was invented to..." or better, '"The lovely GHZ example was invented to...") and upon resubmission, it should be

published without further refereeing or delay.

My second example, I rashly suggest, serves as a metaphor for how not to think about quantum nonlocality. After a conference on foundations of quantum mechanics in Urbino about a dozen years ago, Danny and I spent a couple of days traveling around Tuscany and Umbria. The moment of the creation of the finest spontaneous joke in human history finds us in the Academy in Florence, standing in front of Michaelangelo's David:

> *Mermin*: Look, he's not circumcised.

> *Greenberger* [instantly]: Ah, what do these *goyim* know about sculpture?

I would like to suggest that the same kind of exuberantly setting sail in entirely the wrong direction takes place when one is led to infer physical nonlocality from the strange correlations in EPR or GHZ or Hardy states. The straight response to my remark about David's puzzling anatomy would have been "What do these *goyim* know about Jewish ritual?" That lunatic last-minute switch from Levitical legislation to Michaelangelo's qualifications as an artist is analogous to the breathtaking switch we make from the inability of physics to deal with the individual properties of an individual system, to the miraculous creation of such properties from afar.

This, of course, is just a hunch. I do not establish it here by direct argumentation, but try at least to undermine the notion that nonlocality is a sensible inference from quantum entanglement, by a method which, inspired by Danny, I make bold to call a process of successive exasperations. I give the main *reductio ad absurdum* in Sec. 3. I describe in Sec. 2 some attitudes toward nonlocality, give a preliminary, familiar, run through the successive exasperations, and describe a pertinent theorem (whose elementary proof is in the Appendix) which deserves to be more widely known

2   AGAINST NONLOCALITY: THE OBJECTIVE NATURE OF MIXED STATES

Sir Rudolf Peierls didn't believe Bell's theorem established nonlocality. It only showed that any attempt to complete quantum mechanics with hidden

variables would be necessarily nonlocal.[1] On one of his visits to Ithaca I put it to him that he must therefore take the view that the real lesson of EPR-Bell is not that there is objective nonlocality, but that there is no objective difference between a $50 - 50$ ensemble of horizontally $(90°, H)$ and vertically $(0°, V)$ polarized photons, and a $50 - 50$ ensemble of diagonally $(\pm 45°, D \ and \ \bar{D})$ polarized photons. He said he did.

The view that different realizations of one and the same mixed-state density matrix describe objectively identical ensembles is one of those propositions whose validity can flicker on and off like an optical illusion. On the one hand, there is obviously no objective difference between a $50 - 50$ mixture of H and V polarizations and a $50 - 50$ mixture of D and $\bar{D}$, since the statistics of any measurements you can make on each of the two ensembles are identical.

On the other hand, there obviously is an objective difference, since the photons in the two ensembles have different histories, having been prepared in different ways. In one case Bob can give the ensemble to Alice by sending her a series of photons each of which has emerged from a polarizer randomly chosen each time to transmit either horizontal or vertical polarizations; in the second case his polarizer randomly transmits either of the two diagonal polarizations. So the photons have quite distinct past experiences in the two cases, which makes the two ensembles objectively different. Furthermore Bob can demonstrate the difference to Alice by telling her, in the first case, the results she would get if she were to measure the horizontal-vertical polarization of each photon, or, in the second case, the results she would get for their diagonal polarizations.

But on further reflection, there obviously is not an objective difference since Bob can also do this trick by preparing pairs of photons in the polarization state $1/\sqrt{2}(|H, H\rangle + |V, V\rangle) = 1/\sqrt{2}(|D, D\rangle + |\bar{D}, \bar{D}\rangle)$, and sending one member of each pair to Alice. He can then get the information he needs to inform Alice of the results she will get for either set of experiments by measuring the appropriate polarization of the member of each pair that he keeps for himself, i.e., without doing anything whatever to Alice's photons, whose past experiences are identical in the two cases.

Yet on subsequent reconsideration, if one believes in quantum nonlocality, then, after all, there still *is* an objective difference, for by subjecting his

member of each pair to a polarization measurement, Bob is, in fact, altering the character of Alice's corresponding member in a way that depends on his choice of which polarization to measure. The alteration takes place through the well-known mechanism of spooky action at a distance, which might better be called the SF mechanism, in recognition of Einstein's original term, *spukhafte Fernwirkungen.*[2]

Even so, however, if you look a little more carefully, then, in the final analysis there clearly is not an objective difference, since Alice can measure either the horizontal-vertical or the diagonal polarizations of all her photons, indelibly recording the data in a notebook, long before Bob ever starts to measure any of his corresponding photons. Bob's acts of measurement cannot possibly alter the character of the data in Alice's notebook, nor can they alter the character of her photons, which can be destroyed in the process of her polarization measurements long before Bob attempts to alter them via the SF mechanism.

This last flip-flop strikes me its definitive. While it may be reasonable to contemplate the possibility that Bob's measurement can alter the character of something as wraith-like as the potentialities of Alice's photons for subsequently behaving in one way or another, if the behavior has already taken place and been recorded in ink before Bob takes any action, then nobody (except those who think that it's looking in the box that kills Schrodinger's cat) would maintain that the SF mechanism can affect the notebook entries.

And indeed, if you believe in SF, then the least implausible thing to say by way of refutation is that if Alice makes her measurements long before Bob makes his, then she is at that very time inadvertently and spookily transmitting to him the very information he will later need to deceive her into thinking that he prepared the ensemble, which she herself disposed of some time before, as either a horizontal-vertical or a diagonal ensemble. At this point SF takes on the character of a clever (if not uproariously funny) joke.

I tell in Sec. 3 a somewhat more subtle (still- sorry, Danny-not rib-tickling) version of this joke, which, at least for me, makes SF look even more like science fiction.

This apparent ability remotely to produce distinct ensembles with the same

density matrix W might appear to stem from the degeneracy of W, but in fact the trick (I use the word "trick," because the situation is reminiscent of stage magic, which works by a process of self-deception, skillfully helped along by the magician. When the trick is SF, the magician is Nature herself.) can be done quite generally. Take as a simple example the nondegenerate density matrix

$$W = p\,|H\rangle\,\langle H| + q\,|V\rangle\,\langle V| \tag{1}$$

with $p \neq q$, which, in spite of its nondegeneracy has many alternative representations in terms of *nonorthogonal* polarization states, for example,

$$W = \frac{1}{2}\,|R\rangle\,\langle R| + \frac{1}{2}\,|L\rangle\,\langle L| \tag{2}$$

where $|R\rangle$ and $|L\rangle$ are

$$|R\rangle = \sqrt{p}\,|H\rangle + \sqrt{q}\,|V\rangle \tag{3}$$

and

$$|L\rangle = \sqrt{p}\,|H\rangle - \sqrt{q}\,|V\rangle \tag{4}$$

Representation (1) of W describes an ensemble of photons that are horizontally polarized with probability p and vertically polarized with probability q; representation (2) describes an ensemble of photons that have with equal probability one of two linear polarizations along axes tilted away from $\mathbf{z}$ along $\mathbf{x}$ or $-\mathbf{x}$ through an angle $\theta = \cos^{-1}\left(\sqrt{p}\right)$.

One can produce either one of these ensembles from afar, in the EPR manner, by introducing a second noninteracting two-state system and starting with an ensemble of pairs, each in the pair state

$$|\Psi\rangle = \sqrt{p}\,|H\rangle \otimes |H\rangle + \sqrt{q}\,|V\rangle \otimes |V\rangle \tag{5}$$

This state can equally well be written

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\,|R\rangle \otimes |D\rangle + \frac{1}{\sqrt{2}}\,|L\rangle \otimes |\bar{D}\rangle \tag{6}$$

since $|D\rangle$ and $|\bar{D}\rangle$ are explicitly

$$|D\rangle = \frac{1}{\sqrt{2}}\,|H\rangle + \frac{1}{\sqrt{2}}\,|V\rangle$$

$$\tag{7}$$

$$|\bar{D}\rangle = \frac{1}{\sqrt{2}}\,|H\rangle - \frac{1}{\sqrt{2}}\,|V\rangle$$

If he prepares pairs in the state $|\Psi\rangle$, Bob can persuade Alice that he has given her the ensemble associated with the representation (1) of W by measuring horizontal-vertical polarization on his own photons, while he can persuade her that her ensemble is the one suggested by (2) by measuring diagonal polarization. So if one is predisposed against quantum nonlocality, one is required to acknowledge that there is no objective difference between the two ensembles associated with the representations (1) and (2) of the density matrix W.

Einstein was so predisposed:

> *On one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system $\mathcal{S}_2$ is independent of what is done with the system $\mathcal{S}_1$, which is spatially separated from the former.*[3]

His eminently reasonable supposition has received bad press because of what Einstein did with it. He combined it with his strong intuition about what constituted a real factual situation, to conclude that quantum mechanics offers an incomplete description of physical reality. Now that John Bell has shown us that it is impossible to complete the quantum mechanical description of physical reality along the lines Einstein seemed to call for, we ought to explore the converse approach to saving his supposition: assume that quantum mechanics *does* provide a complete description of physical reality, *insist on Einstein locality*, and see how this constrains what can be considered *physically real*.

This gives a rule of thumb for when an internal property of an isolated physical system can be certified to be *objectively real*: a necessary condition for an individual system to have an objective internal property is that the property cannot change in immediate response to what is done to a second far-away system that may be correlated but does not interact with the first.

With this view the reduced density matrix W of a subsystem of a correlated system can be objectively real, but there can be no objectively real differences among any of the different interpretations (it will have (infinitely) many, unless the system is in a pure state) of W in terms of ensembles of systems in different (not-necessarily orthogonal) pure states, associated with

representations of the form

$$W = \sum_\mu p_\mu \left| \phi_\mu \right\rangle \left\langle \phi_\mu \right| \tag{8}$$

This is the content of a theorem of Gisin, Hughston, Jozsa, and Wootters,[4,5] which makes a powerful case for the fundamental nature of mixed states.

The GHJW theorem establishes that a system (Alice's) can always be correlated with another system (Bob's) in such a way that when Alice and Bob have an ensemble of such identically correlated noninteracting pairs of systems and Alice specifies any particular representation (8) of the density matrix of her system, then Bob can measure an appropriate observable on his, the result of which will enable him to instruct Alice how to identify fractions p_μ of her systems which have internal properties appropriate to the state $\left| \phi_\mu \right\rangle$. Thus Bob can certify as the "actual" ensemble any possible realization of Alice's ensemble in the form of a collection of pure states $\left| \phi_\mu \right\rangle$ with probabilities p_μ. He does this by giving Alice the information necessary to sort out her collection of subsystems into any one of those realizations. In the Appendix, I provide this important fact with an elementary proof, accessible to senior citizens like Danny and me.

If you want to take Einstein locality seriously, then the GHJW theorem requires you to acknowledge that there can be no more objective reality to *any* of the different possible realizations of a density matrix, then there is to the different possible ways of expanding a pure state in terms of different complete orthonormal sets. The density matrix of a system must in general be viewed as a fundamental and irreducible objective property of that system, containing all information about its internal properties, whether or not the state the density matrix describes is mixed or pure.

3   QUANTUM NONLOCALITY: A FABLE

Here is a more striking version of the "good-news-bad news" view of nonlocality given in Sec. 2 (What follows is based on unpublished lecture notes for a talk I gave at Columbia University in March, 1996. A similar story, with a somewhat different moral, is told by Kern and Zeilinger[6]). This time Carol provides Alice and Bob with an ensemble of labeled pairs of spin-$\frac{1}{2}$ particles.

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Alice gets one member of each pair and Bob, the other. Then they each measure the spin of their own member of pair 1 along a common direction $\mathbf{n}_1$, pair 2 along a common direction $\mathbf{n}_2$, etc., where in each case they randomly select $\mathbf{n}_i$ from one of the three orthogonal directions $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$. They find that regardless of their choice of directions, each spin behaves randomly and independently of the the other. Is it possible to discriminate between the following two (One can complicate the tale by adding four more options in which up and down in cases (I) and (II) refer to the x-and y-directions, but I prefer to break the rotational symmetry and keep the story simple) explanations for such behavior?

(I) The ensemble Carol gave them consisted of an equally weighted mixture of spins in the four completely uncorrelated states (up and down are with respect to the z-direction, a and b identify Alice's spin and Bob's)

$$|\uparrow_a\uparrow_b\rangle \ , \ |\uparrow_a\downarrow_b\rangle \ , \ |\downarrow_a\uparrow_b\rangle \ , \ |\downarrow_a\downarrow_b\rangle \tag{9}$$

(II) he ensemble Carol sent was an equally weighted mixture of pairs in the four states consisting of the singlet state

$$|0,0\rangle_{ab} = \frac{1}{\sqrt{2}}(|\uparrow_a\downarrow_b\rangle - |\downarrow_a\uparrow_b\rangle) \tag{10}$$

and three triplet states

$$
\begin{aligned}
|1,1\rangle_{ab} &= |\uparrow_a\uparrow_b\rangle \\
|1,\bar{1}\rangle_{ab} &= |\uparrow_a\downarrow_b\rangle \\
|1,0\rangle_{ab} &= \frac{1}{\sqrt{2}}(|\uparrow_a\downarrow_b\rangle + |\downarrow_a\uparrow_b\rangle)
\end{aligned}
\tag{11}
$$

Two of these four states are highly correlated – indeed, they are maximally entangled.

Note that in either case the global density matrix for the pairs of Alice and Bob is just the four-dimensional unit matrix.

Zygmund and Yvonne have a debate about whether there is any objective difference between Case (I) and Case (II). Are the ensembles of Alice-Bob pairs objectively different in the two cases?

Zygmund: Both ensembles are described by exactly the same two-spin density matrix. This means there is no way Alice and Bob can tell the difference. Therefore there is no difference.

Yvonne: You've overlooked the fact that Carol can help Alice and Bob tell the difference. If it was Case (I), then Carol can tell them what their individual results were in every single one of the cases (about a third of them) in which they happened to have picked the z-direction along which to make their measurements. She could not possibly do this in Case (II). But if the ensemble was Case (II), then Carol knows which of the pairs were singlets, so she can identify for Alice and Bob a subset (about a quarter) of their data for which all their measurements are perfectly anticorrelated, regardless of whether they were along $\mathbf{x}$, $\mathbf{y}$, or $\mathbf{z}$. She could not possibly do this in Case (I). So in each case she can identify a subset of Alice and Bob's ensemble of pairs with properties that uniquely identify that case.

Zygmund: But in either case, the collection of data Alice and Bob accumulate for the runs in which they both happen to pick $\mathbf{z}$ is *completely random*. Carol's trick supposedly demonstrating that Alice and Bob are dealing with a Case (I) ensemble has nothing to do with the objective character of what happens in Alice's and Bob's experiments. It consists merely in her being able to sort out, without actually having seen it, the zz portion of their random data into the four (about equally populated) subsets associated with the four possible outcomes.

Furthermore, in either case about half the data collected by Alice and Bob will be perfectly anti-correlated by *sheer chance*. Carol's trick supposedly demonstrating that Alice and Bob are dealing with a Case (II) ensemble consists only in her being able to identify about half of those perfectly anticorrelated pairs without having access to their actual data. The existence of perfect anticorrelations for *some* subset of Alice and Bob's data, is again an objective feature of that data, present in both cases.

Since the data Carol need to convince Alice and Bob that their ensemble is either Case (I) or Case (II) are already there in either case, all you have demonstrated is that Carol's relation to the Alice-Bob ensemble – the kinds of information she can have about it – can take various

forms. You have not shown that there is any objective difference in the two forms the ensemble itself might have.

Yvonne: You can't just talk abstractly about "Carol's relation" to the Alice-Bob ensemble. The fact is that in preparing the ensemble for Alice and Bob she would actually have had to do entirely different things to the spins, depending on which ensemble she prepared. It's only because Carol didn't provide Alice and Bob with enough information about what she actually did, that Alice and Bob are required to describe their ensemble by a density matrix which is the same in both cases. In Case (I) Carol had to prepare pairs of spins with definite values of their z-components, while in Case II she had to prepare pairs of spins with definite total angular momentum (and, in the triplet case, with the appropriate z-components of total spin.) The pairs of spins Alice and Bob possess had objectively different past histories so it clearly makes sense to characterize them as objectively different.

Zygmund: On the contrary, Carol can prepare Alice and Bob's ensemble in a way that makes it obvious that Case (I) and Case (II) are not objectively different. She can produce each pair she sends to Alice and Bob by taking two singlet pairs, sending a member of one pair to Alice, a member of the other pair to Bob, and keeping the remaining spins herself, labeled so she knows which had its partner sent to Alice and which to Bob, as well as which Alice-Bob pair each of her own pairs is associated with.

If Carol sends Alice and Bob their ensemble of pairs in this way, then each quartet of spins is in the (rotationally invariant) state

$$
\begin{aligned}
|\Psi\rangle &= |0,0\rangle_{ca} \otimes |0,0\rangle_{cb} \\
&= \frac{1}{\sqrt{2}}(|\uparrow_{c1}\downarrow_a\rangle - |\downarrow_{c1}\uparrow_a\rangle) \otimes \frac{1}{\sqrt{2}}(|\uparrow_{c2}\downarrow_b\rangle - |\downarrow_{c2}\uparrow_b\rangle) \qquad (12)
\end{aligned}
$$

Carol can build up the Alice-Bob ensemble by doing this many times. Then she can do one of two things: either (I) she measures the individual z-spins of all the pairs she kept or (II) she measures the total spin and total z-component of each of her pairs of spins. In the first case she acquires the information necessary to persuade Alice and Bob that

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their ensemble is of type (I), since the state $|\Psi\rangle$ has the expansion

$$|\Psi\rangle = \frac{1}{2}(|\uparrow_{c1}\rangle\,|\uparrow_{c2}\rangle \otimes |\downarrow_a\rangle\,|\downarrow_b\rangle + |\downarrow_{c1}\rangle\,|\downarrow_{c2}\rangle \otimes |\uparrow_a\rangle\,|\uparrow_b\rangle$$
$$- |\uparrow_{c1}\rangle\,|\downarrow_{c2}\rangle \otimes |\downarrow_a\rangle\,|\uparrow_b\rangle - |\downarrow_{c1}\rangle\,|\uparrow_{c2}\rangle \otimes |\uparrow_a\rangle\,|\downarrow_b\rangle) \qquad (13)$$

But in the second case she acquires the information needed to persuade Alice and Bob that their ensemble is of type (II), since the state $|\Psi\rangle$ also has the expansion

$$|\Psi\rangle = \frac{1}{2}(|1,1\rangle_{cc} \otimes |1,\bar{1}\rangle_{ab} + |1,\bar{1}\rangle_{cc} \otimes |1,1\rangle_{ab}$$
$$- |1,0\rangle_{cc} \otimes |1,0\rangle_{ab} + |0,0\rangle_{ab} \otimes |0,0\rangle_{ab}) \qquad (14)$$

Since Carol can decide which properties to measure long after Alice and Bob are in full possession of all their pairs, the two ensembles cannot be objectively different.

Yvonne: A lovely trick on Caro's part, to be sure, but you have entirely overlooked the phenomenon of quantum nonlocality. Carol's measurements actually do change the objective character of Alice's and Bob's pairs by the SF-process: *spukhafte Fernwirkungen*.

Zygmund: Even if I grant you the existence of such nonlocal action, Carol can easily evade your claim that what she does spookily alters the character of Alice's and Bob's pairs. Her action need be taken only after every pair in the Alice-Bob ensemble has been measured and the data entered in Alice's and Bob's notebooks. Clearly if Carol does nothing until Alice and Bob have already collected and recorded their data, nothing she does can alter the character of the Alice-Bob ensemble from which those data were extracted.

The only difference between the two cases is that in Case (I) Carol has collected the information she needs to tell Alice and Bob the results in the third of their data coming from their z-spin measurements, thereby sacrificing the possibility of getting the information required to identify to them a quarter of their data that contains perfect anticorrelations. In Case (II), on the other hand, she has collected the information enabling her to identify an anticorrelated quarter, while sacrificing the possibility of identifying for them the results of their individual z-spin

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measurements. To maintain that the two ensembles are objectively different you have to maintain that Carol's measurement alters the objective character of the Alice-Bob pairs in a manner that is not only nonlocal in space but backward in time.

Yvonne: There is no need for anything to go backward in time. If Carol does not make her measurement until long after Alice and Bob have collected their data then the nonlocal SF action has gone forward in time. But it has gone not from Carol to Alice's and Bob's ensemble, but from Alice and Bob to the ensemble of pairs kept by Carol.

As Alice and Bob measure the individual spins of each pair in their ensemble, SF converts the corresponding pair in Carol's possession into a pair in a product of eigenstates strictly correlated through (13) with the results they found for that pair. [Because $|\Psi\rangle$ is rotationally invariant it has the same structure (13) whether the spin quantization axis is taken to be $\mathbf{z}$, $\mathbf{x}$, or $\mathbf{y}$.] For Carol later to do her Case (I) trick it is necessary only for her to measure the z-components of the spins of each of her pairs. The results will agree with what Alice and Bob found in that third of their runs in which they measured along $\mathbf{z}$. On the other hand, for her to do her Case (II) trick she need only identify about a quarter of her pairs whose spin eigenvalues are opposite, which will enable her to identify a quarter of Alice's and Bob's data that is perfectly anticorrelated.

Zygmund: But how can she do the Case (II) trick when she doesn't know the direction along which her spins are polarized, since she doesn't know whether Alice or Bob measured the spins of any particular pair along $\mathbf{x}$, $\mathbf{y}$, or $\mathbf{z}$? To ask them would surely arouse their suspicions!

Yvonne: No problem. Every one of Carol's pairs (about half of them) that are polarized the same way along their unknown direction (as a spooky consequence of Alice's and Bob's earlier measurements) are in a state orthogonal to the singlet state (whatever that unknown direction may be). So, if Carol measures the total spin of her pairs and rejects those - about three-fourths of them - that have total spin one, she will be able to identify about half of the Alice-Bob pairs for which they measured opposite spins. This provides her with just the 25%

of perfectly anti-correlated pairs she needs to fool Alice and Bob into thinking that she had given them an ensemble of type (II).

Zygmund: Clever of Carol, to be sure. But all this shows is that even in your own terms, the *spukhafte Ferwirkungen* has nothing to do with the objective difference between ensemble I and ensemble II. It has to do only with how Alice and Bob inadvertently telegraph to Carol the information that enables her to fool them into thinking that she prepared an ensemble of one or the other type. The fact is that the objective character of Alice and Bob's ensemble is not altered from afar by Carol. Nor do Alice and Bob spookily send Carol information about their ensemble. The fact that the same actions by Alice, Bob, and Carol can be interpreted either as Carol altering the character of Alice and Bob's ensemble from afar or as Alice and Bob inadvertently transmitting to Carol the information necessary to enable her ingeniously and deceptively to persuade them of a particular objective character of their ensemble, illuminates the fundamental absurdity of either of these two quite different interpretations.

All that is going on here is that because Carol's pairs of spins are strongly correlated with the pairs in Alice and Bob's ensemble, Carol has mutually exclusive options of extracting from her spins various kinds of information about Alice and Bob's ensemble. She has the option of identifying a quarter of their pairs for which they found opposite spins. But she also has the option of identifying the specific results they found in all their zz runs.

Yvonne: [slinking off but muttering under her breath]: *Sie wirken doch fern* ... and what about Bell's theorem ...?

4   INCONCLUSIONS(Inconclusions are to conclusions as inconclusive is to conclusive): FASHION AT A DISTANCE?

What, indeed, about Bell's theorem? I am not prepared to spell out here how Alice's trick in Section 2 or Carol's in Section 3can be reconciled with the absence of spooky action at a distance and the completeness of the quantum

mechanical description of physical reality. I believe that the reconciliation is to be found in a view of physical reality as comprised only of correlations between different parts of the physical world, and not at all of unconditional properties possessed by those parts. In that case the straight answer to the question posed in my title, like the straight answer to Danny's inspired but lunatic question in Florence, is "everything." And the straight question - to which the straight answer is indeed "nothing" - is, "What do these correlations know about individual properties."(I have expanded on such ideas under the noncommittal rubric of "the Ithaca interpretation of quantum mechanics".[7,8]

Underlying such a view will be a stronger sense than we currently possess, of the absurdity of trying to reconcile our perceived particularity of what actually happens under actual conditions, with an imagined particularity of what might have happened under conditions that might have held, but did not. My guess is that Bohr's not entirely transparent views on such matters[10] might be elucidated[10] by rephrasing them in a broader context where the correlated systems are not, as they invariably are for Bohr, limited to a microscopic specimen and a classical apparatus. Such a broader view might reveal that the problem of "nonlocality" and "the measurement problem" both stem from the same weakness in current preconceptions about what constitutes physical reality. I hope to have more to say about this (or to be disabused of my current sense of partial illumination) by Dannys 75th.

All I have tried to do here (aside from offering up a proof of the GHJW theorem that is accessible to those of us over 60 who don't understand explanations cast in terms of POVM's) is to suggest that people may have become too facile in their readiness to blame everything on (or credit everything to) "quantum nonlocality". Nonlocality seems to me to offer "too cheap" a way out of some deep conundra (to appropriate Einstein's remark to Born about Bohm theory). If you push hard on it you can force "nonlocality" into offering some explanations that strike me as just plain silly.

When Abner Shimony first came up with the term "passion at a distance" to characterize the spooky actions at a distance required by the notion of quantum nonlocality, I thought that by replacing "action" with "passion" he was emphasizing that the action was on the interpretive instincts of people (who can experience passion), and not among the correlated subsystems

themselves (which cannot). But this "passion" has since been taken by just about everybody (including, I believe, Abner himself) to signify a weaker, spookier kind of action of the physical systems on each other.

I would like to suggest that people have been a little too quick in talking themselves into this widely held position. To salvage what I had thought was a very good suggestion of Abner, and as an act of homage to Danny Greenberger, without the tutorial example of whose many (superior) *bon mots* I could never have had the inspiration, I would like to suggest that the time has come to consider the possibility that quantum nonlocality is nothing more than fashion at a distance(the acronym has not escaped my attention).

APPENDIX

Let Alice have a system with a density matrix W having a variety of different expressions in terms of projection operators on finite or countably infinite collections of (not necessarily orthogonal) pure states. Each such expression describes Alice's system as if it were in a pure state $|\phi_\mu\rangle$ with probability p_μ:

$$W = \sum_\mu p_\mu |\phi_\mu\rangle \langle\phi_\mu| \tag{15}$$

The GHJW theorem establishes that for any such collection of alternative interpretations of W, it is possible to provide Bob with a system of his own for which the joint Alice-Bob system has a pure state $|\Psi\rangle$ with the following properties:

(i) W is the reduced density matrix for Alices subsystem.

(ii) associated with *any* interpretation (15) of Alices density matrix W, there is a corresponding observable B of Bobs subsystem whose measurement in the state $|\Psi\rangle$ gives a result b_μ with probability p_μ. If Bob measures B and gets the result b_μ, he is able correctly to inform Alice that the result of any measurement she makes on her own system will be as if her system were in the state $|\phi_\mu\rangle$ (In less guarded and clumsy but decidedly more spooky language, if Bob measures B and gets the result b_μ, then Alices system collapses into the state $|\phi_\mu\rangle$).

The proof that follows is Chris Fuchs's simplification[11] of John Preskill's simplification[12] of my simplification[7] of the proof of HJW.[5]

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The crucial thing to notice is that any two states $|\Psi\rangle$ and $|\Psi'\rangle$ of a joint Alice-Bob system leading to the same reduced density matrix for Alice,

$$W = Tr_{Bob} |\Psi\rangle \langle\Psi| \tag{16}$$

can be related by a unitary transformation

$$V = 1 \otimes U \tag{17}$$

that acts nontrivially only on Bob's subspace. This follows from expanding the density matrix W of Alices subsystem in terms of a complete orthonormal set of eigenvectors (Some of the w_i can be zero or the same, so the complete set is not uniquely determined by W. Any one will do.):

$$W = \sum_i w_i |\alpha_i\rangle \langle\alpha_i| \tag{18}$$

Because the $|\alpha_i\rangle$ are complete in Alice's subsystem, an arbitrary state $|\Psi\rangle$ of the Alice-Bob system has an expansion of the form

$$|\Psi\rangle = \sum_i |\alpha_i\rangle \otimes |\beta_i\rangle \tag{19}$$

(where the $|\beta_i\rangle$ are in general neither normalized nor orthogonal). The general reduced density matrix (16) associated with the state (19) can thus be put in the form

$$W = \sum_{ij} \langle\beta_j \,|\, \beta_i\rangle \, |\alpha_i\rangle \langle\alpha_i| \tag{20}$$

But if W is of the special form (18), then since the $|\alpha_i\rangle$ are an orthonormal set, the $|\beta_i\rangle$ appearing in (19) are constrained to satisfy

$$\langle\beta_j \,|\, \beta_i\rangle = w_i \delta_{ij} \tag{21}$$

Therefore the $|\beta_i\rangle$ can be nonzero only if $w_i \neq 0$, and when $w_i \neq 0$ they are of the form

$$|\beta_i\rangle = \sqrt{w_i} \, |\gamma_i\rangle \tag{22}$$

where the $|\gamma_i\rangle$ for $w_i \neq 0$ are orthonormal and can be extended (arbitrarily) to a complete orthonormal set. Hence any $|\Psi\rangle$ yielding the reduced density matrix (16) must be of the form

$$|\Psi\rangle = \sum_i \sqrt{w_i} \, |\alpha_i\rangle \otimes |\gamma_i\rangle \tag{23}$$

Since the $|\gamma_i\rangle$ are a subset of a complete orthonormal set, all such forms (23) do indeed differ only by a unitary transformation of the form (17).

Now consider that representation (15) of Alice's density matrix W containing the largest (possibly infinite) number of distinct states $|\phi_\mu\rangle$. Let Bob's subsystem have a dimension at least as large as that number, so that it possesses an orthonormal set of states $|\psi_\mu\rangle$ that can be put in one-to-one correspondence with Alices states $|\phi_\mu\rangle$. If the composite Alice-Bob system is in the state

$$|\Psi\rangle = \sum_\mu \sqrt{p_\mu}\, |\phi_\mu\rangle \otimes |\psi_\mu\rangle \tag{24}$$

then Bob can acquire the information necessary to identify which of the states $|\phi_\mu\rangle$ characterizes Alices subsystem, by measuring a nondegenerate observable B of the form

$$B = \sum_\mu b_\mu\, |\psi_\mu\rangle \langle\psi_\mu| \tag{25}$$

Consider now any other expansion of Alice's density matrix,

$$W = \sum_\mu p'_\mu\, |\phi'_\mu\rangle \langle\phi'_\mu| \tag{26}$$

Bob can do the same trick using a different state

$$|\Psi'\rangle = \sum_\mu \sqrt{p'_\mu}\, |\phi'_\mu\rangle \otimes |\psi_\mu\rangle \tag{27}$$

for the Alice-Bob system. But since $|\Psi\rangle$ and $|\Psi'\rangle$ yield the same reduced density matrix W, they must be related by

$$|\Psi\rangle = 1 \otimes U\, |\Psi'\rangle \tag{28}$$

Applying $1 \otimes U$ to (27) we learn that the original state (24) of the Alice-Bob system has the alternative expansion ?

$$|\Psi\rangle = \sum_\mu \sqrt{p'_\mu}\, |\phi'_\mu\rangle \otimes |\psi'_\mu\rangle \tag{29}$$

where

$$\left|\psi'_\mu\right\rangle = |\psi_\mu\rangle \tag{30}$$

Therefore with the composite system in the original state $|\Psi\rangle$, Bob need only measure an observable of the form

$$B = \sum_{\mu} b_{\mu} \left|\psi'_{\mu}\right\rangle \left\langle \psi'_{\mu}\right| \tag{31}$$

to acquire the information necessary to persuade Alice that her system is in the state states $\left|\psi'_{\mu}\right\rangle$ with probability p'_{μ}.

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